Teacher's Guide Level 5

Metric Edition

SRA
MATHEMATICS
LEARNING SYSTEM TEXT

Teacher's Guide
Level [illegible]

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Thank You

Science Research Associates, Inc., and the authors want to say THANK YOU to a group of people who helped tremendously in the development of this system. The whole thing started way back in 1969.

Juanita Tolson, experienced public school and college educator in Washington, D.C., and J. Fred Weaver, author and professor at the University of Wisconsin, were asked to review SRA's existing basal mathematics program, tell us bluntly what was wrong with it, and recommend what should be in the next basal series. They did. We listened.

Jeremy Kilpatrick, Columbia University Teachers College, was asked to share his knowledge of problem solving in elementary mathematics and tell us what could be done to improve problem-solving materials. This was a hard job. He tried. We tried.

Classroom teachers Norma Jean Cheek, Joy Craig, Fran Engelbrecht, and Jane Hawley tried out new ideas before manuscript was prepared for field testing.

John C. Egsgard, well-known Canadian educator, helped us think through the best way to develop the geometry strand so that applications would be natural and valid. He brought to our attention the outstanding work done by the Ontario Institute of Curriculum Studies and unselfishly shared his own thoughts and efforts.

George T. Duncan, University of California at Davis, was asked to do a huge job: to construct the probability and statistics strand for the entire program. His approach was refreshingly simple and understandable. It's not every day that you can find a specialist in this area who can have empathy for the young child and his thinking process and yet handle the subject with expertise.

Jerome D. Kaplan guided the work of gifted educator-writers from Educational Analysis and Evaluation, Inc., who prepared some of the chapters.

Irving Morrisett, of the Social Science Educational Consortium, was asked to react to an approach involving economics. He said don't. We didn't.

Teachers from the Montreal Catholic School Commission and William Bober, supervisor of mathematics for the Edmonton Catholic School Board, tackled the huge job of reviewing all of the final, revised manuscript used in the verification study. Their efforts gave us still more information to consider as the pages received their finishing touches.

The Mary Beck School, Elkhart, Indiana, put the verification-study materials into their nongraded, personalized education structure. They gave us insight into the various management techniques that could be used with these materials. Charles Walker assisted us from an administrator's point of view. Leo Anglin, consultant in the developmental tryout, transferred his efforts and enthusiasm to the verification study.

Ralph W. Tyler, trustee and director emeritus of the Center for Advanced Study in the Behavioral Sciences, has given generously of his time and ideas to the people directly responsible for developing this program. His knowledge of today's problems and his dream of tomorrow's education encouraged us every step of the way.

We couldn't possibly find the right words to thank all the people in the developmental field-study schools who worked so hard and responded so honestly throughout 1971 and 1972. It's hard to tell who was the most blunt, the 75 teachers or the 2700 kids. They told us what was wrong—and what was right.

The information from nearly 700 teachers and 20 000 students in schools throughout the United States and Canada who participated in the 1972/73 verification study gave confidence that the revision of the first year's testing was effective.

Other teachers dug into their bag of tricks and contributed tried and tested classroom activities. Many thanks to James K. Bidwell, Philip Cox, Thomas S. Davis, Frances Greenberg, Muriel Greig, Bettye Hall, Donald Kamp, Evelyn Kozar, Kay Nebel, David O'Neil, Louise Petermann, Madolyn Reed, William Swart, and Judy Tate.

Literally hundreds of people in SRA have shared in the development of the SRA MATHEMATICS LEARNING SYSTEM. They all cared enough to do their very best. The editorial staff did even more. Special thanks to those exceptionally talented and dedicated people.

Preface

How was the SRA Mathematics Learning System *developed?* Writing was almost the last step. We started by listening.

We visited schools of all kinds, from the inner city to remote rural areas. We sat in classrooms. Teachers, children, and parents told us about what they liked and didn't like about math programs.

We asked an independent research organization to interview supervisors and administrators. We talked to SRA Staff Associates about the needs they saw.

We reviewed all major basal math series. We studied standardized tests to see what children might be expected to know at various ages. Consultants evaluated existing SRA programs.

We also analyzed recommendations and reports from curriculum study groups, state and city adoption committees, and researchers in a variety of fields.

Then our authors, editors, and consultants worked together to prepare the rationale and learning objectives for the SRA Mathematics Learning System. Writing of the program did not begin until the entire scope and sequence had been defined by the learning objectives.

The manuscript was continuously reviewed, discussed, and revised by our development team. But it's arrogant for adults to sit in an office and predict what will work in the classroom. We needed answers to three questions:

- Will pupils attain the objectives?
- Will pupils develop positive attitudes toward the program?
- Will teachers find the program easy to use?

Our next step was to undertake two years of prepublication tryouts. We carefully selected classrooms across the United States and Canada to represent the broad range of pupil abilities, family backgrounds, and teaching styles. We visited, surveyed, listened, and tested. We rewrote and revised and retested before going to press with the program you see today.

We're confident that you'll find the SRA Mathematics Learning System effective, enjoyable, and easy to use. One of our tryout pupils wrote:

I liked the program and nothing was hard or dificalt. I think the book should come out to the world.

What makes the SRA Mathematics Learning System *a "system"?* The word *system* has many definitions. We call the SRA Mathematics Learning System a system because of the following five characteristics:

1. The entire program is based upon well-defined learning objectives.
2. Although the program can be enriched in many ways, the texts are complete in themselves. The teacher is not required to use any other materials.
3. There is a comprehensive evaluation program in each text.
4. Learning alternatives are provided for teachers and pupils who wish to use them.
5. The program provides information about the learners that will guide the teacher in altering or expanding a learning sequence.

What kind of objectives are there? Objectives are given for each chapter. Key to the program, however, are the year-end mastery objectives. These are the goals toward which instruction is directed.

Distinguished educator Ralph Tyler helped with the difficult task of defining learning objectives. He told us: "Remember the purpose of objectives. They are to guide, not dictate. Think of them as goals to be reached as a result of the teaching-learning process.

"Keep the number of objectives for any level under thirty, if possible. The teacher should be able to remember them all. A teacher who has to search through hundreds of objectives to figure out what to do cannot be free to teach anything more than bits and pieces.

"Avoid fashionable formulas for writing objectives. Fashions change. Avoid jargon. Keep the language simple. Objectives have to say something or their value is lost."

I liked the whole book because it told me things that I didn't even know about and it was very interesting

What are the texts like? The SRA Mathematics Learning System focuses on the real world and develops many concepts from real-world situations.

Compared with other programs, this program spends a longer time on an idea and its related skills before introducing another topic. This gives skill competency a better chance to develop.

I like the S.R.A. math book because it's not just plane old 5 x 3 it you can gives you more time to think.

There are many invitations for pupils to *think* as well as to do. Not all questions are meant to be answered. Some questions have many answers; some have none.

I liked this section. They had fun math sentences And they maid you think.
Math is getting a lot eazer and its funner to; math realy is fun

The language is informal. Purists may object to this departure from standard textbook English, but our tryout pupils responded enthusiastically to the style. Instead of stressing technical vocabulary and symbols, the program emphasizes skills and nonverbal understanding.

No pupil should have to moan, "Aw! It's the same old stuff," when he flips through his math book. The pages of the SRA MATHEMATICS LEARNING SYSTEM are varied. They're lively. They look like fun.

How are the texts organized? There are three major types of chapters.

Exploratory In an exploratory chapter pupils play with a big idea, think about it, and share their own ideas. The necessary vocabulary is introduced, along with some of the notation and operations related to the major idea.

Instructional In this type of chapter pupils begin the serious business of acquiring skills. Ideas are carefully sequenced into learning steps, and each learning step is accompanied by practice.

Review Here pupils must demonstrate understanding and skills. There is a review of the learning sequence, along with opportunities to explore applications.

No one text contains all three types of chapters for a single concept strand. For example, there are exploratory and instructional chapters on addition of whole numbers in level 1. The instructional chapters continue through level 4, and the review chapters start at level 5.

How are pupils evaluated? The evaluation program is built into each text. It allows a learner to check his own progress and determine his own strengths and weaknesses.

The first pages of an instructional chapter contain an informal survey to find out what the learner knows about the chapter to come. These pages indicate what the chapter is about, and they help to define the learning goal of the chapter.

The tests within an instructional chapter are called Progress Checks. A Progress Check identifies the knowledge that is a prerequisite for further work.

After learning from your math book I found this test very easy I found it easy because I learned from your book this is what made it easy.

A Progress Check can serve two functions:

1. If a pupil has gone through the preceding instructional pages, a Progress Check tells whether or not he has acquired the appropriate knowledge and skills. If he hasn't, he should try other kinds of instruction.
2. If a pupil seems to have prior knowledge of the chapter, as indicated by the survey at the start of the chapter, he may go directly to the Progress Check. It will determine if the preceding instructional pages can be skipped. If he's not successful on the test, he simply goes back to pages he skipped.

At the end of every chapter a Checkout lets the pupil and teacher know whether the learning objectives of the chapter have been reached.

What learning alternatives are there? There's a limit to the number of pages in a textbook. Beyond a certain length the book becomes too expensive and hard to use. So only a few pages can be devoted to extras for pupils who need more help or who would benefit from additional activities. To make sure these important extras are available, we've supplied a wealth of reinforcement and extension activities in the Teacher's Guides.

The SRA MATHEMATICS LEARNING SYSTEM *was built on some simple convictions.* Mathematics is relevant and vital. It's useful and interesting. Everyone should relax and enjoy it.

I like math a lot more than I did last year.

Some of the greatest learning opportunities are found in everyday things and are discovered when people talk together.

I liked the chapter becouse I think we learned the most and becouse we had a lot of discussions.

Mathematics doesn't have to be formal and abstract to be good mathematics.

I think this math is very good because I know what I'm doing

Everyone – students and teachers – should succeed in mathematics.

I liked it very much I never learned this kind of meagures but I think it is good that you wrote it in the book because soon America will learn ~~the kin~~ a new kind of measure and you already are teaching it to us.

A textbook can't transmit the joy of learning as well as an enthusiastic teacher, but it can help.

Notes & Things

We have asked a lot of questions. We have listened to the answers. One set of questions had to do with the organization of a teacher's guide.

Teachers told us that they wanted the guide pages numbered the same as the pupil pages. That was not an easy request to handle, since there are teacher resource pages at the end of each chapter and some pages of introduction to the next chapter. But the request made sense, so we put the alphabet to work.

The pupil page number repeats at the end of a chapter like the tune of a broken record; but attached to the number is a letter, which changes to accommodate those extra guide pages. When we finally get to the first pupil page of the next chapter, the guide page number will once again match the pupil page number. This whole thing sounds terrible. But flip through the book. Keep an open mind. It won't be so bad once you get used to it.

As we listened to teachers we found that there were many possible classroom organizations. They varied from large, heterogenous grade-level groups to small, homogeneous groups composed of pupils of different ages. It was impossible to tailor a guide to fit all kinds of organizations. But it was possible to let you know the special features of each page so that you can quickly decide how you want to use that page.

Each pupil page appears, slightly reduced in size, on the guide page. And the answers are right in place. You will find comments beside each pupil page. They will be in categories like these:

lesson The pages indicated here are related but need not all be presented on the same day. They provide the continuity of experience necessary to get an idea established.

goal The words needed to turn this goal into a behavioral or performance objective have not been printed, although they could have been. The goal simply helps you pinpoint the learning task for the page.

memo These words will tell you if a discussion is needed to get the learners started in the right direction. Or they may warn you about a potential problem or provide an explanation of why something is done the way it is. Or they may be simply a suggestion to make your work easier.

things All materials that you'll need for the page activities are listed here.

warm-up You can guess where this label came from. If the mathematics idea is a new one or maybe a hard one, a suggestion is made that will help get the learners ready to learn.

page 1 The comments about the page itself may be introduced with traffic-light colors and will mean about the same things.

Caution—everyone needs most of this information, but the use of the page will vary according to the ability of individual pupils.

Stop—think about which pupils should do this activity. Your independent learners will have no trouble, but your pupils at the other end of the performance scale may benefit much more by doing other, more appropriate activities.

As you flip through the pages you will see a few sentences in *italic type*. These sentences are intended to give you some ideas about the words you might use to talk about the page. (How many times have we all thought, "I understand, but I don't know what words to use to explain it"?) In talking, probably nobody would use the exact words that are given. They are not meant to be a script. They are offered as a guide, nothing more.

You will find the new concept-development words in SMALL CAPITAL LETTERS, and some other functional words that deserve emphasis will be in **this kind** of type.

Now flip through the book again. Notice the copy below the pupil pages. There are 3 types of activities. Each type has its special symbol—one of the international traffic signals.

The old familiar red *stop* sign will signal an activity for the youngster who simply needs more work before he goes on to a new learning goal.

The yellow *divided highway* sign signals an activity for those youngsters who can accept a challenge beyond the learning expectations of the page.

The new, blue *rest area ahead* sign signals an activity you can use with any group. This type of activity departs from the standard math work and gives everyone a chance to have a break and hopefully some fun.

Here you will find the ideas that will extend the lessons and suggestions that offer specific help for those children who need it. Each activity was carefully selected so that you could have something special to personalize each child's learning.

You'll find a Resource Section at the end of each chapter. This provides alternate forms of all progress checks and the checkout and still more activities. Not every Resource Section is the same, but you will find references to additional learning aids that may be in your school and are just right to use. At the end of some chapters you will find a description of these learning aids. At the end of the guide you'll find a bibliography that lists children's books and film medium references. These features were planned to help you with your job of teaching and the children's job of learning.

A glossary of mathematical terms used in the text appears at the end of each pupil book. It is also reproduced at the end of each of the Teacher's Guides. Why not take time to look at it now, so you can get an overview of how the SRA Mathematics Learning System uses math words?

Canadian SI Edition of the SRA Mathematics Learning System

The Canadian SI pupil texts of the SRA Mathematics Learning System have been revised to be fully in accord with the specifications of the Canadian Metric Commission, the Canadian Government Specifications Board, and with teaching requirements of Canadian schools. In revising the Teacher's Guides, we have been governed by considerations of economy. In order to keep costs to schools at a reasonable level, the revision of the Teacher's Guides has been limited to what is necessary for teachers.

Where significant changes have been made on pages of the pupil's book, the Teacher's Guide has been revised accordingly. Where only minor changes have been made in the pupil's book, the Teacher's Guide has not been changed. You will find occasional references in the Teacher's Guide which are not applicable to schools in Canada. While the pupil's texts are fully metric, in the Teacher's Guides there are refences to non-metric units of measure or to working in both systems. In the pupil's book, all numbers of five or more digits are separated by spaces between millions, thousands, and so on, instead of the traditional commas; this change has not been made on pages of the Teacher's Guide that needed no other revision. Similarly, in the pupil's book there is always a zero before the decimal point where there is no other number; again, this change has not been made on otherwise unchanged pages of the Teacher's Guide. In the text, questions have been revised to avoid reference to non-metric units, but when the answers have remained numerically the same, the Teacher's Guide has not always been altered. Pages that have been revised to assist the teacher and avoid ambiguity will be easily recognized; these pages are printed in black and red.

Special Features of

Mathematics Learning System — Canadian SI Edition

- A *complete* instructional system which includes clear objectives, built-in evaluation, and alternate instructional and assessment procedures
- *Extensive field testing* before final publication to ensure a program that works
- A *real-world approach* that involves students in practical, everyday mathematics
- *Three types of chapters* to develop each content strand:
  - (i) *Exploratory* chapters in which major concepts are introduced
  - (ii) *Instructional* chapters in which specific learning and skill building takes place
  - (iii) *Review* chapters in which the student's mastery of skills is asssessed and challenged
- *Complete SI* (International System of Measures) metrication, Levels 1 through 6
- Well-defined *mastery objectives* for each level and specific objectives at the beginning of each chapter
- *Flexible learning sequence* by which a teacher can meet the needs of individual students by selecting sequences of chapters that differ from the order in the text
- *Evaluation within the text* by means of
  - (i) *Survey questions* leading into each chapter
  - (ii) *Progress checks* - periodic trouble shooting within each chapter
  - (iii) *Checkouts* - measurement of student achievement at the end of each chapter
  - (iv) *Alternate checkouts* for each chapter in the Teacher's Guide
- *Practice Sheets* for each level that contain additional materials for skill building and evaluation
- *Teacher's Guides* giving comments and suggestions for every page of the text. Resources at the end of each chapter provide
  - (i) alternate evaluation for Progress Checks
  - (ii) activities
  - (iii) additional learning aids, including specific card references to many related SRA educational materials, as well as to those of other suppliers
  - (iv) in some chapters, games and and exercises, with permission to reproduce the pages for class use
- *Level placement tests* by which pupil readiness for a given level can be determined, or by which year-end mastery can be assessed
- *Chapter placement tests* by which enrichment or remedial needs can be determined

the curriculum

In order to maintain a balance of concept development and drill, each SRA Mathematics Learning System chapter is built around one big idea.

The three kinds of chapters discussed in the preface – exploratory, instructional, and review – serve to organize major mathematical ideas into continuous strands. The big-idea chapters in strand organization give you more control over the learning sequence. In cooperative planning with other teachers, you can safely change the order of chapters across several levels in order to emphasize a particular content strand at a given level.

The following chart shows the chapter organization for levels 3 through 7. Think of this chart as a road map. You will use only the part you need to get you where you want to go. The strands of content are listed down the side. The levels are listed across the top. The numeral in each rectangular shape tells the chapter number and the location of the rectangle itself signals what part of the strand's content is featured in each chapter.

If you want to emphasize the whole numbers for example, the chart will show you the sequence of the strand. If you wish to use a measurement chapter in some order other than that in the book, it's O.K. The chart indicates that certain chapters can be used much earlier than that designated by the printed sequence. Most important, the chart lets you see the flow of content from one level to another.

whole numbers	concept operations + − × ÷
fractions	concept operations + − × ÷ decimal notation operations + − × ÷
measurement	length mass capacity time money
geometry	3-d, 2-d shapes 1-dimensional measurement
statistics	collecting/recording interpreting information
probability	
rational and real numbers	concept operation
graphs and functions	concept application

In planning your year's work with the SRA Mathematics Learning System, be sure to consult your local or provincial curriculum guidelines.


level 3
level 4
level 5
level 6
level 7

OBJECTIVES

This program was built upon learning objectives. Each objective clearly states what pupil behavior is to be observed, the conditions under which the pupil is to perform, and the criteria for acceptable performance to be demonstrated at the end of this level.

whole-number notation

1. Given any one of the roman numerals from I to M, the learner can write the equivalent standard numeral.
2. Given any number with 9 or less digits, the learner can tell the value of each digit.
3. Given a problem in estimation, the learner can round the numbers appropriately.

whole-number operations

1. Given any 3-digit number and any 2-digit number, the learner can estimate and find the product.
2. Given any 1-digit number and any multiple of 10 or 100, the learner can name the product without using pencil and paper.
3. Given any 4-digit number and any 2-digit number, the learner can estimate and find the quotient and the remainder (if any).
4. Given a completed division computation, the learner can use multiplication to check the accuracy of the quotient.

sentences

1. Given a one-step word problem with extraneous information and any one of the arithmetic operations, the learner can solve it.
2. Given a one-step word problem, the learner can write a math sentence to summarize the problem and verify the computation.

fractional-number notation

1. Given any two frequently used fractions, the learner can find equivalent fractions that have a common denominator.
2. Given a frequently used fraction, the learner can determine if it is in simplest form; if it is not, the learner can write it in simplest form.
3. Given an appropriate fraction, the learner can express it as a whole or mixed number; given a whole or mixed number, the learner can express it as a fraction.

fractional-number operations

1. Given any two fractions with denominators of 2, 3, 4, 5, 6, 8, 9, 10, or 12, the learner can find their sum or their difference.
2. Given any two mixed numbers with denominators of 2, 3, 4, 5, 6, 8, 9, 10, or 12, the learner can find their sum or their difference.
3. Given two decimal fractions less than 100 expressed in tenths or hundredths, the learner can find their sum or their difference.
4. Given two frequently used fractions, the learner can find their product.

geometry concepts

1. Given a set of polygons, the learner can sort them according to the number of sides.
2. Given a plane figure, the learner can determine its line(s) of symmetry by folding.

measurement

1. Given an object to be measured, the learner can select an appropriate unit to measure the object's length, mass, or capacity.
2. Given common units of length, mass, or capacity, the learner can convert measurements to related units by shifting the decimal point.

measurement of events

1. Given a chart or a table used in everyday situations, the learner can find information.
2. Given a tally chart, table, or bar graph showing the frequency distribution of data, the learner can find the mean.

1 NUMERATION AND NUMBERS

before this chapter the learner has—

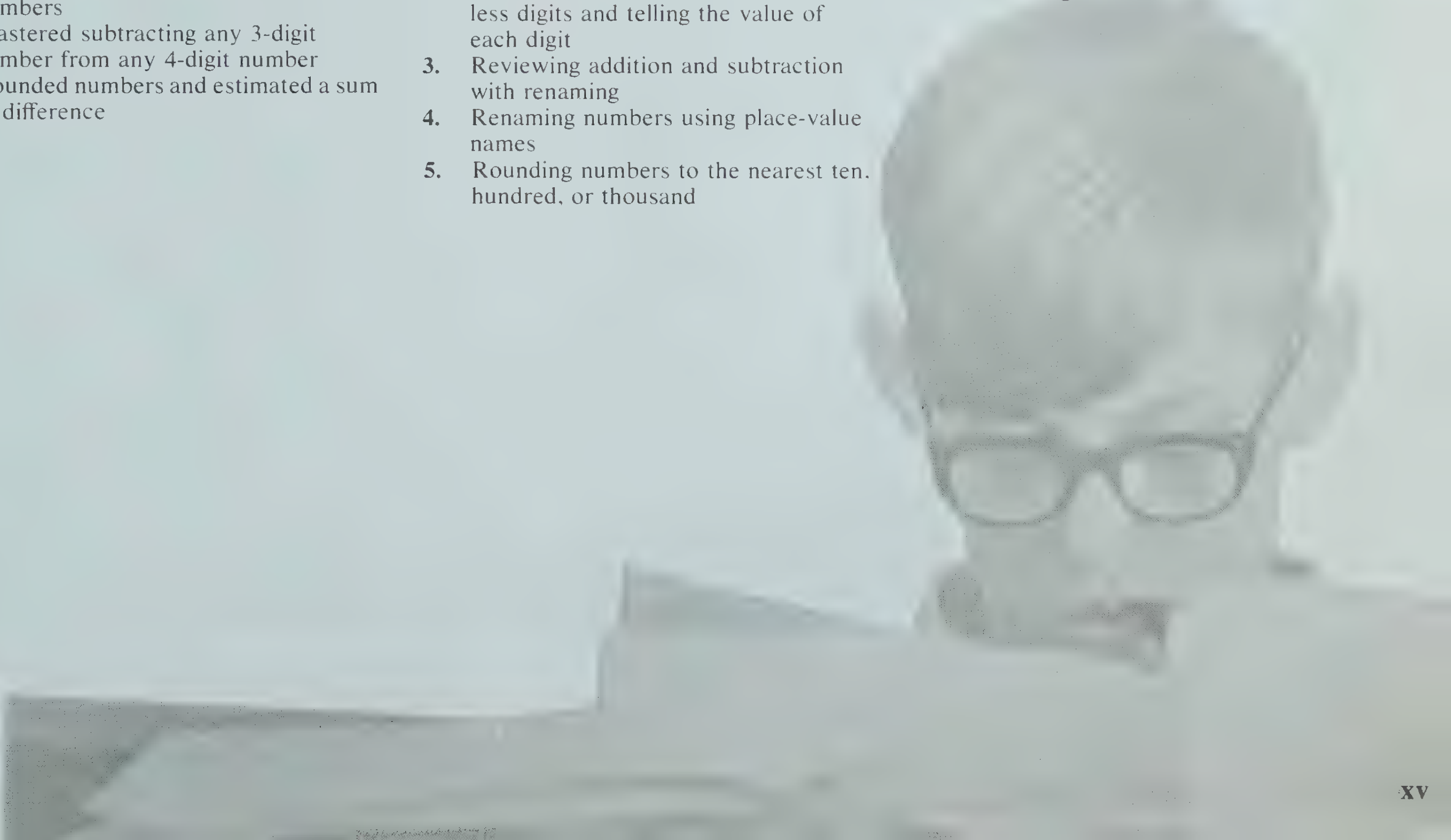
1. Read aloud or written any of the numerals 0, 1, 2, ..., 999,999
2. Told the value of each digit of a number with 6 digits or less
3. Mastered adding any two 4-digit numbers
4. Mastered subtracting any 3-digit number from any 4-digit number
5. Rounded numbers and estimated a sum or difference

in chapter 1 the learner is—

1. Examining the roman system of numeration and writing the equivalent standard numeral for any roman numeral from I to M
2. Reading aloud any number having 9 or less digits and telling the value of each digit
3. Reviewing addition and subtraction with renaming
4. Renaming numbers using place-value names
5. Rounding numbers to the nearest ten, hundred, or thousand

in later chapters the learner will—

1. Master telling the value of each digit of any number having 9 or less digits
2. Master rounding numbers and estimating answers



Notes & Things

This first chapter kicks off a new level of work with a review. The study of place value starts with a search for patterns among roman numerals. Getting the youngsters in the mood of inquiry should allow the patterns in our own number system to be viewed in a fresh way. The Romans could write a lot of numbers with their few symbols and so can we! The 10 digits are arranged to write numbers as large as a hundred million. Renaming numbers in a numeration study provides the perfect excuse to review subtraction when the notion of renaming is applied to computation situations.

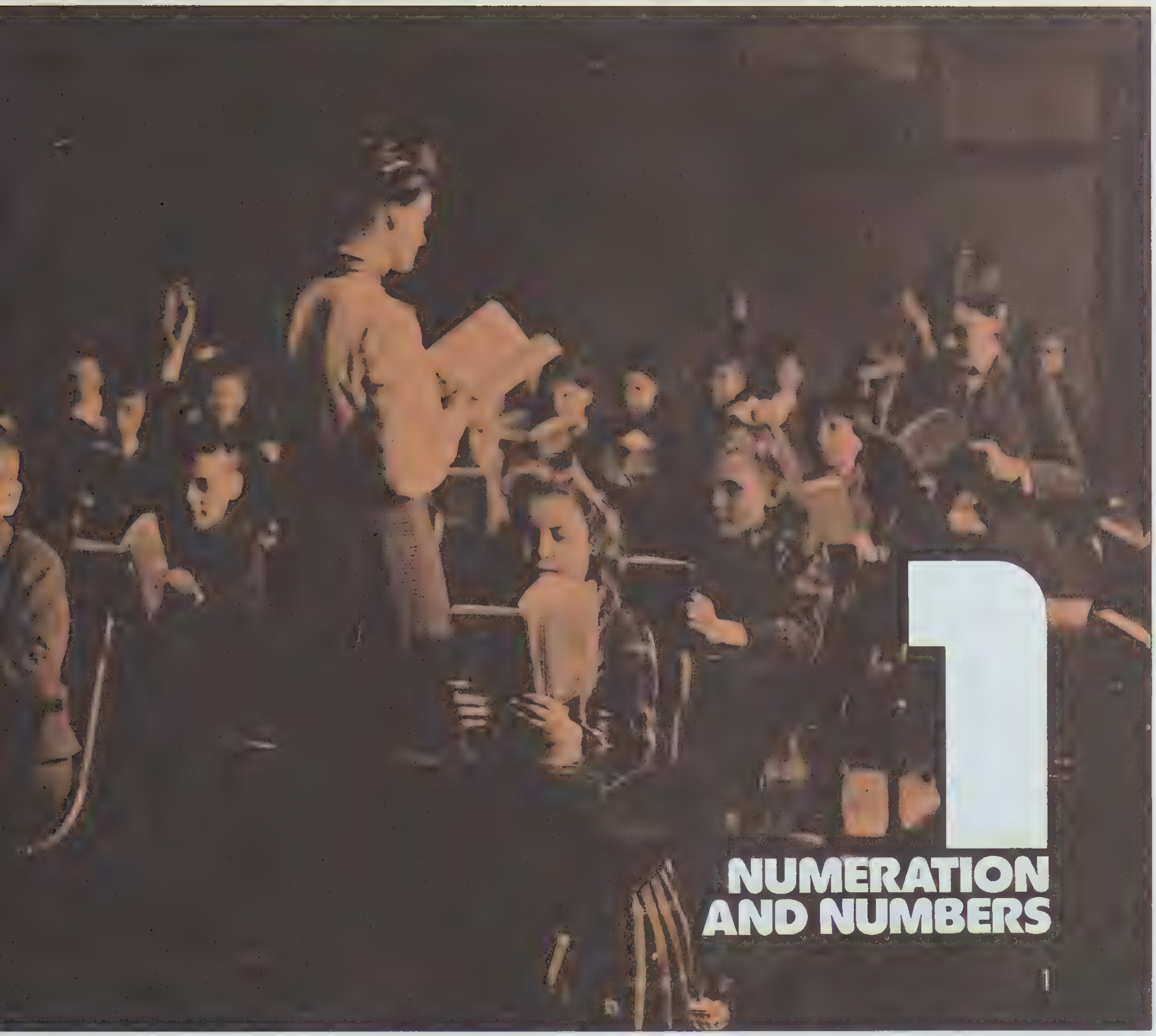
You will have a chance to comprehensively inventory each pupil's subtraction skills in one of the Progress Checks. An addition review is introduced when addition is used to check subtraction. The whole thing sounds backward, doesn't it? The youngsters have been working with the operations of addition and subtraction for years. A change in the learning sequence shouldn't be troublesome at all. In fact, it may be a welcome relief. You will be able to do a complete inventory of each pupil's addition skills with still another Progress Check.

The chapter comes to a close with work in rounding and estimation. Estimation skills are viewed by many math educators as some of the most important skills to be learned. We agree. Estimation is a skill that can be used every day—both in and out of school. You will see a lot of it in this program.

things

spirit master of place-value chart (see page 72c)

For the extra activities you will want to have these things available:
- circle compass or spirit master (see page 72e)
- 10 same-size boxes (milk cartons)
- wood cubes



goal Think about and explore ideas through a picture clue

page 1 Each chapter opens with a full-page photograph. Such a page contains much more than the identification of a chapter number and a chapter title. Each photograph will provide an opportunity to discuss ideas related to the theme of the chapter and thereby provide a direction for study.

Surely you must have some youngsters who enjoy stories of the early days. Both television and movies have created a romantic appeal for the 1800s. Find out how much your pupils know about schools of those days. The classroom pictured certainly is different from their own. What do they think is different? Is anything the same? What subjects do they think were studied about 100 years ago? Has anyone seen any really old schoolbooks? Does the library have one or two that are available to look at? Check with senior citizens. Have the youngsters create an oral picture of what attending the school pictured would be like. Were there electric lights? furnaces to supply heat in the winter? school buses? movies about social studies? as many people as there are in your school? as many teachers? ballpoint pens?

This discussion may motivate some pupils to do independent research and report back to the group in a week or so. You can anticipate what the last question should be. *Do you think they had anything in math such as we have?* As the page is turned, they will find out for themselves.

lesson Page 2

goal **Survey** – knowledge of place value and numeration

page 2 This is a fun page to discuss together. Examine the reproduced table of contents. With which topics are the youngsters familiar?

Questions 2 and 3 will signal those pupils who may already know the content of this chapter. Focus on problem 3 of the page taken from the old arithmetic book. The pupil should provide at least three names for each number to prove that he knows the essentials of numeration. For example, 368 can be named as

3 hundreds	6 tens	8 ones
	36 tens	8 ones
		368 ones
	35 tens	18 ones
2 hundreds	16 tens	8 ones
2 hundreds	15 tens	18 ones

Summarize the page by looking for similarities between math books of 1883 and of today. Do the pages of the old book look very different from those of today's books?

8 *WHITE'S NEW COMPLETE ARITHMETIC.*

An **Abstract Number** is not applied to a particular thing or quantity; as, 4, 6, 20.

A concrete number is composed of concrete units; and an abstract number, of abstract units.

ART. 6. A **Problem** is a question proposed for solution.

ART. 7. An **Example** is a problem used to illustrate a process or a principle.

ART. 8. A **Rule** is a general description of a process.

ART. 9. In the **Written Solution** of a problem, the results of the successive steps are written.

In the **Mental Solution** of a problem, the results of the successive steps are not written.

NUMERATION AND NOTATION.

ORAL EXERCISES.

We use ones. It means the same.

1. How many tens and how many units in 37? 65? 84? 90? 75? 18? 60? 80? Example: 3 tens, 7 units
2. Read 29; 47; 85; 70; 77; 90.
3. How many hundreds, tens, and units in 368? 427? 549? 608? 680? 600? 806? 860? 800?
4. Read 452; 506; 560; 600; 784; 690; 900; 909.
5. How many ten-thousands and how many thousands in 48500? 83250? 50400? 60070? 82405?
6. Read 37500; 84250; 70840; 92080; 90900.
7. How many hundred-thousands, ten-thousands, and thousands in 456048? 707803? 680435? 700450? 650048? 805347? 170480?

CONTENTS.

(v)

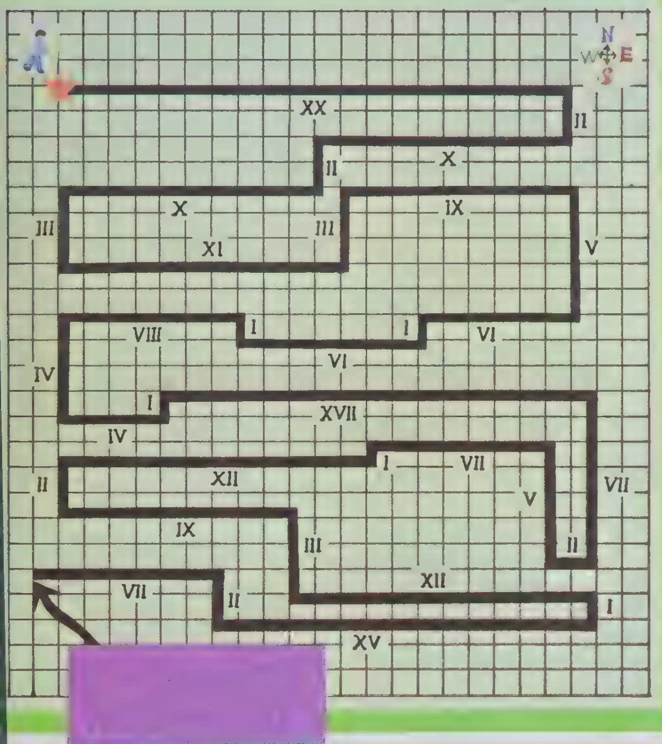
Your goal is to know at least as much about numbers as people your age in 1883. Maybe a lot more!

Here is a copy of two pages from a math book used in 1883. It was written for people your age.

1. About how old is this book? 90 years
2. This page is marked *v*. What does *v* stand for? 5
3. How would you have done back in those days? Answer the problems on the page.

Don't expect everyone to be able to answer thes questions. Just have a good time exploring answ

2



A radio station in Roma had a treasure hunt. The treasure map told the direction to go. There was also a code to tell how far. Sam decided to join the hunt. He walked all over the city. He started at the star and walked XX blocks east. Can you figure out how many blocks is XX? Then he walked 20 II blocks south. How many blocks is II? Then he walked 2 X blocks west. How many blocks is X? 10

How far did Sam walk to get to the directions? Decode the 209 directions. Follow Sam to the prize.

How far did poor Sam walk in all? Where did he pick up his 229 blocks prize? Right back where he started.

3

lesson Pages 3, 4, 5, 6

goal Introduction to roman numerals for 1 through 20 as a code

memo This lesson provides an experience in decoding as well as an introduction to roman numerals. Roman numerals should be thought of simply as a system people invented to record numbers. Do not look for a direct application to our system of place value; there is very little carry-over from one system to the other. But because roman numerals do appear in our society, the child should be acquainted with the basic symbols.

page 3 Review the direction of north, east, south, and west on a map. It will be a challenge to the pupils to answer all the decoding questions. Please do not introduce the words **roman numerals** until page 5. Treat these symbols simply as a code.

You may have to give hints to those pupils who can't get started. Show them that the count of the first 20 blocks east corresponds to the symbol XX. The 2 blocks south correspond to the symbol II. Don't give them the remaining parts of the code yet. Everyone should be able to count the spaces along the path to find the total blocks walked. After what you consider sufficient time, tell the remaining searchers where the prize was picked up.

Have everyone organize the symbols in order from least to greatest number. Ask if they can find a pattern.

goal Exploration of the additive and subtractive features of writing roman numerals

page 4 You decide how best to use this page. Encourage pupils to use the map on page 3 for help. Have them list the code names in a column and write the standard numerals alongside. Continue the list through problem 4 on page 5. This list should help them discover the pattern.

1. What does IV stand for? 4
   IV
   V 5
   VI
   VII
   $5 - 1 = 4$
   What is the pattern here? The I changes places.
   $5 + 1 = 6$
   $5 + 2 = 7$

2. What does IX stand for? 9
   IX
   X 10
   XI
   $10 - 1 = 9$
   What is the pattern here?
   $10 + 1 = 11$

   What do you think XII stands for? $10 + 2 = 12$

3. What does XIX stand for? 19
   XIX
   XX 20
   XXI
   XXII
   $10 + (10 - 1) = 19$
   What is the pattern here?
   $10 + 10 + 1 = 21$
   $10 + 10 + 1 + 1 = 22$

4. What does X stand for? XX? XXX? 10; 20; 30

5. The code name for 50 is L. The code name for 40 is XL.
   XLI
   XLII
   XLIII
   XLIV
   XLV
   $(50 - 10) + 1 = 41$
   $(50 - 10) + 1 + 1 = 42$
   What is the pattern here?
   $(50 - 10) + 1 + 1 + 1 = 43$
   $(50 - 10) + (5 - 1) = 44$
   $(50 - 10) + 5 = 45$

6. What does LX stand for? LXX? LXXX? 60; 70; 80

7. The code name for 100 is C. What is the code name for 90? for 95? 99? 101? 200? XC; XCV; XCIX; CI; CC

8. The code name for 500 is D. The code name for 400 is CD.

Have you found the master code yet? There is one.

The value of a roman numeral is the sum of the values of letters. When a symbol for a lesser number is written to the left of a symbol for a greater number, the lesser number is subtracted from the greater number.

4

1. Look for the pattern here.

CD	500 − 100 = 400
CDX	(500 − 100) + 10 = 410
CDXX	(500 − 100) + 10 + 10 = 420
CDXXX	(500 − 100) + 10 + 10 + 10 = 430
CDXL	(500 − 100) + (50 − 10) = 440

2. What is the code name for—

a 450? CDL **b** 499? CDXCIX **c** 500? D **d** 501? DI **e** 600? DC **f** 700? DCC **g** 800? DCCC

3. The code name for 900 is CM. What is the code name for 950? 999? CML; CMXCIX

4. What is the code name for 1000? M

5. The code that we've been using is called the *roman numeral* system. Thousands of years ago the Romans used it for their arithmetic.

Copy and complete the following roman numeral chart.

standard numeral →	1	2	3	4	5
roman numeral →	I	II	? III	? IV	V

6	7	8	9	10
VI	? VII	? VIII	IX	? X

11	12	13	14	15
XI	? XII	? XIII	? XIV	? XV

49	50	51	90	99	100	101	400	450	460
XLIX	? L	LI	? XC	? XCIX	C	CI	CD	CDL	? CDLX

499	500	501	900	999	1000	1001
? CDXCIX	D	DI	? CM	? CMXCIX	M	? MI

ID ← makes sense → IM
But unfortunately they are not written that way.

5

goal Examining the roman system for writing numerals through MI (1001)

memo Do not try for mastery in writing roman numerals. This skill is not important enough to be given a great deal of time.

page 5 Study should be continued in the same spirit as page 4: explore—decode—examine the patterns. Problem 5 signals a good place for sharing ideas.

Group those pupils who still do not fully understand the roman system. Make a number line on the board, showing only roman numerals. Have the standard numerals assigned to the line. Stress that the line shows two different symbols that name the same point on the number line.

Look for the patterns found in roman numerals. Ask if someone would like to explain the code. Try to get across the understanding summarized in the following rules:

- If a symbol for a lesser or an equal number appears to the right of a symbol for a greater or an equal number, the numbers are to be added. For example, X < L, so LX represents L + X, or 60.
- If a symbol for a lesser number appears to the left of a symbol for a greater number, the lesser number is to be subtracted from the greater number. For example, I < V, so IV represents V − I, or 4.
- A symbol appears no more than three times in succession.

See activity 1, page 22b.

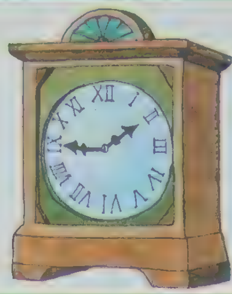
Students who have shown they understand the roman system may be sent to the library to research a report. The report should include such information as when these numerals were used, what kind of society was probably in existence at that time, and how people probably used numbers.

goal Examining real-world uses of roman numerals

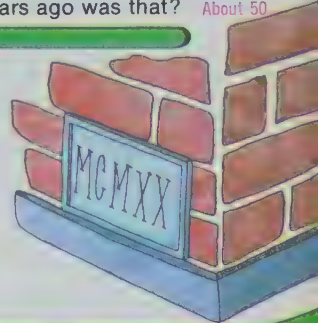
page 6 Has anyone seen roman numerals anywhere else? The great seal on the United States one-dollar bill, for example, has a roman numeral.

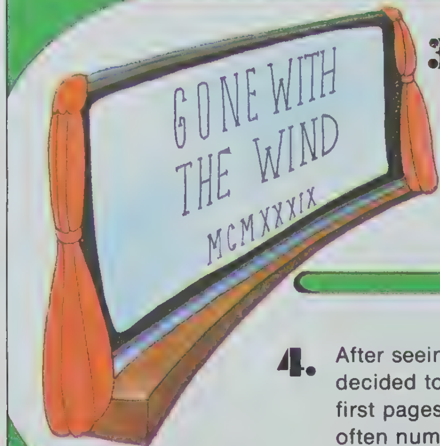
Expand problem 4. Examine some of the books from the library. Do any have lowercase roman numerals in the introductory material? Are roman numerals used as chapter numbers? What about the appendixes at the back of the book?

1. Rosa went to a movie. On the way she passed the jewelry shop. She noticed that some of the clocks had roman numerals on their faces. What time is it when the big hand points to II and the little hand points to IX? 9:10



2. The building the theater was in had a cornerstone. On it was the roman numeral MCMXX. This numeral tells when the building was built. How many years ago was that? About 50





3. The movie Rosa saw was *Gone with the Wind.* At the beginning of the film, at the bottom of the credits, she saw the roman numeral MCMXXXIX. This tells when the film was made. When was *Gone with the Wind* made? 1939

4. After seeing the movie, Rosa decided to read the book. The first pages in a book are often numbered with roman numerals written with little letters. What page comes just before page *xix*? xviii

5. Write each standard numeral as a roman numeral.

a 7 VII **b** 15 XV

c 29 XXIX **d** 105 CV

e 1982 MCMLXXXII

Our number system can be thought of as a code too. A visitor from another planet might try to figure it out the way you tried to figure out the Romans' system.

They would look at this pattern.

0	10	20	30	40	100
1	11	21	31	41	101
2	12	22			102
3	13	23			
4	14	24			
5	15	25			
6	16	26			
7	17	27			
8	18	28			
9	19	29			

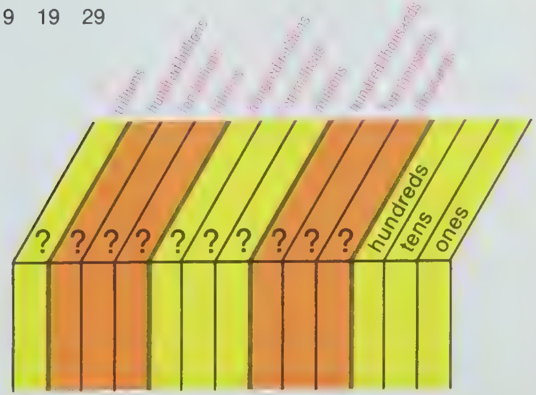
They would get to 100 and think, "Aha! a new key to the code."

What would be the next numeral that would help them break our code? 1000

If our visitor were very smart, he might start to organize his information like this.

Copy the chart. Complete as many of the headings as you can.

The chart can be called a place-value chart.



7

lesson Pages 7, 8, 9, 10, 11, 12, 13

goal Examining our system of numeration

memo You may want to prepare a spirit master for a place-value chart—it will come in handy. Or pupils can turn a sheet of lined paper sideways to get the needed column reminder. Either way, they should have something like the following.

page 7 This is another good place to think together. Heaven only knows what those first place-value charts will look like. Accept even the most humble effort for now. If the pupil compares his chart with the ones on page 9, accuracy and completeness can be self-checked.

goal Examining writing and using large numbers

page 8 There are many words and many ideas here. You'd better have a buzz session for this page. Most youngsters will need a place-value chart for these really big numbers or they will never be able to name them correctly.

1. Your chart can hold a very large number. How large?
Write the largest numeral that will fit, using only the digit 9.
13 digits—trillions; 9,999,999,999,999

2. Pretend that is the number of dollars you have in the bank.
Can you read that number? Nine trillion, nine hundred ninety-nine billion, nine hundred ninety-nine million, nine hundred ninety-nine thousand, nine hundred ninety-nine

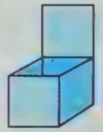
3. Is that number the largest number in the world? No!

4. Could the chart be made bigger? Where would you add columns? Yes, to the left of the trillions column (You can add columns to the right also to sh
decimal fractions. The chart would be larger, but the numbers shown would be smaller.)

Big numbers are a part of our lives. We see
them when we read about the— size of the national debt
distances traveled in outer space
amount of energy generated in a power plant
population explosion

And you're full of big numbers too. When you cut yourself, there is proof that you have blood inside you. How much? An adult has about 5000 millilitres in his body. (That's not many quarts, but it's a lot of millilitres.)

Now think about a tiny box—very tiny. Think about a box that is the size of a cubic centimetre. Now think about a smaller box. So small that 1000 of them would fit into the cubic centimetre box. Fill one of these 1000 boxes with a speck of blood.



Do you know there are about **5,400,000** red cells in that box?

An adult has about 35,000,000,000,000 red cells in all.

Each of us has a lot of a lot of things.

mbers like 35,000,000,000,000 are so large they ally don't mean anything to us. It's hard to imagine tually counting that number of things. We can get me idea of "how many" if we study our number de. A place-value chart helps.

Here is part of a place-value chart. It has places for thousands.

thousands: hundred	ten	one	ones: hundreds	tens	ones
		6	4	2	3
9	2	1	0	6	0
6	5	4	0	0	0
7	0	9	6	0	0

What is the value of 4 in 6423? hundreds
What is the value of 9 in 921,060? hundred-thousands
What is the value of 5 in 654,000? ten-thousands
What is the value of 9 in 709,600? thousands

millions: ten	one	thousands: hundred	ten	one	ones: hundreds	tens	ones
	5	6	4	2	0	0	0
0	9	4	5	6	2	7	3
2	6	1	4	4	0	0	0

How is this chart different? Millions are included here

What is the value of 5? millions
What is the value of 7? tens
What is the value of 2? ten-millions

millions: one	thousands: hundred	ten	one	ones: hundreds	tens	ones
1	0	0	0	0	0	0
2	2	4	5	0	0	0
	6	0	1	0	0	1
6	8	0	0	0	2	5
		1	0	1	0	0
1	3	0	0	0	0	4
					7	2
1	1	0	0	1	0	0

1. Draw a place-value chart with nine columns. Put each digit of the following numerals in the correct column.

   a 1,000,000 **b** 792,245,000 **c** 601,001

2. Put the following numerals into the same chart.

   a Six million, eight hundred thousand, twenty-five
   b Ten thousand, one hundred
   c Twenty-one million, three hundred thousand, four
   d Seventy-two
   e One million, one hundred thousand, one hundred

9

goal Examining place value

things spirit master of place-value chart (see page 72c)

page 9 Understanding place value is the key to correctly computing problems that require renaming. Don't rush over this page.

The youngsters will need place-value charts. Or have them make their own charts by turning lined paper sideways.

Contest: Who can find the largest number?

Contest Rules:

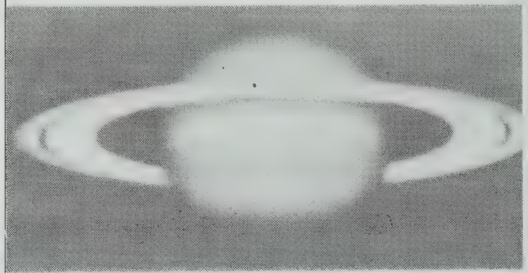
- There must be proof of what that largest number is if it is to count in the contest; therefore, there must be some reference source included. A newspaper clipping, a magazine article, or perhaps the name of a book – whether it is a specific, specialized reference or an encyclopedia – must be submitted along with the number.
- The pupil must be able to read the number aloud before the number is eligible for the contest.

goal Reading large numbers with the help of spaces

warm-up Write 23 651 on the board. Have a volunteer read this numeral. Listen for a pause where the space appears—and listen for the word **and** where it doesn't belong. Make sure that the pupils understand that we do not read the space as **and**. Provide additional practice in reading aloud numerals up through the hundred-millions place.

page 10 To start you will want to work with the pupils here. Have them read aloud the distances of the various planets from the sun. Everyone will agree that spaces make large numerals easier to read. Explain that we don't usually use a space in a 4-digit numeral, because it just isn't necessary. It's not wrong, however, if a space is used.

Pupils will likely have met the symbol **km** before. Make sure that they are familiar with it and have a general idea of the length of a kilometre.



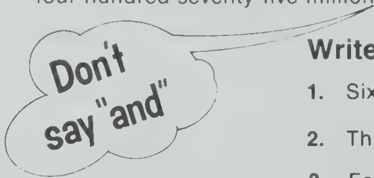
Mercury is about **58 000 000** km from the sun,
Venus is about **107 000 000** km from the sun,
and Jupiter is about **768 000 000** km from the sun.

Notice the spaces in the numerals.
What place-value groups do the spaces separate?
Millions from thousands, thousands from ones
Spaces make numerals for large numbers easier to read.
Spaces separate millions from hundred-thousands,
and thousands from hundreds.
Spaces are seldom used in a 4-digit numeral.

How many spaces would be needed for each of the following numerals?

820069423 2 **6124971** 2 **97431275** 2

When we place numerals in a place-value chart, we must have a digit in every column. But when we read numerals aloud, we don't always say every place-value label. For example, we read 475 607 000 as "four hundred seventy-five million six hundred seven thousand."



Write standard numerals for the following:

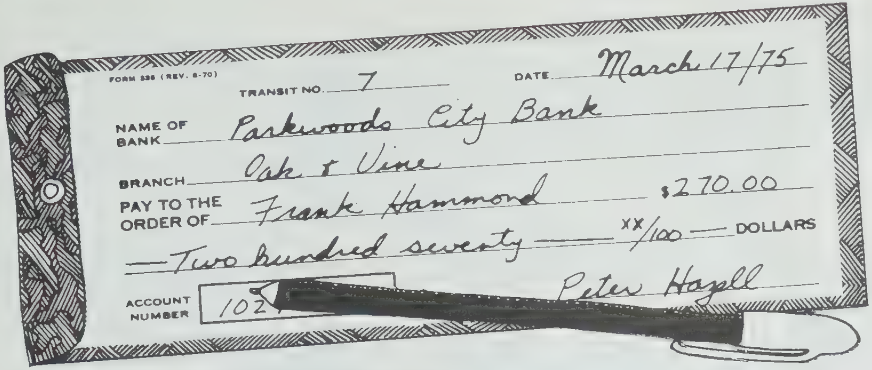
1. Sixty-two million, four hundred seventy thousand, seven hundred. 62 470
2. Three hundred ninety million, one hundred thirty-one thousand, fifteen. 39
3. Four million, eight hundred sixty thousand, five hundred two. 4 860 502

Be able to read the following numerals.
4. six hundred twenty million, nine hundred seventy-five
5. sixty-six million, three hundred thousand, sever
6. two million, thirty thousand, five hundred

4. 620 975 000 **5.** 66 300 700 **6.** 2 030 500

About the only time you need to write the words for numerals is when you write a cheque to pay for something.
Then it may be very important that you know how. Why?
Written words are less easily misread; they are more difficult to forge or alter.

11



his is a standard cheque form. Notice the amount of
he cheque has been written in words and with a numeral.
Vhy do you think this is? The $\frac{xx}{100}$ means no cents.
ut a cheque for $1.25 would be written out as One and $\frac{25}{100}$ dollars.

To prevent errors or someone illegally making a change in the amount

uppose you had inherited a million dollars. You put the money
n the bank. Now suppose you wanted to buy some of the things
sted below. Draw cheque forms and write out some cheques.
o not spend more than $1 million. Answers will vary. Have choices compared and answers checked by a peer.

Food for everyone in town from the Foodery Co. $119 000.00	Island from Gilligan Water Products $36 595 000.00	New clothes from Emperor's Emporium $29.67
A new home from Jiff Construction Co. $499 427.00	Five golden rings from Partridge Pear Co. $463.56	Ten shoes for the right foot from the Prince Shoe Co. $65.58
Stereo and records from the Dip-Disc Record Co. $3095.00	Accountant from Lax Tax & Company $10 270.00	TV knobs from Base TV Service $4.75

goal Practice in writing cheques

memo You may wish to duplicate blank cheques. You are free to reproduce the sample included in the Resource Section. (See page 72d.)

page 11 It is interesting to note that one of the few times we need to spell the numeral words is in writing a cheque. This doesn't happen often, but when it does we must know how to do it correctly. Point out the problems that our language causes. **Four** versus **forty** is troublesome spelling, and knowing when to use a hyphen is a real monster. You may not want to take time out for a language lesson, but the rules for putting hyphens in number names are not nearly as neat or as easy to understand as putting spaces in numerals.

Let the kids role-play. Remember that the directions say not to spend more than $1 million.

Ask the pupils to bring in samples of the cheque forms their parents use. They can then compare them and discuss the features that are common to all, and the reasons for the differences they find.

goal Examining place-value names for a number

memo It takes too much space and time for anyone to write out the words for place-value chart headings. This page shows a symbol substitute for each place-value word.

- **o** Lowercase letter **o** stands for ones.
- **t** Lowercase letter **t** stands for tens.
- **h** This stands for hundreds.
- **T** Capital **T** stands for thousands.
- **tT** This combination stands for ten-thousands.
- **hT** This combination stands for hundred-thousands.
- **m** This stands for millions. And so on, according to the same pattern.

Make sure that the youngsters understand what these letters at the top of the place-value chart stand for.

page 12 Depending on responses to problems 1 and 2, you may want to practice with several more examples—both oral and written.

Lined paper turned sideways will work for a simple place-value chart. The simplified headings will make things easier for everyone.

Numbers have many names. The focus on this page is on a number's place-value name.

What does 2345 mean? One thing it means is 2000 + 300 + 40 + 5. It could be written this way:

$$\begin{array}{r} 2000 \\ 300 \\ 40 \\ +\quad 5 \\ \hline 2345 \end{array}$$

and we are back where we started.

1. Look at another number and one name for it.

What does this stand for?

92 875

- **a** How many ten-thousands? 9
- **b** How many thousands? 2
- **c** How many hundreds? 8
- **d** How many tens? 7
- **e** How many ones? 5

tT	T	h	t	o
9				
	2			
		8		
			7	
				5

2. One more time.

a 8000—How many thousands? 8

Get another name.

b 8000—How many hundreds? 80

Get still another name.

c 8000—How many tens? 800

And another.

d 8000—How many ones? 8000

T	h	t	o
8			
8	0		
8	0	0	
8	0	0	0

3. Make a | T | h | t | o | chart of your own. Answer these questions.

- **a** How many thousands in 1000? 1
- **b** How many hundreds in 1000? 10
- **c** How many tens in 1000? 100
- **d** How many ones in 1000? 1000

	T	h	t	o
a	1			
b	1	0		
c	1	0	0	
d	1	0	0	0

Zeros the c

Now let's really get rolling.
The place-value names will be spelled out this time.

	thousands	hundreds	tens	ones
How many names are there for— 3254 →	3	2	5	4
Is this a correct name? Yes	2	12	5	4
Is this a correct name? Yes	3	1	15	4
How about this one? Yes	2	12	4	14

If you wonder, take time to add them.
Add the last one, for example.

$$\begin{array}{r} 2 \\ 12 \\ 4 \\ +\ 14 \\ \hline ????\ \\ 3254 \end{array}$$

GREAT! You did get the sum of 3254, didn't you?

PROGRESS CHECK

Skill: Reading and telling value of each digit

It's your turn. Make a chart with the word names for each place value. Find at least two other place-value names for each of these.

Accept any good answers. Examples:

1. 5279 2. 2621 3. 4836 4. 7065

	thousands	hundreds	tens	ones
1.	5	2	7	9
	4	12	7	9
	4	11	17	9
2.	2	6	2	1
	2	5	11	11
	1	15	11	11
3.	4	8	3	6
	3	18	3	6
	3	17	13	6
4.	7	0	6	5
	6	10	6	5
	6	9	15	15

You use this sort of renaming a lot when you subtract.
Here's a really hard one to use as an example.

*5. $\begin{array}{r} {\scriptstyle 1\,9\,9\,10} \\ \cancel{2000} \\ -\ 789 \\ \hline 1211 \end{array}$

There is no doubt about it—2000 must be renamed. Can you get the job done? Try it.

13

goal Renaming numbers with different place-value names; **Progress Check**—renaming numbers

things spirit master of a place-value chart

page 13 The top of the page is a bit wild. Take time to discuss the examples together. Use additional examples if you feel it's necessary.

Once directions are understood, the Progress Check is independent work. If you made a spirit master of a place-value chart earlier, you'll want to distribute the charts again for the Progress Check.

STOP See activity 2, page 22b.

See activity 3, page 22b.

lesson Pages 14, 15

goal Review of subtraction requiring renaming

page 14 Getting a child to know when and where renaming is necessary is a big job. But it is important. Extending problem 3 will get you started on appropriate discussion. Have each pupil check his thinking as he does problem 4. Additional examples with zeros may be needed before the renaming skill can be pinned down.

If anyone is in trouble, write some examples with errors on the board and challenge the pupil to find the errors. Finding someone else's mistakes is much easier and more fun than finding one's own.

---

1. Rename 342 for subtraction.

a $\begin{array}{r} 342 \\ -\ 134 \\ \hline \end{array}$ What name should be used for 342?
3 ? hundreds 3 ? tens 12 ? ones

b $\begin{array}{r} 342 \\ -\ 161 \\ \hline \end{array}$ What name should be used for 342?
2 ? hundreds 14 ? tens 2 ? ones

c $\begin{array}{r} 342 \\ -\ 274 \\ \hline \end{array}$ What name or names should be used for 342?
2 ? hundreds 13 ? tens 12 ? ones

2. In this problem, 63 has been renamed as 5 tens ? ones. 13

$$\begin{array}{r} {\scriptstyle 5\ 13} \\ \cancel{6}\,\cancel{3} \\ -\ 2\ 4 \\ \hline 3\ 9 \end{array}$$

3. Which numbers in the following problems require renaming?

a $\begin{array}{r} 65 \\ -\ 24 \\ \hline \end{array}$ **b** $\begin{array}{r} 82 \\ -\ 49 \\ \hline \end{array}$ **c** $\begin{array}{r} 92 \\ -\ 81 \\ \hline \end{array}$ **d** $\begin{array}{r} 56 \\ -\ 17 \\ \hline \end{array}$ **e** $\begin{array}{r} 48 \\ -\ 39 \\ \hline \end{array}$ **f** $\begin{array}{r} 96 \\ -\ 95 \\ \hline \end{array}$

4. Look at these renamed numbers. Where did the 9s come from? 10 renamed

$$\begin{array}{r|r|r} {\scriptstyle 3} & {\scriptstyle 9} & {\scriptstyle 14} \\ \cancel{4} & \cancel{0} & \cancel{4} \\ -\ 3 & 8 & 5 \\ \hline ? & ? & ? \\ & 1 & 9 \end{array} \qquad \begin{array}{r|r|r|r} {\scriptstyle 0} & {\scriptstyle 9} & {\scriptstyle 9} & {\scriptstyle 10} \\ \cancel{1} & \cancel{0} & \cancel{0} & \cancel{0} \\ - & 4 & 2 & 5 \\ \hline ? & ? & ? & ? \\ & 5 & 7 & 5 \end{array}$$

Do these subtraction problems.

5. $\begin{array}{r} 965 \\ -\ 49 \\ \hline 916 \end{array}$ **6.** $\begin{array}{r} 847 \\ -\ 49 \\ \hline 798 \end{array}$ **7.** $\begin{array}{r} 5486 \\ -\ 677 \\ \hline 4809 \end{array}$ **8.** $\begin{array}{r} 1000 \\ -\ 737 \\ \hline 263 \end{array}$ **9.** $\begin{array}{r} 900 \\ -\ 86 \\ \hline 814 \end{array}$ **10.** $\begin{array}{r} 1000 \\ -\ 999 \\ \hline 1 \end{array}$

If you can figure out this renaming, you'll probably never have a problem with subtraction. Think about what you are doing as you compute.

$$\begin{array}{r} {\scriptstyle 15\ 9\ 15} \\ \cancel{1}605 \\ -\ 957 \\ \hline 648 \end{array}$$

Be honest with yourself. Was it a very hard problem for you? If so, you may need some special help.

PROGRESS CHECK

Subtract. Skill: Subtraction — no renaming

	a	b	c	d	e
Group (1)	67 − 4 63	58 − 23 35	239 − 29 210	356 − 56 300	482 − 162 320

Skill: Subtraction — renaming tens

	a	b	c	d	e
Group (2)	20 − 8 12	38 − 9 29	65 − 47 18	173 − 54 119	184 − 78 106

Skill: Subtraction — renaming hundreds and tens

	a	b	c	d	e
Group (3)	295 − 76 219	368 − 59 309	526 − 88 438	540 − 93 447	505 − 27 478
Group (4)	671 − 184 487	506 − 254 252	701 − 355 346	800 − 262 538	400 − 367 33

Skill: Subtraction — renaming thousands, hundreds, and tens

	a	b	c	d	e
Group (5)	5273 − 648 4625	4672 − 569 4103	3598 − 1489 2109	6042 − 5256 786	9004 − 1425 7579

15

STOP See activity 4, page 22b.

See extension of activity 4, page 22b.

goal **Progress Check** – subtraction requiring renaming

page 15 The example is at the top to give individuals a chance to ask for help. Now is the time to set the tone for the year. When someone asks for help early in the game, it's easy to give the right kind of practice. It doesn't take much time then to get a pupil back on the right track. Giving your own time or finding a peer tutor right now will really pay off later.

The Progress Check will help you pinpoint each pupil's strengths and weaknesses in subtraction. Each row checks one particular skill. These skills are identified on the answer key.

Group (1) – Errors here mean only one thing: the pupil doesn't know the subtraction facts. Flash cards or some other form of drill must be used.

Group (2) – The simplest renaming skills are here. Watch for the following:

$$\begin{array}{r} 20 \\ -\ 8 \\ \hline 28 \end{array}$$

Probable reversal of digits. Check by giving other problems with zero in the top number.

$$\begin{array}{r} {\scriptstyle 1} \\ 65 \\ -\ 47 \\ \hline 28 \end{array}$$

Did not rename the tens.

These same errors can be found in any of the subsequent groups also.

lesson Pages 16, 17

goal Review of addition requiring renaming

page 16 Discuss the examples as a group. Emphasize the balance between the subtraction problem and its addition check in the renaming steps. Then review renaming in addition.

Problems 1 through 10 will be the pupils' first chance to show off their addition skills. Provide help for pupils who are having trouble. Don't hesitate to use capable buddies to provide additional help.

Do you remember how to check a subtraction problem?

$$\begin{array}{r} 523 \\ -\,179 \\ \hline 344 \end{array} \qquad \begin{array}{r} 344 \\ +\,179 \\ \hline ? \end{array}$$

523

Subtraction is "take away"; so to check, just "put it back."

Look at the relation between subtraction and addition again.

$$\begin{array}{r} 7\overset{5}{\cancel{6}}\overset{13}{\cancel{3}} \\ -\,149 \\ \hline 614 \end{array}$$

Renaming was required.

$$\begin{array}{r} \overset{1}{6}14 \\ +\,149 \\ \hline 763 \end{array}$$

Renaming happened here too.

Here is a model of the renaming situation when we *add*.

tens	ones		tens	ones		tens	ones		tens	ones
2	9		2	9		$\overset{1}{2}$	9		$\overset{1}{2}$	9
+ 2	8		+ 2	8		+ 2	8		+ 2	8
?	?		?	17		?	7		5	7

17 → to be renamed 1 ten and 7 ones

$$\begin{array}{r} \overset{1}{4}\overset{1}{6}\overset{1}{3}9 \\ +\ \ 587 \\ \hline 5226 \end{array}$$

16 is renamed ? ten ? ones. 1 6

120 is renamed ? hundred ? tens. 1 2

1200 is renamed ? thousand ? hundreds. 1 2

ADD

1. $\begin{array}{r} 28 \\ +\,61 \\ \hline \end{array}$ 89	**2.** $\begin{array}{r} 58 \\ +\,37 \\ \hline \end{array}$ 95	**3.** $\begin{array}{r} 99 \\ +\,89 \\ \hline \end{array}$ 188	**4.** $\begin{array}{r} 156 \\ +\ \ 72 \\ \hline \end{array}$ 228	**5.** $\begin{array}{r} 238 \\ +\ \ 98 \\ \hline \end{array}$ 336
6. $\begin{array}{r} 107 \\ +\ \ 59 \\ \hline \end{array}$ 166	**7.** $\begin{array}{r} 164 \\ +\,136 \\ \hline \end{array}$ 300	**8.** $\begin{array}{r} 573 \\ +\,427 \\ \hline \end{array}$ 1000	**9.** $\begin{array}{r} 4327 \\ +\ \ 894 \\ \hline \end{array}$ 5221	**10.** $\begin{array}{r} 6367 \\ +\,3975 \\ \hline \end{array}$ 10,342

16

The secret of accuracy in addition is knowing the addition facts.

$$\begin{array}{r} 5276 \\ +\,4512 \\ \hline \end{array}$$

If you have to rename, there is still no problem for you.

$$\begin{array}{r} 6879 \\ +\,5624 \\ \hline \end{array}$$

PROGRESS CHECK

Add.

	a	b	c	d	e
Skill: Addition — no renaming					
Group 1	62 + 7 69	54 + 25 79	438 + 61 499	506 + 91 597	729 + 160 889
Skill: Addition — renaming ones					
Group 2	58 + 9 67	83 + 7 90	76 + 18 94	257 + 25 282	562 + 418 980
Skill: Addition — renaming tens and ones					
Group 3	156 + 76 232	627 + 99 726	518 + 95 613	231 + 59 290	472 + 28 500
Group 4	664 + 187 851	743 + 298 1041	526 + 634 1160	462 + 336 798	275 + 795 1070
Skill: Addition — renaming hundreds, tens, and ones					
Group 5	1467 + 294 1761	2527 + 695 3222	2762 + 9258 12,020	5514 + 2486 8000	8006 + 1994 10,000

17

goal **Progress Check** — addition requiring renaming

page 17 Here's another example. Does anyone need help?

The Progress Check is independent work. Watch for errors in group 1. Note that no renaming is required. Check on the addition facts quickly. This diagnosis will again enable you to pinpoint problems. See the answer key for specific skills checked in each group.

STOP See activity 5, page 22c.

See activity 6, page 22c.

goal Practice in rounding to the nearest ten, hundred, or thousand

memo Estimating answers to problem situations is one thing that we grownups do nearly every day. There are times when we must compute actual answers. (We would all like to use a calculator for our income tax because the computations must be correct.) There are more times when we estimate answers. (We always try to keep the grocery bill reasonable.) The estimating skill should be learned by youngsters so that they can develop some number sense. Rounding is a prerequisite skill, and the next pages are devoted to that skill. We'll get to estimation later.

page 18 Number lines are the best way for all to see and then understand what rounding is all about. Level 4 lessons had lots of work on rounding, so this lesson may be a review. You will soon know how much more practice is needed.

1. 0 10 20 30 40 50 60 70 80 90 100

- **a** Is 86 closer to 80 or 90? So round 86 to ■. 90
- **b** Is 38 closer to 30 or 40? So round 38 to ■. 40
- **c** Is 72 closer to 70 or 80? So round 72 to ■. 70
- **d** Is 24 closer to 20 or 30? So round 24 to ■. 20
- **e** Is 65 closer to 60 or 70? It's halfway. Agree to round up to the nearest ten. 70

2. 100 200 300 400 500 600 700 800 900 1,000

- **a** Is 240 closer to 200 or 300? So round 240 to ■. 200
- **b** Is 779 closer to 700 or 800? So round 779 to ■. 800
- **c** Is 516 closer to 500 or 600? So round 516 to ■. 500
- **d** Is 991 closer to 900 or 1000? So round 991 to ■. 1000
- **e** Is 350 closer to 300 or 400? It's halfway. So round 350 to ■. 400

3. 1,000 2,000 3,000 4,000 5,000 6,000 7,000 8,000 9,000 10,000

- **a** Is 3500 closer to 3000 or 4000? So round 3500 to ■. It's halfway. 4000
- **b** Is 5750 closer to 5000 or 6000? So round 5750 to ■. 6000
- **c** Is 1470 closer to 1000 or 2000? So round 1470 to ■. 1000
- **d** Is 9612 closer to 9000 or 10,000? So round 9612 to ■. 10,000

4. Round each of these to the nearest ten.

a 53 50 **b** 15 20 **c** 46 50 **d** 97 100 **e** 55 60

5. Round to the nearest hundred.

a 138 100 **b** 670 700 **c** 465 500 **d** 851 900 **e** 949 900

6. Round to the nearest thousand.

a 2901 3000 **b** 7499 7000 **c** 4099 4000 **d** 8610 9000 **e** 1490 1000

18

The ability to round numbers will increase your ability to estimate. Ability to estimate will increase your computation skills, **and** increased computation skills will make everybody happy.

If you have trouble imagining a number line, try this method:

Round 87 to the nearest ten.
You want a zero in the ones place.
8(7) – Look at the ones digit.
Five or more?
Round up to 9 tens.

Round 664 to the nearest ten.
You still want a zero in the ones place.
66(4) – Look at the ones digit. Five or more? No
Round down to 66 tens.

Now round 664 to the nearest hundred.
Tens and ones places have zeros.
6(64) – Look at the tens and ones. Fifty or more?
Round up to 700.

Round 949 to the nearest hundred.
You want zeros in the ones and tens places.
9(49) – Look at the tens and ones. Fifty or more? No
Round down to 900.

Round 950 to the nearest hundred.
9(50) – Fifty or more? Round up to 10 hundreds or 1000.

Numbers can be rounded to the nearest multiple of 10, of 100, of 1000, of 10,000, of 100,000, of 1,000,000, and so on.
For example –
Round 15,401 to the nearest thousand.
15,(401) Look at the digits to the right of the place value you want. Five hundred or more? No
Round down to 15,000.

9,(599) Five hundred or more? Round up to 10,000.

Now you should be able to guess how you can round to the nearest multiple of 100,000 and 1,000,000.

19

goal Developing skill in rounding

memo The technique advanced on this page is rather mechanical. BUT the youngsters need this skill. If the number lines didn't work, hope that this approach will.

page 19 Here are some rules for rounding. Make sure that the notion of rounding **up** and rounding **down** is clearly understood. Try a few additional examples of each type to verify understanding – or the need for more practice.

goal **Progress Check** – rounding numbers, adding or subtracting rounded numbers

page 20 Look out for the directions for problems 5 through 16. Emphasize that the pupils are to add or subtract only the rounded numbers, not to compute the exact answer. (They are really estimating their answers, but more about that later.)

Problems 14, 15, and 16 have been marked with an *. These problems extend the skills developed thus far. Don't expect anyone to answer them. And there is certainly no need to provide additional practice with 7-digit numbers at this time.

PROGRESS CHECK

Skill: Reading any number with 10 or less digits

Write the standard numeral.

1. Five hundred fifty-three million, six hundred thousand, twenty-three. 553,600,023
2. Twenty million, nine hundred ninety thousand, nine hundred nine. 20,990,9
3. One million, eight hundred fifty-eight thousand, two hundred five. 1,858,205
4. Thirty-two hundred fifty-one million, seventeen thousand, fourteen. 3,251,0

Skills: Rounding to nearest ten; estimating sums

Round each number to the nearest ten. Then add the rounded numbers.

5. 194 + 62 → 190 + 60 = 250
6. 285 + 10 → 290 + 10 = 300
7. 136 + 100 → 140 + 100 = 240

Skills: Rounding to nearest hundred; estimating sums and differences

Round each number to the nearest hundred. Then compute the rounded numbers.

8. 897 − 499 → 900 − 500 = 400
9. 569 − 389 → 600 − 400 = 200
10. 423 + 899 → 400 + 900 = 1300

Skills: Rounding to nearest thousand; estimating sums and differences

Round each number to the nearest thousand. Then compute the rounded numbers.

11. 9958 − 9857 → 10,000 − 10,000 = 0
12. 5574 + 9857 → 6,000 + 10,000 = 16,000
13. 8523 − 7000 → 9000 − 7000 = 2000

Skills: Rounding to nearest million; estimating sums and differences

Round each number to the nearest million. Then compute the rounded numbers.

*14. 4,950,000 + 1,692,400 → 5,000,000 + 2,000,000 = 7,000,000
*15. 5,271,190 + 6,108,000 → 5,000,000 + 6,000,000 = 11,000,000
*16. 67,809,100 − 3,955,000 → 68,000,000 − 4,000,000 = 64,000,000

20

STOP See activity 7, page 22c.

See activity 8, page 22c.

City	POPULATION 1891 census	1971 census
Calgary	4 000	403 319
Halifax	38 495	122 035
Montreal	216 650	1 214 352
Ottawa	37 269	602 510
Vancouver	18 229	426 256

You will get reasonable answers if you round these large numbers to thousands. Operate with estimated answers.

1. Which of the cities on the 1971 list has the greatest population? Which has the least? Halifax
Montreal
2. By how much did the population increase in Calgary between 1891 and 1971? About 400 000
3. How many more thousands of people lived in Halifax than in Vancouver in 1891? About 20 000
4. Was the difference between the population of Halifax and the population of Vancouver greater in 1891 than in 1971? No
5. If someone who lived in Ottawa in 1971 said the population of his city was about 600 000 people, would he be reporting correctly? Yes
6. If a person in Vancouver in 1971 said the population of his city was about 1 000 000, would he be reporting correctly? What if he said the population was about 500 000? What would be the best rounded number for him to report? Do you suppose there was a period of one month when the population was exactly 426 256? No No 400 000 Probably not (Discuss)

21

lesson Pages 21, 22

goal Application of rounding and estimation skills to real-world situations

page 21 You decide how best to use this page with your pupils. It is not easy. You may want to use it with your more able students while you take time to help others who need your attention.

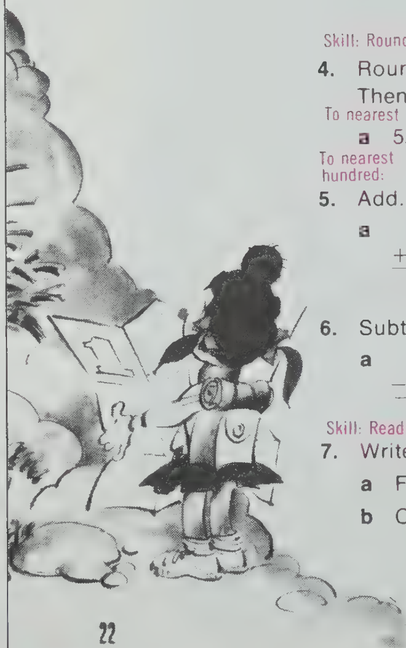
goal **Checkout** – skills developed in the chapter

page 22 This Checkout is independent work. Skills are identified on the answer key.

After identifying any errors, refer pupils needing further help back to the pages on which the skill was developed or practiced. Have them review the models shown.

Rows 1 and 2: pages 3–6
Row 3: pages 7–13
Row 4: pages 20–21
Row 5: pages 16–17
Row 6: pages 14–15
Row 7: pages 7–13

CHECKOUT



Skill: Writing equivalent standard numeral

1. Write the standard numeral for each of these roman numerals.

a III 3 **b** IV 4 **c** XII 12 **d** XX 20 **e** C 100

Skill: Writing equivalent roman numeral

2. Write a roman numeral for each of these standard numerals.

a 4 IV **b** 10 X **c** 15 XV **d** 30 XXX **e** 100 C

Skill: Reading and telling value of each digit

3. Name the value of any underlined digit in this numeral.

1 528 346

6 (ones)
40 (tens)
300 (hundreds)
8000 (thousands)
1 000 000 (millions)

Skill: Rounding to nearest specified place value

4. Round each number to the nearest ten.
Then round each to the nearest hundred.

	a	b	c	d	e
To nearest ten	530	160	350	860	970
	526	162	354	856	965
To nearest hundred:	500	200	400	900	1000

5. Add. Skill: Adding 2-, 3-, and 4-digit numbers – renaming

a	b	c	d	e
539	678	515	864	6826
+ 93	+ 102	+ 385	+ 479	+ 1275
632	780	900	1343	8101

6. Subtract. Skill: Subtracting 3- and 4-digit numbers – renaming

a	b	c	d	e
598	653	400	6728	8005
− 49	− 186	− 261	− 1619	− 1216
549	467	139	5109	6789

Skill: Reading any number with 9 or less digits

7. Write each of these as a standard numeral.

a Five hundred thousand, six hundred seventy-two 500 672

b One thousand, one hundred one 1101

22

See activity 9, page 22d.

Early systems of numeration – Babylonian, Mayan, Egyptian – hold fascination for many youngsters. Encourage them to research the symbols used in each of these systems. The symbols can be written in soft clay, simulating the writing of numerals on clay tablets in ancient times.

RESOURCES

another form of evaluation

for progress check–page 13

Make a chart with the word names for each place value. Find at least two other place-value names for each of these numbers.

1. 4163 2. 2395 3. 8107 4. 5748

Examples:

	T	h	t	o
1.	4	1	6	3
	3	11	6	3
	3	10	16	3
2.	2	3	9	5
	1	12	18	15
	2	2	18	15
3.	8	1	0	7
	7	11	0	7
	7	10	10	7
4.	5	7	4	8
	4	17	4	8
	5	6	13	18

for progress check–page 15

Subtract.

Group 1.

(a)	(b)	(c)	(d)	(e)
45	68	548	257	865
− 3	−22	− 37	− 57	−233
42	46	511	200	632

Group 2.

35	40	47	284	173
− 7	− 8	−28	− 65	− 19
28	32	19	219	154

Group 3.

625	312	267	320	405
− 57	− 24	− 89	− 41	− 37
568	288	178	279	368

Group 4.

321	564	403	600	900
−232	−375	−256	−548	−479
89	189	147	52	421

Group 5.

4267	3857	2732	3022	8005
− 679	− 968	−1984	−2354	−5837
3588	2889	748	668	2168

for progress check – page 17

Add.

Group 1.

(a)	(b)	(c)	(d)	(e)
95	38	624	207	824
+ 4	+21	+ 35	+ 92	+175
99	59	659	299	999

Group 2.

56	67	43	429	343
+ 7	+ 4	+17	+ 51	+319
63	71	60	480	662

Group 3.

391	725	906	857	394
+ 39	+ 86	+ 97	+ 64	+ 87
430	811	1003	921	481

Group 4.

177	713	101	327	531
+259	+387	+596	+175	+489
436	1100	697	502	1020

Group 5.

1472	7184	3345	6009	2238
+ 458	+ 937	+2986	+1993	+7782
1930	8121	6331	8002	10,020

for progress check–page 20

Write the standard numeral.

1. Two million, one hundred sixty-seven thousand, seven hundred thirty-six. 2,167,736
2. Five hundred twenty-two million, five hundred twenty thousand, three hundred seventeen. 522,520,317
3. Four hundred eighty-two million, fourteen thousand, two hundred fifty. 482,014,250
4. Thirty-six million, five hundred four thousand, six hundred. 36,504,600

Round each number to the nearest ten. Then compute the rounded numbers.

5.			6.			7.		
	326	330		125	130		271	270
	+ 10	+ 10		+ 42	+ 40		+100	+100
		340			170			380

Round each number to the nearest hundred. Then compute the rounded numbers.

8.			9.			10.		
	468	500		472	500		324	300
	−279	−300		+361	+400		−169	−200
		200			900			100

Round each number to the nearest thousand. Then compute the rounded numbers.

11.		12.		13.	
	1916		8756		9768
	+1383		−7422		+ 9576
	2000		9000		10,000
	+1000		−7000		+10,000
	3000		2000		20,000

*Round each number to the nearest million. Then compute the rounded numbers.

14.		15.		16.	
	7,990,000		5,742,420		25,237,300
	+ 6,205,000		+ 8,101,000		− 3,016,000
	8,000,000		6,000,000		25,000,000
	+ 6,000,000		+ 8,000,000		− 3,000,000
	14,000,000		14,000,000		22,000,000

for checkout–page 22

1. Write the standard numeral for each of these roman numerals.
   a) IV b) XV c) L d) XI e) VII
   4 15 50 11 7
2. Write the roman numeral for each of these standard numerals.
   a) 6 b) 11 c) 30 d) 100 e) 27
   VI XI XXX C XXVII
3. Name the value of any underlined digit in this numeral.
   4,353,786
   80 (tens)
   50,000 (ten-thousands)
   300,000 (hundred-thousands)
   4,000,000 (millions)
4. Round each number to the nearest ten. Then round each to the nearest hundred.

a) 345	b) 682	c) 419	d) 738	e) 976
340	680	420	740	980
300	700	400	700	1000

5. Add.

(a)	(b)	(c)	(d)	(e)
682	634	274	427	3786
+ 28	+188	+417	+ 696	+1556
710	822	691	1123	5342

6. Subtract.

(a)	(b)	(c)	(d)	(e)
321	287	786	6018	7004
− 49	−128	−498	−3419	−1475
272	159	288	2599	5529

7. Write each of these as a standard numeral.
   a) Three thousand, four hundred thirty-three
   3433
   b) Two hundred twenty thousand, seventy-four 220,074

activities

1. Compare the roman system to our system of numeration. How many symbols are needed to write the roman numerals through 20? (3) How many digits are needed to write the numerals through 20 in our system? (10) Compare the number of places needed to write such numerals as 8, 14, 18. Try writing your school enrollment in each system of numeration and the national debt in roman numerals. What are the advantages of each system?

2. things 8 sets of numeral cards 0 through 9

Players divide into teams of two players. Each team will need 4 sets of cards. These are sorted into 10 stacks—one for each digit. A leader calls out any number through 9999. The teams race to arrange the numeral, using the digit cards. The first team to arrange the numeral earns a point. Ten points wins the game. The number of place-value positions can be increased by providing each team with additional sets of numeral cards—1 set for each place-value position.

3. Send your researchers to the library to investigate the development of early number systems. The focus is on understanding that numbers were developed as an adaptation to the environment.

Possible research questions include:
1. What was the first number system ever used?
2. Why were new number systems invented?
3. What number systems are in use today? How are these systems alike? different?

Sources of information include:
1. An encyclopedia
2. Adler, Irving. *The Giant Golden Book of Mathematics.*
3. Adler, Irving and Ruth. *Numbers Old and New.*

4. things spirit master or compass (see page 72e)

Have the pupils prepare their own figures as shown, or provide them with a duplicated sheet of the figure only. The youngsters will write in their own numbers as they work.

Pupils work in pairs. The first pupil writes 4 numbers from 7 through 18 on alternate spokes of the innermost ring. Using this set of numbers will emphasize the basic subtraction facts. Other numbers can be used. The second pupil then writes the difference for successive pairs of numbers on the alternate spokes of the second ring. This procedure continues until the 4 differences become 0.

Extension activity: The number of rings required depends on the first 4 numbers chosen. Challenge more capable pupils to begin with any 4 numbers having 4 digits or less.

5. Individual activity (Provide the pupil with the following directions.)

1. Add these problems.

1257	76	99	5261	28
+ 2148	+ 67	+ 102	+ 2142	+ 46
(3405)	(143)	(201)	(7403)	(74)
(DEAF)	(BED)	(CAB)	(HEAD)	(HE)

2. Use the letter selector and the digits in each sum to spell a word. If you worked carefully, each sum will spell a word you know.

Letter Selector

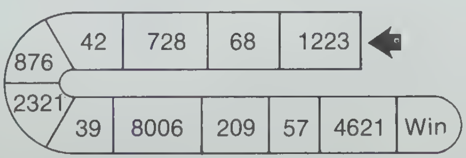
A	B	C	D	E	F	G	H	I	J
0	1	2	3	4	5	6	7	8	9

3. Make up some problems like these. Challenge a friend.

6. things game board; wood cube; markers

Any path divided into spaces will serve as a game board. Write a series of 2-, 3-, and 4-digit numbers—one in each space—until all the spaces on the path are filled. Write the numbers 1000, 2000, 3000, 4000, 5000, 6000 on the faces of a cube.

In turn, each player rolls the cube. The thousands digit that lands faceup on the cube indicates how many spaces the player may move, provided the sum of the numbers in these spaces is not greater than the faceup number. Suppose the player rolls 3000. The sum of the first three spaces is 2019. The sum 2019 is less than 3000. The player places his marker in the third space.



7. things 10 same-size boxes (milk cartons); slips of paper

Show a different multiple of 10 on one face of each box.

On another face show a multiple of 100 (100 through 1000). On a third face show a multiple of 1000 (1000 through 10,000). On the fourth, show a multiple of 10,000 (10,000 through 100,000).

For practice in rounding to the nearest ten, turn the boxes to show the multiples of 10. Place them side by side in order. Write 2-digit numerals on the slips of paper, one per slip. The pupil is to select a slip, round the number to the nearest ten, and then place it in the appropriate box.

Change the set of numbers and turn the boxes for practice in rounding to hundreds, thousands, ten thousands—whatever type of rounding practice is needed.

For practice in estimating sums or differences write an appropriate problem on each slip of paper. Mix up the slips. The pupil selects a slip, rounds appropriately, mentally estimates the answer, and then places the problem in the box labeled with the estimated answer. answer.

For example:

$$\begin{array}{r} 237 \\ +\,129 \\ \hline \end{array} \text{ rounds to } \begin{array}{r} 200 \\ +\,100 \\ \hline \end{array}$$

The problem is placed in the 300-box. Turn the boxes to accommodate the type of problem used for practice.

8. things for each pupil: 5-by-5 square game board; small squares of paper for markers

Roundo Game: Have each player make a 5-by-5 square game board, select 5 numbers from each column given below, and write one number in each square in the corresponding column on his game board.

100,000	10,000	1000	100	10
200,000	20,000	2000	200	20
300,000	30,000	3000	300	30
400,000	40,000	4000	400	40
500,000	50,000	5000	500	50
600,000	60,000	6000	600	60
700,000	70,000	7000	700	70
800,000	80,000	8000	800	80
900,000	90,000	9000	900	90

A "caller" is selected from the group. The caller can use the list provided below or make a list of 50 numbers ranging from 10 to 940,000 in any order. The caller reads one number at a time. Each player rounds the number called to the largest place value possible. If he has the rounded number on his board, he covers it with a marker. First player to fill a row, a column, or a diagonal wins the game.

817,858	7301	26,663	328	10
32	87	97	84,563	445
2563	944,763	819	8446	812,905
37,193	6708	2567	51	99,907
145,778	145	943	901,205	7354
5831	78,193	376,326	54,456	78
72,608	21,549	8999	670	13,378
62	40	92	86	1490
392	635,488	58,160	404,172	767
160	120	929,695	2021	306,548

9. Have the pupil find the sum of the first problem. Was renaming necessary? Change the digits in one number from a 1 to a 2 as in the second problem. Is renaming necessary?

1111	1111	1111	4444
1111	2222	1111	1111
1111	1111	3333	1111
+ 1111	+ 1111	+ 1111	+ 1111

Continue changing the digits of one number until renaming becomes necessary.

Challenge the youngster to write a problem having four 4-digit numbers and a sum equal to 9999. Is renaming necessary?

additional learning aids

notation – chapter objectives 1, 2, 4, 5

SRA products

Mathematics Learning System, Activity Masters, level B, SRA (1974)
Spirit masters: W-1, 2, 21, 22, 23

diagnosis: an instructional aid– Mathematics Level B, SRA (1972)
Probe: M-13
Mathematics Involvement Program, SRA (1974)
Card: 114

other learning aids (described on page 72j)– Abacus, Chip Trading, Place Value I and II

operation – chapter objective 3

SRA products

Mathematics Learning System, Activity Masters, level B, SRA (1974)
Spirit masters: W-3, 4, 10, 11, 12, 17, 18, 29; P-1

Arithmetic Fact Kit, SRA (1969)
Addition cards: 2–29
Subtraction cards: 2–27
Computapes, SRA (1972)
Module 2, Lessons: AS 24, 25, 33, 34, 36
Computational Skills Development Kit, SRA (1965)
Diagnostic Tests: 1, 2
Addition cards: 2, 6, 10–20
Subtraction cards: 3, 4, 7–16
Cross-Number Puzzles (Whole Numbers), SRA (1966)
Addition cards: 1–14
Subtraction cards: 1–13
diagnosis: an instructional aid– Mathematics Level B, SRA (1972)
Probe: M–1, 2
Math Applications Kit, SRA (1971)
Appeteasers card: 5
Sports and Games card: 32
Mathematics Involvement Program, SRA (1971)
Cards: 165, 26, 126
Skill through Patterns, level 5, SRA (1974)
Spirit masters: 2, 3, 5, 14, 17, 18, 38, 40

other learning aids – Dial-A-Matic* Adding Machine, Good Time Mathematics, I Win (sets 1, 2, and 3), Veri-Tech Senior (addition and subtraction books)

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2 WHOLE NUMBERS × AND ÷

before this chapter the learner has—

1. Mastered the multiplication facts
2. Mastered estimating and finding the product of any two 2-digit numbers
3. Mastered estimating and finding the quotient and remainder (if any) for any 3-digit number and any 1-digit number

in chapter 2 the learner is—

1. Exploring prime and composite numbers
2. Reviewing multiplication of two 2-digit numbers
3. Practicing multiplication of a 3-digit number by a 2-digit number
4. Reviewing dividing by a 1-digit divisor
5. Learning to divide by a 2-digit multiple of 10
6. Checking division computation with multiplication

in later chapters the learner will—

1. Master the multiplication of any 3-digit number by any 2-digit number
2. Practice dividing a 4-digit number by a 2-digit number and finding the remainder (if any)
3. Master checking a division computation with multiplication

Notes & Things

Estimation continues to be the point of emphasis as both multiplication and division are reviewed in this chapter. There are very few assumptions made about the learner's prior knowledge of these two operations, even though the pupil who has been in the MLS program has had a good deal of instruction and practice.

The chapter begins with some work on arrays and number patterns. That means a review of the multiplication number facts. The new topic of prime and composite numbers provides still more basic computation review. Finding multiples of a factor allows the multiplication algorithm to be reviewed. There is lots of practice multiplying 3-digit numbers by 2-digit numbers.

It's plain to see that with this much multiplication, the multiplication facts need to be known. If a pupil's records show that the multiplication fact-memorization task has been tried again and again but is still not a skill that has been mastered, now is the time to try something different. That child should be allowed to complete a multiplication table. Let him use this completed table whenever he needs or wants to use it. This learning crutch is much more desirable than frustration and repeated defeat.

The chapter's thrust on multiplication changes to division by relating the two operations. The operation of long division requires that the child estimate, multiply, and subtract. The top stacking of partial quotients requires that addition skills also be put to use. The top-stacking approach has been used consistently in this program because it allows the pupil to estimate freely and at the same time keep track of the place value of the quotient. Side stacking of partial quotients can also be justified. If you prefer that method, please use it. Explain frankly to the pupils the small differences between the two methods and go on from there.

Quotients with remainders are not treated as a special case of division. The children intuitively know about them because they happen all the time when division is used in the real world. It is also believed that by using estimation and seeing the repeated subtraction used in division, the idea of remainders won't be hard at all even if the child has not had any real-world experiences with them.

Most of the division computation in this chapter has a 1-digit divisor, but the quotients progress from 2 digits to 3 digits to 4 digits. Some work is done with 10 or a multiple of 10 as a divisor, but the problems are easy and they simply set the stage for further work in division in chapter 5.

Don't worry if the skills of division presented in this chapter are not mastered. There is really a lot more work with this operation in this level.

things

paper squares or wood cubes
spirit master of multiplication table
(see page 72f)

For the extra activities you will want to have these things available:

- graph paper (see page 72g)
- pictures showing a large number of objects
- milk carton, box
- weight scale

goal Think about and explore ideas through a picture clue

page 23 What better place could you be than in a candy store like this one if you have some money in your pocket?

Let the discussion be free. Give direction with some questions such as these. Has anyone ever been in a store like this? With the candy in those big baskets, how would you go about buying some? Do you think it would be cheaper or more expensive than prepackaged candy? Why? (There's no right answer to that question.) If you owned that store, what would you have to know? If you were a customer what would you have to know?

Make up just a few problems to find out who are your fast computers. Let the youngsters use scratch paper to find an answer if necessary. Assign a price to one of the kinds of candy. Buy 2 pounds, or 4 pounds, or 10 pounds. How much would that amount cost in all? Switch to division. You paid ______ for ______ pounds. How much for each pound? What kind of candy did you buy?

Keep this a light, fun exercise rather than hard work. Your pupils will have their share of that later.

lesson Page 24

goal **Survey**—ability to multiply and divide

memo Help your pupils understand the purpose of a survey page. It will help determine just how much of the work within the chapter they already know and will indicate what concepts will be covered. Are there pages that can be skipped? Is it necessary to back up in order to get off to a running start? This page will help each learner identify his strengths and weaknesses. One goal for the learner is to be proud of his strengths as well as to use them. Another goal is to turn his weaknesses into strengths as soon as possible.

page 24 Discuss the directions together. Make sure that your pupils understand how to fold their papers. The page should be completed independently.

You need not set a time limit for completing these problems, but do be aware of anyone who takes an exceptionally long time. Although his answers may be correct, that slow pupil needs additional work with the instructional pages and perhaps the additional activities suggested. Accuracy **and** speed indicate mastery.

The page surveys learner ability in the following skills:

- Multiplying a 2- or 3-digit factor by a 1-digit factor
- Multiplying a 3-digit factor by a 2-digit factor
- Dividing by a 1-digit divisor
- Dividing by a multiple of 10

Your goal is to be able to multiply and divide just about any two numbers. And you will get the right answer, too!

HOW ACCURATE IS YOUR WORK?

Don't copy the multiplication problem itself. Put a sheet of paper under the first row of problems. Write only your answer. When you finish the row, fold your answers under so that you have a clean surface. Do the next row. Fold your sheet again to have writing space for the last row.

1. 34×2 = 68	**2.** 43×3 = 129	**3.** 211×6 = 1266	**4.** 423×3 = 1269	**5.** 46×5 = 230	**6.** 38×3 = 114
7. 213×7 = 1491	**8.** 524×7 = 3668	**9.** 368×18 = 6624	**10.** 194×26 = 5044	**11.** 507×59 = 29,913	**12.** 680×47 = 31,960

You will have to copy these problems and then compute.

13. $6\overline{)126}$ 21 **14.** $7\overline{)342}$ 48 R6 **15.** $9\overline{)5783}$ 642 R5 **16.** $60\overline{)2227}$ 37 R7

Why do you think you were asked to do these problems *before* you started your study of multiplication and division?

This is a survey of what is known and what isn't. Difficulties can be pir start and you can direct your attention to these area

24

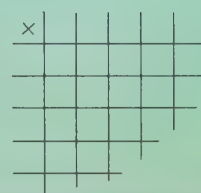
An array is a picture of rows and columns of objects. The picture usually forms a rectangular shape.

This array has 2 rows and 3 columns. The number of rows and the number of columns represent the factors of a multiplication problem. The number of squares in all represents the product.

1. Use squares or blocks. See if you can form an array that has more than one row for any of these numbers. 1, 3, 5, 7, and 9 You can't for 1, 3, 5, or 7. 9 can be shown as

2. The numbers you have just worked with are the odd numbers between 0 and 10. You should have been able to make an array for only one of the odd numbers listed above. Which one? 9
3. Look at the factors of the odd number that formed an array. Are the factors odd numbers? Yes (3 and 3)
4. An even number is 2. All multiples of 2 are also even numbers. In other words, every even number has a factor of 2. Write the even numbers between 2 and 10. 4, 6, 8
5. Make a multiplication table for factors 1 through 9. Use three different colors in the table.

   Use one color for the grid.
   Use the second color for every even number that you write.
   Use the third color for every odd number that you write.



25

×	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

lesson Pages 25, 26, 27

goal Review of the multiplication facts; making arrays for numbers; looking for patterns

things paper squares or wood cubes
spirit master of multiplication table (see page 72f)

warm-up Check to make sure that everyone knows how to distinguish **even** numbers from **odd** numbers. You might try writing several numbers randomly on the chalkboard. For example, 356, 39, 135, 300, 15, 7, 6, 429. Ask a pupil to name the odd numbers, another to name the even numbers. Review the signal given by the ones digit. Ask who can name the even digits; the odd digits.

page 25 You'll want to try these activities together. Does everyone know what an **array** is? Make sure that **rows** and **columns** are clearly understood. Rows run across, columns down.

Yes, it is possible to arrange the same number of objects in patterns of other shapes. **But,** to help with multiplication, the array must form a rectangular shape. Each of the numbers 1, 3, 5, and 7 will form an array, but not with **more than** one row.

A spirit master of the multiplication table will save some time on the last problem. Save the table for page 26.

goal Examining patterns of products on the multiplication table; introduction to prime numbers

page 26 Maybe you can get a good argument started by talking about the answers on this page.

Focus on the definition of a PRIME NUMBER. The definition in the book does not allow the number 1 to be a prime.

Use the MULTIPLICATION TABLE you just made for these questions

1. Are there more odd numbers or more even numbers in your table?
2. Is the product of two even numbers an odd number? an even number?
3. Can every even number be divided by 2? Yes
4. Is the product of two odd numbers an odd number? an even number?
5. Can every odd number be divided by 1? by 3? Yes No
   (It shouldn't take you long to answer that.
   The answer is in the first row of products)
6. Is the product of every odd and even number an even number? Yes
7. What pairs of factors name each of these products?
   a 4 **b** 9 **c** 16 **d** 25
   e 36 **f** 49 **g** 64 **h** 81
   a 1×4, 2×2 b 1×9, 3×3 c 2×8, 4×4
   e 4×9, 6×6 f 7×7 g 8×8
8. Are all other products named by more than one pair of factors? No
   How many pairs of factors? 2 at most
9. Is 11 a product of more than one pair of factors?
   Is 13? Is 17? Is 19? No—(1×11 and 11×1 are the same pair); no—13×1 or 1×13; n

The numbers 2, 3, 5, 7, 11, 13, 17, and 19 are called prime numbers. There are more, but these are all you need for now.

A prime number is a whole number greater than 1 that has only the factors of 1 and itself.

28

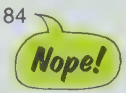
All products in your multiplication table except 1, 2, 3, 5, and 7 are called composite numbers. The word *composite* means "made up of parts." A composite number is made up of parts called factors. Every composite number has at least one pair of factors besides 1 and itself.

82 is the next-largest number outside your table. It is a composite number. You know that because it is an even number. All even numbers have 2 as a factor.

83 is the next number to consider.
Is it prime or composite? Prime
It's an odd number, so 2 is not a factor.
How many 3s in 83?
There are more than 10, because $10 \times 3 = 30$.
There are more than 20, because $20 \times 3 = 60$.
There are less than 30, because $30 \times 3 = 90$.

Try 25 threes.	$25 \times 3 = 75$	There are more than 25.
	$26 \times 3 = 78$	There are more than 26.
	$27 \times 3 = 81$	There are more than 27.
	$28 \times 3 = 84$	Too many! 3 is not a factor of 83.

Go on. Investigate the next prime factor. 83 does not end in 5 or 0, so you know 5 is not a factor.
How about 7? How many 7s in 83? 11 (with a remainder)
There are more than 10, because $10 \times 7 = 70$.
Try 11 sevens. $11 \times 7 = 77$
$12 \times 7 = 84$



7 is not a factor of 83. So 83 must be a prime number.

There are more prime numbers. But multiplication skills must be developed first.
And we really need division to do this work efficiently.
We will stop for now. We'll come back to primes later.

goal Introduction to divisibility tests for determining primes

page 27 This is strictly a page for discussion. To reinforce the definition of a COMPOSITE NUMBER, turn back to problem 7 on page 26. For each number given, name a pair of factors other than 1 and the number itself. Slip in the numbers 13, 17, and 19. Can these numbers be named as a pair of factors? Is the number 1 one of the factors? Then what kind of numbers are these? (Prime)

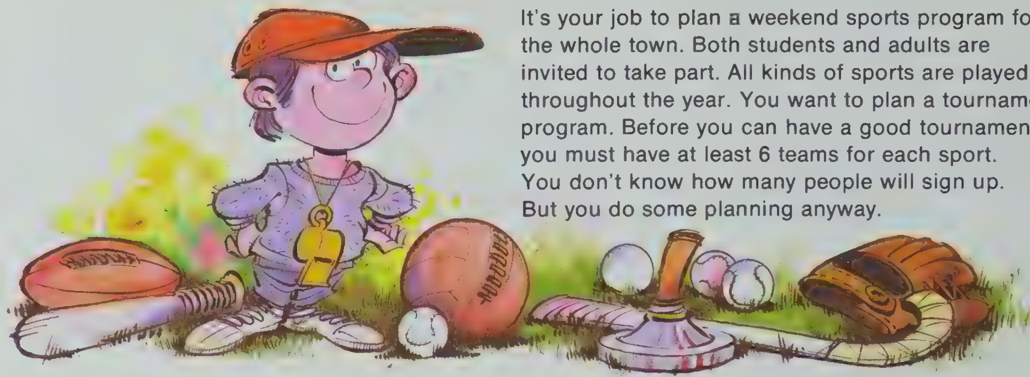
Introduce the divisibility tests for 2 and 5. All even numbers have 2 as a factor and therefore can be divided by 2. Numbers with 0 or 5 as the ones digit have 5 as a factor and can be divided by 5. Are numbers with 0 in the ones digit also divisible by 2? Time will be spent on more tests later.

lesson Page 28

goal Application of multiplication skills

page 28 Talk about the theme of the page together. What is a tournament? Why are there at least six teams? How many games might be needed to find the champion team? Ask thought-provoking questions that need not be answered immediately. Provide time for more research so that information on curling and rugby can be found.

Note that the problems emphasize the hard multiplication facts. Check especially on problem 4. For some reason, 10×11 quite often is miscomputed to be 111. Check to make sure that there is no difficulty multiplying by 10, since this skill is used in the estimation work that follows. If there's trouble, review and provide additional practice. In fact, youngsters who have trouble will need special attention throughout this chapter.



It's your job to plan a weekend sports program for the whole town. Both students and adults are invited to take part. All kinds of sports are played throughout the year. You want to plan a tournament program. Before you can have a good tournament, you must have at least 6 teams for each sport. You don't know how many people will sign up. But you do some planning anyway.

Copy and complete the charts.

1 For baseball

Number of teams	1	6	7	8	9	10
Least number of people	9	?	?	?	?	?
		54	63	72	81	90

2 For volleyball (Same number as needed for women's basketball.)

Number of teams	1	6	7	8	9	10
Least number of people	6	?	?	?	?	?
		36	42	48	54	60

3 For men's basketball

Number of teams	1	6	7	8	9	10
Least number of people	5	?	?	?	?	?
		30	35	40	45	50

4 For field hockey (Same number as needed for football.)

Number of teams	1	6	7	8	9	10
Least number of people	11	?	?	?	?	?
		66	77	88	99	110

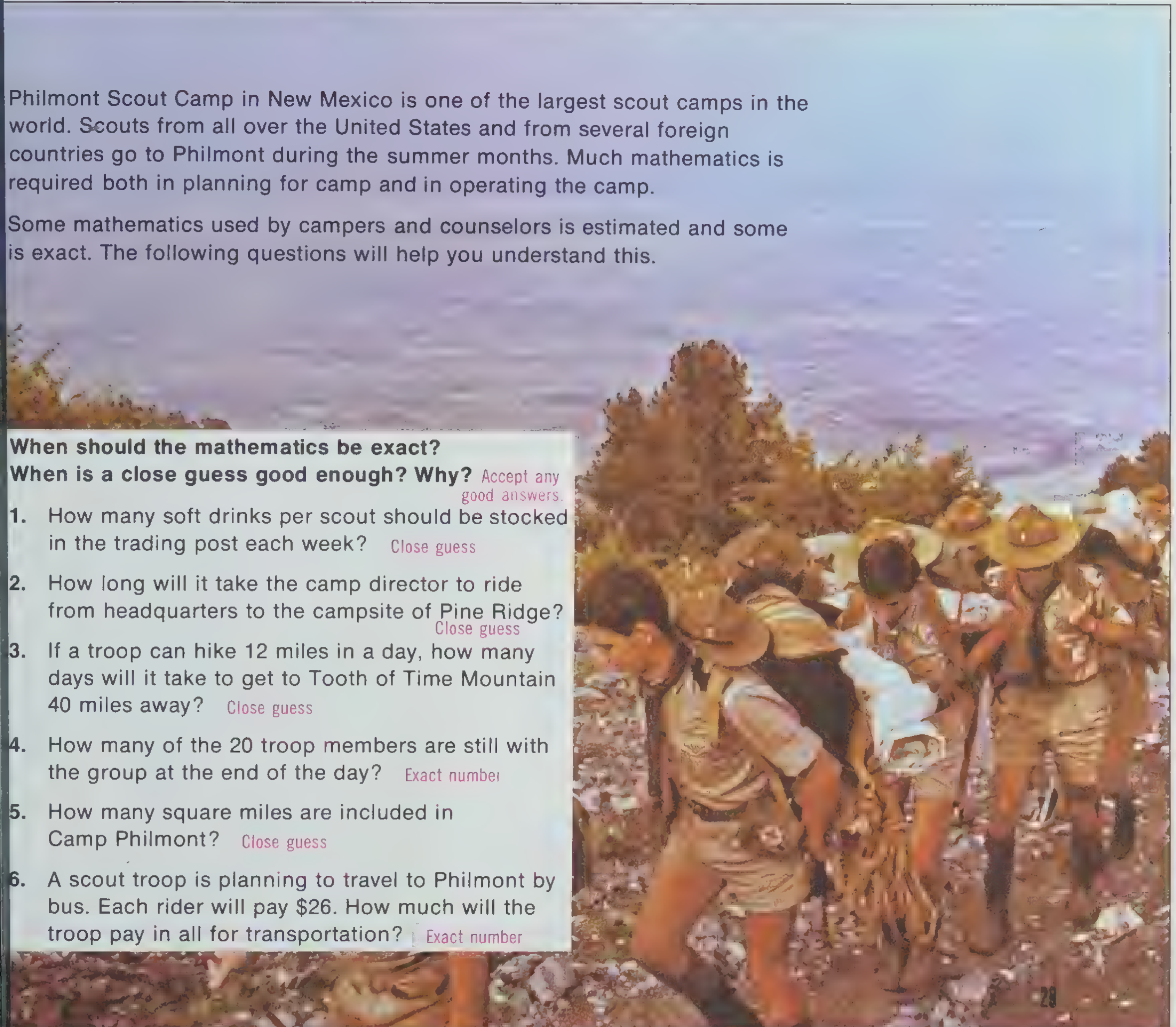
5 For curling (If you don't know what this ice game is, look it up.)

Number of teams	1	6	7	8	9	10
Least number of people	4	?	?	?	?	?
		24	28	32	36	40

6 For rugby (This is another game you might have to check on.)

Number of teams	1	6	7	8	9	10
Least number of people	15	?	?	?	?	?
		90	105	120	135	150

28



Philmont Scout Camp in New Mexico is one of the largest scout camps in the world. Scouts from all over the United States and from several foreign countries go to Philmont during the summer months. Much mathematics is required both in planning for camp and in operating the camp.

Some mathematics used by campers and counselors is estimated and some is exact. The following questions will help you understand this.

When should the mathematics be exact?
When is a close guess good enough? Why? Accept any good answers.

1. How many soft drinks per scout should be stocked in the trading post each week? Close guess
2. How long will it take the camp director to ride from headquarters to the campsite of Pine Ridge? Close guess
3. If a troop can hike 12 miles in a day, how many days will it take to get to Tooth of Time Mountain 40 miles away? Close guess
4. How many of the 20 troop members are still with the group at the end of the day? Exact number
5. How many square miles are included in Camp Philmont? Close guess
6. A scout troop is planning to travel to Philmont by bus. Each rider will pay $26. How much will the troop pay in all for transportation? Exact number

lesson Pages 29, 30, 31

goal Investigating whether an estimated answer is good enough

page 29 The label "a good guess" is more easily accepted by children than the label "estimated answer." Use whatever words are necessary to emphasize that sometimes counting is just plain impractical. For example, it would be silly to try to get an exact census figure. But if you are going on a field trip, you'd better know how many are going so that you know how many should come back. If you are going shopping, you can probably guess how much money you will need; but if you buy something, you'd better have at least the exact amount needed. Consider the number of books in the whole school versus the number of books needed for a specific group of students so that each person can have one.

For each numbered question, have the pupils jot down on paper either the word **exact** or the word **guess.** When they finish, go through the questions one at a time and ask why it is necessary, impossible, or not necessary to find the exact answer. (The exact total in problem 6 should be found because it asks for a specific amount.)

goal Developing a good guess—estimation skills

page 30 There are no wrong guesses or estimates—some are just better than others. Rounding first will help to make a guess closer. Youngsters who have trouble rounding should go back to a number line on page 18.

Some pupils may have difficulty seeing when the exact answer will be more than or less than the estimate. In problem 1, 22 is between 20 and 30. 22 is nearer 20. **But,** 20 is **less than** 22; therefore the exact answer will be **more than** the estimate.

For any number that is halfway, round up. You may have to point out in problem 2 that $2\frac{1}{2}$ is halfway between 2 and 3; therefore we round up. Since we rounded up from the exact number for the estimate, the exact answer will be less than the estimate.

You decide who is ready to work independently.

A good guess can be called an estimate.
An estimate is based on some thinking.

1. The camp wants to develop 6 new trails. About 22 scouts will be needed to work on each trail. About how many scouts will be working on all 6 trails?
   Think 22 is close to 20. 20×6 is 120.
   An answer of 120 scouts would be a good estimate.
   Would there be more or less than 120 scouts?

2. The supply truck can average about 50 miles per hour. The truck has to travel for at least $2\frac{1}{2}$ hours to get supplies. About how far does it travel?
   Think $2\frac{1}{2}$ hours is close to 3 hours.

   In 1 hour it goes about 50 miles.
   In 3 hours it should go about 3×50, or 150 miles.
   Would it be more or less than 150 miles?

3. Estimate these.
   - **a** $3 \times 33 = ?$ Think $3 \times 30 = 90$, so 90 would be a good estimate. Would 3×33 be more than or less than 90?
   - **b** $2 \times 87 = ?$ Think $2 \times 90 = 180$. Would 2×87 be more or less than 180?
   - **c** $5 \times 24 = ?$ Think $5 \times 20 = 100$. Would 5×24 be more or less than 100?
   - **d** $3 \times 412 = ?$ Think $3 \times 400 = 1200$. Would 3×412 be more or less than 1200?
   - **e** $9 \times 279 = ?$ Think $9 \times 300 = 2700$. Would 9×279 be more or less than 2700?

You have been estimating products. Even if you have to find the exact product, an estimate should be made. An estimate will help you avoid silly mistakes.

You will usually round one number before you estimate. You will round so that you can estimate in your head. An estimate is just more work if you have to figure it out with pencil and paper.

1. Compute these numbers in your head.

a 2×40 80	**b** 4×80 320	**c** 5×20 100	**d** 6×70 420
e 8×50 400	**f** 3×60 180	**g** 7×90 630	**h** 8×30 240
i 2×200 400	**j** 5×400 2000	**k** 8×300 2400	**l** 4×600 2400

2. Tell if the exact product will be more or less than the estimate that is written.

a	2×38	more than / less than	2×40	**b**	4×59	more than / less than	4×60
c	6×18	more than / less than	6×20	**d**	7×22	more than / less than	7×20

A ? will be in the place of the words *more than / less than*. But you will give the same kind of answer as you did before.

e 9×47 ? 9×50 less than (<)   **f** 8×63 ? 8×60 more than (>)

g 5×393 ? 5×400 less than (<)   **h** 4×715 ? 4×700 more than (>)

31

goal Practice with prerequisite skills for estimating products

page 31 Estimation should be a mental skill. Have the youngsters record only products for problem 1. Watch for those who have trouble handling the zeros. Focus their attention first on the basic facts, then on place value, and finally on the zeros.

$2 \times 4 = ?$ 2×4 tens $= ?$ $2 \times 40 = ?$

2×4 hundreds $= ?$ $2 \times 400 = ?$

Provide as much practice of this kind as is necessary.

If more help is needed, use the technique developed on page 30.

lesson Pages 32, 33, 34

goal Developing the idea of estimation as a check on the reasonableness of a computed answer

page 32 Discuss the first pair of examples. Assign problems 1 through 8. Compare the exact products and the estimates. Find any patterns? Continue the discussion with the next pair of examples. Assign problems 9 through 16. Note that these problems require renaming to find the exact product.

Next discuss the meaning of **reasonableness.** Write on the board the problem shown below. Ask the pupils if 618 is a reasonable answer. How would an estimate help?

$$\begin{array}{r} 26 \\ \times\ 3 \\ \hline 618 \end{array}$$

Encourage learners who show lack of confidence during the rounding practice to sketch a number line.

42 is closer to 40 than 50. If this practice is unnecessary for some pupils, have them skip problem 17 and go on.

These two examples show an estimated product and an exact product for 3×32.

ESTIMATE	EXACT
$3 \times 30 = 90$	$\begin{array}{r} 32 \\ \times\ 3 \\ \hline 96 \end{array}$

Round the two-digit factor and estimate the product. Then find the exact product. How does the exact product compare with the estimate? Each estimate is less than the exact product. Estimated answer in parentheses.

1. 2×42 (80) 84 **2.** 3×13 (30) 39 **3.** 2×24 (40) 48 **4.** 4×21 (80) 84

5. 2×23 (40) 46 **6.** 4×12 (40) 48 **7.** 3×22 (60) 66 **8.** 6×11 (60) 66

These two examples show an estimated product and an exact product for 4×28. Notice that the answer in both cases has three digits.

ESTIMATE	EXACT
$4 \times 30 = 120$	$\begin{array}{r} 28 \\ \times\ 4 \\ \hline 112 \end{array}$

Write an estimate. Then find the exact product. Estimated answer in parentheses

9. 3×26 (90) 78 **10.** 7×39 (280) 273 **11.** 4×17 (80) 68 **12.** 5×38 (200) 190

13. 2×18 (40) 36 **14.** 9×48 (450) 432 **15.** 6×16 (120) 96 **16.** 8×25 (240) 200

Are your answers reasonable? How do you know? Because they are close to the estimate
Yes

17. Having trouble estimating? Practice rounding skills on these.

a Is 42 closer to 40 or 50? **b** Is 39 closer to 30 or 40?
c Is 33 closer to 30 or 40? **d** Is 68 closer to 60 or 70?
e Is 54 closer to 50 or 60? **f** Is 94 closer to 90 or 100?
g Is 76 closer to 70 or 80? **h** Is 96 closer to 90 or 100?
i Is 25 closer to 20 or 30? It's halfway. We round up to 30.

32

These two examples show estimated and exact products. The problem is 3 × 123.
To what number should 123 be rounded? 100

ESTIMATE

3 × 100 = 300

EXACT

123	or	123
× 3		× 3
9		369
60		
300	(You can use either form	
369	of multiplication.)	

Estimate. Then find the exact products. Estimated answer in parentheses

1. 2 × 214 (400) 428 2. 3 × 312 (900) 936 3. 2 × 443 (800) 886 4. 3 × 232 (600) 696

5. 3 × 221 (600) 663 6. 2 × 124 (200) 248 7. 4 × 411 (1600) 1644 8. 3 × 302 (900) 906

Are the answers reasonable? How do you know? Yes They have the same number of digits as the estimates, and they are close to the estimates.

The next examples show estimated and exact products for 3 × 286.
To what number is 286 rounded? 300

ESTIMATE

3 × 300 = 900

EXACT

286	or	286
× 3		× 3
18		858
240		
600		
858		

Estimate. Then find the exact products. Estimated answer in parentheses.

9. 3 × 237 (600) 711 10. 6 × 456 (3000) 2736 11. 5 × 216 (1000) 1080 12. 4 × 462 (2000) 1848

13. 8 × 564 (4800) 4512 14. 2 × 765 (1600) 1530 15. 7 × 839 (5600) 5873 16. 9 × 278 (2700) 2502

17. 5 × 483 (2500) 2415 18. 3 × 514 (1500) 1542 19. 6 × 973 (6000) 5838 20. 9 × 596 (5400) 5364

Are the exact answers close to your estimates? Yes

33

goal Practice in estimating to check the reasonableness of a computed answer

page 33 The focus is on rounding to the nearest hundred for estimating. You will want to talk about the examples. You may want to provide a little verbal practice in rounding each 3-digit factor in problems 1 through 8 before assigning these problems.

Please encourage each pupil to use the algorithm that he likes best. Success—not the computational form—is what counts.

Note that problems 9 through 20 are considerably more difficult—renaming is required. Review of this skill may be necessary with some.

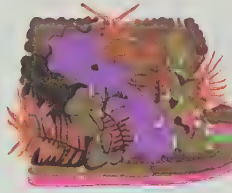
goal **Progress Check**—estimating and computing products

page 34 Take time to thoroughly review the directions for each set of problems. Note that pupils are asked to **estimate** only for problems 1 through 24.

Pupils who need help can use number lines showing multiples of 10. Have them mark approximately where each 2-digit number given should be and decide which multiple it is closer to. Then they can concentrate on the multiplication.

Trouble with problems 13 through 24? Substitute number lines showing multiples of 100. Skill in rounding is prerequisite to estimating. Time spent now in perfecting this skill will pay off during the rest of the year.

You will want to work individually with those students who made errors in computing the exact products for problems 25 through 32. Look for errors in basic facts as well as in renaming.



PROGRESS CHECK

Skill: Estimating 1-digit times 2-digit number

Estimate these products. Do *not* compute the exact product.

1. 4×65 280 2. 6×28 180 3. 9×47 450
4. 3×59 180 5. 7×31 210 6. 2×18 40
7. 8×71 560 8. 5×94 450 9. 1×16 20
10. 8×39 320 11. 7×77 560 12. 5×89 450

Skill: Estimating 1-digit times 3-digit number

Estimate these products. Do *not* compute the exact product.

13. 3×382 1200 14. 7×745 4900 15. 2×621 1200
16. 8×914 7200 17. 9×381 3600 18. 7×861 6300
19. 6×298 1800 20. 5×109 500 21. 1×580 600
22. 4×475 2000 23. 8×659 5600 24. 9×344 2700

Skill: Finding exact product of 1-digit times 3-digit number

Find the exact products.

25. 914×8 7312 26. 621×2 1242 27. 774×7 5418 28. 382×3 1146
29. 109×5 545 30. 298×6 1788 31. 475×4 1900 32. 381×9 3429

Do you think your answers are correct?
Are they reasonable?
Probably; yes—each is close to estimate made (see problems 13, 15, 16, 17, 19, 20, and 22)

34

See activity 1, page 52a.

See activity 2, page 52b.

Write the answers.

1. $3 \times 20 = ?$ 60 · $30 \times 20 = ?$ 600 · $300 \times 20 = ?$ 6000

2. $6 \times 30 = ?$ 180 · $60 \times 30 = ?$ 1800 · $600 \times 30 = ?$ 18,000

3. $7 \times 40 = ?$ 280 · $70 \times 40 = ?$ 2800 · $700 \times 40 = ?$ 28,000

4. $5 \times 40 = ?$ 200 · $50 \times 40 = ?$ 2000 · $500 \times 40 = ?$ 20,000

What is the pattern? Would the pattern help you find the answer to $600 \times 40 = ?$ Multiply the two nonzero digits; annex the total number of zeros in the two factors; yes

5. $8 \times 70 = ?$ 560 · $60 \times 70 = ?$ 4200 · $300 \times 70 = ?$ 21,000

6. $3 \times 50 = ?$ 150 · $50 \times 50 = ?$ 2500 · $400 \times 50 = ?$ 20,000

7. $9 \times 80 = ?$ 720 · $60 \times 80 = ?$ 4800 · $300 \times 80 = ?$ 24,000

8. $4 \times 90 = ?$ 360 · $30 \times 90 = ?$ 2700 · $200 \times 90 = ?$ 18,000

What differences are there between this set of problems and the first set of problems? The first factors in problems 5, 6, 7, and 8 have different digits.

9. $8 \times 40 = ?$ 320 · $50 \times 20 = ?$ 1000 · $200 \times 80 = ?$ 16,000

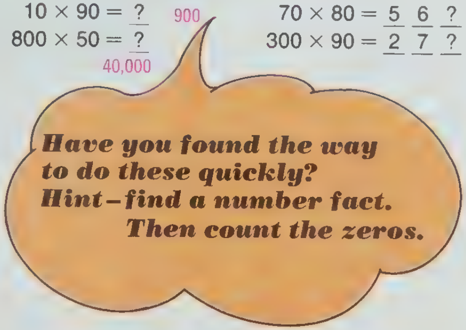
10. $6 \times 30 = ?$ 180 · $20 \times 40 = ?$ 800 · $500 \times 60 = ?$ 30,000

11. $3 \times 70 = ?$ 210 · $80 \times 30 = ?$ 2400 · $800 \times 70 = ?$ 56,000

12. $2 \times 60 = ?$ 120 · $40 \times 60 = ?$ 2400 · $600 \times 20 = ?$ 12,000

13. $4 \times 60 = ?$ 240 · $10 \times 90 = ?$ 900 · $800 \times 50 = ?$ 40,000

14. $6 \times 40 = 2\ 4\ ?$ 0 · $70 \times 80 = 5\ 6\ ?\ ?$ 00 · $300 \times 90 = 2\ 7\ ?\ ?\ ?$ 000



35

lesson Pages 35, 36, 37, 38

goal Practice in multiplying multiples of 10 and 100

page 35 You can use this page as written work or in a quick oral drill—depending on the abilities of your pupils and the type of practice needed. If problems 1 through 4 go well, then the whole page should be a snap.

The skill practiced is necessary for estimating products, for the long multiplication algorithm, and for division. The emphasis is on place value. Watch for youngsters who are confused by the zeros.

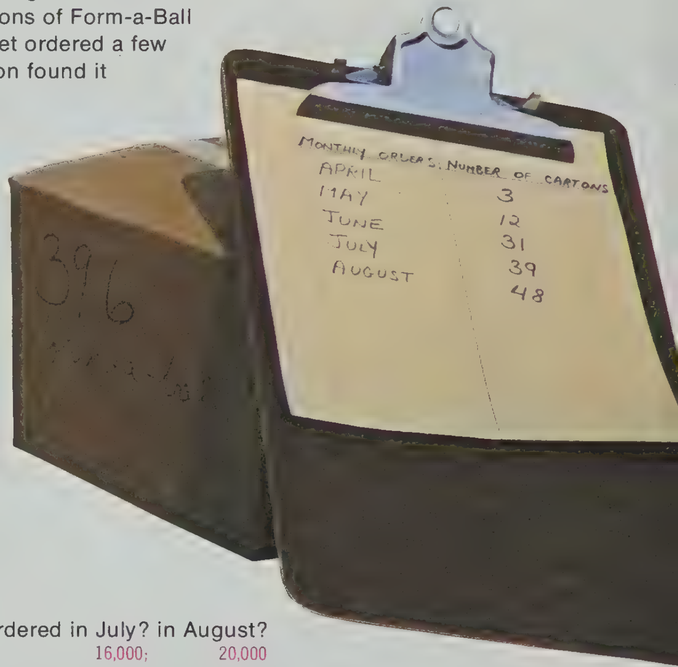
goal Practice in estimating products

page 36 You will need to talk about the Form-a-Ball exercise. Have pupils complete the order for July and August. If the number of zeros is still causing difficulty, take time to make a summary chart.

Example	Types of numbers being multiplied	No. of zeros in product
8×30	ones $\times$ tens = tens	1
8×400	ones $\times$ hundreds = hundreds	2
40×80	tens $\times$ tens = hundreds	2
60×700	tens $\times$ hundreds = thousands	3

Note that exact products are **not** required for problems 1 through 9.

A small puttylike ball of material is packaged and sold by the Redi-Play Game Company. Cartons of Form-a-Ball contain 396 balls. The Shoppers' Market ordered a few cartons for early in the season, but soon found it necessary to order many more.



About how many Form-a-Balls were ordered in April? 396 is about 400, to the nearest hundred. 3 cartons of about 400 is about 1200.

$3 \times 400 = 1200$

About how many were ordered in May? To the nearest ten, 12 is 10. 10 cartons of 400 is 4000.

$10 \times 400 = 4000$

How many were ordered in June? To the nearest ten, 31 is 30. 30 cartons of 400 is 12,000.

$30 \times 400 = 12{,}000$

About how many Form-a-Balls were ordered in July? in August?
16,000; 20,000

Estimate the products.

1. 19×412 8000 **2.** 267×91 27,000 **3.** 30×689 21,000

4. 586×59 36,000 **5.** 83×728 56,000 **6.** 61×163 12,000

7. 41×832 32,000 **8.** 342×78 24,000 **9.** 907×22 18,000

34 cartons are stacked in a warehouse.
ach contains 40 bottles of shampoo.
ow many shampoo bottles are there all together?
ne estimate is $40 \times 600 = \underline{?}$ 24,000

$$\begin{array}{rl} 634 & \\ \times\ 40 & \\ \hline 160 & 40 \times 4 \\ 1\,200 & 40 \times 30 \\ 24\,000 & 40 \times 600 \\ \hline 25{,}360 & \end{array} \quad \text{or} \quad \begin{array}{r} {}^{1\,1} \\ 634 \\ \times\ 40 \\ \hline 25{,}360 \end{array}$$

Why do we write 1 above the 3? So as not to forget the renamed tens
What does it represent? Renamed tens

s the product close to the estimate? Yes

Vhat numbers replace the question marks?

$$\begin{array}{rl} 745 & \\ \times\ 30 & \\ \hline 150\ ? & 30 \times 5 \\ 1200\ ? & 30 \times 40 \\ 21000\ ? & 30 \times 700 \\ \hline 22{,}350\ ? & \end{array} \quad \text{or} \quad \begin{array}{r} {}^{1\,1} \\ 745 \\ \times\ 30 \\ \hline ? \\ 22{,}350 \end{array}$$

n estimate of the product is $30 \times 700 = 21{,}000$.
ow close is this to the actual product?
350 off. Also accept "over 1000 less."

stimate. Then find the exact products. Estimated answers are in parentheses.

. 30×61 (1800) 1830	2. 80×92 (7200) 7360	3. 70×49 (3500) 3430
. 60×897 (54,000) 53,820	5. 50×643 (30,000) 32,150	6. 90×357 (36,000) 32,130

ompare each exact product with an estimate.

hink of at least two situations where multiplication s needed to solve a problem. Are exact answers eeded in both situations?
nswers will vary. Example: Nine players on a baseball team need oves that cost \$9.47. You could use an estimate of the cost—until you get ready pay or when you are counting your change from their purchase.

37

goal Review of the long and short multiplication algorithms

page 37 You will want to guide the discussion reviewing the two algorithms (computational forms) for multiplication shown at the top of the page. Proceed slowly if either algorithm is new to your students. The long algorithm uses partial products. Pupils need not write the thinking steps shown in color.

You may want to begin with a simple problem.

$$\begin{array}{rl} 12 & \\ \times\ 2 & \\ \hline 4 & (2 \times 2) \\ 20 & (2 \times 10) \\ \hline 24 & \end{array}$$

The partial-products algorithm should be understood by pupils but not required for multiplying if it is unnecessary. Let them pick the algorithm they want to use.

Pair pupils to work problems 1 through 6. One pupil computes the exact product; the other estimates the product. The products should be compared to determine the reasonableness of the exact product. Roles are switched for the next problem.

Partners should also write responses to the last two sentences on the page. Discuss these situations together.

goal Examining multiplication of two 2-digit numbers; **Progress Check**—estimating products and computing exact products

page 38 You may wish to examine the following examples with your pupils:

$$\begin{array}{rl} & 48 \\ \times & 32 \\ \hline & 16\ (2 \times 8) \\ & 80\ (2 \times 40) \\ & 240\ (30 \times 8) \\ & 1200\ (30 \times 40) \\ \hline & 1536 \end{array} \qquad \begin{array}{rl} & 372 \\ \times & 24 \\ \hline & 8\ (4 \times 2) \\ & 280\ (4 \times 70) \\ & 1200\ (4 \times 300) \\ & 40\ (20 \times 2) \\ & 1400\ (20 \times 70) \\ & 6000\ (20 \times 300) \\ \hline & 8928 \end{array}$$

The string of partial products and the addition problem that results should be sufficient incentives to use the shorter algorithm.

Have pupils complete the example on page 38. Check it together. You may wish to have them compute a 3-digit example (35×507) before the Progress Check.

Discuss the directions for the Progress Check. Look for types of errors—estimation errors and computation errors. If more than two errors are made in either step, the pupil needs individual help.

$32 \times 48 = \underline{?}$ The product will be about $30 \times 50 = 1500$. Can you explain why?

32 rounds to 30, 48 rounds to 50, and $30 \times 50 = 1500$

When both factors are 2-digit numbers, you are really doing two multiplication problems. Then you put the two products together.

$$\begin{array}{r} 48 \\ \times\ 32 \\ \hline 96 \\ 1440 \\ \hline 1536 \end{array} \qquad \begin{array}{r} 48 \\ \times\ 2 \\ \hline 96 \end{array} \qquad \begin{array}{r} 48 \\ \times\ 30 \\ \hline 1440 \end{array}$$

Here is another example.

$$\begin{array}{rr} & 76 \\ & \times\ 29 \\ \hline 684 & ? \quad 9 \times 76 \\ 1520 & ? \quad 20 \times 76 \\ \hline 2204 & ? \end{array}$$

What are the two partial products? 684, 1520

What is the final product? 2204

PROGRESS CHECK

Skill: Estimating products; multiplying by 1- or 2-digit number

Estimate the answers. Complete the multiplication.
Compare your estimates with the exact answers. Estimated answers in parentheses

1. 23×7 (140) 161
2. 87×6 (540) 522
3. 256×9 (2700) 2304
4. 709×2 (1400) 1418
5. 184×10 (2000) 1840
6. 95×20 (2000) 1900
7. 414×80 (32,000) 33,120
8. 806×60 (48,000) 48,360
9. 55×83 (4800) 4565
10. 80×19 (1600) 1520

38

See activity 3, page 52b.

See activity 4, page 52b.

STOP

Multiplication and division are related operations.

$$3 \times 4 = 12$$

This is a factor.
It tells how many sets
you are considering.

This is a factor also.
It tells how many in each set.

This is the product.
It tells how many in all.

Compare this multiplication to a related division equation.

$$12 \div 4 = 3$$

The product.
(How many in all?)

This is one factor.

This is the
second factor.

If you can't remember $54 \div 6$, you can **Think** What number $\times\ 6 = 54$? 9

If you can't remember $36 \div 4$, you can **Think** What number $\times\ 4 = 36$? 9

It saves a lot of time if you can remember the division facts in the first place. How many division facts give you trouble? Find out. See how fast you can write the answers.

	a		b		c		d		e		f		g	
1.	$42 \div 7$	6	$24 \div 6$	4	$64 \div 8$	8	$32 \div 4$	8	$49 \div 7$	7	$54 \div 6$	9	$45 \div 9$	5
2.	$48 \div 6$	8	$72 \div 9$	8	$18 \div 6$	3	$28 \div 7$	4	$24 \div 3$	8	$72 \div 8$	9	$36 \div 6$	6
3.	$54 \div 9$	6	$36 \div 4$	9	$35 \div 7$	5	$27 \div 3$	9	$40 \div 8$	5	$63 \div 9$	7	$28 \div 4$	7
4.	$36 \div 6$	6	$21 \div 3$	7	$30 \div 5$	6	$56 \div 8$	7	$63 \div 7$	9	$42 \div 6$	7	$81 \div 9$	9

39

lesson Pages 39, 40

goal Relating multiplication to division; review of division facts

page 39 Examine together the relationship of multiplication and division. Pupils rely heavily on the multiplication facts to help them in division—but, as stated on the page, "It saves a lot of time if you can remember the division facts in the first place."

Have only answers recorded. Emphasize accuracy **and** speed.

Identify pupils who need a buddy to practice with. Sets of flash cards for specific facts can be made quickly on index cards. Consider using the form $8\overline{)40}$ for the fact card rather than $40 \div 8$. The youngsters are going to see a lot of the $\overline{)}$ symbol. Why not practice the facts using the form that they will be using? When pupils have mastered a fact, remove that card from the set.

goal Continuing the search for prime numbers; review of the division algorithm

memo For the division algorithm, partial quotients are stacked on the top to preserve the concept of place value. As the youngsters improve their estimation skills, the stacked partial quotients will begin to disappear.

page 40 Has anyone forgotten about primes and composites? Then review the definitions, as well as the divisibility tests for the numbers 2 and 5 found on pages 25, 26, and 27. Use the examples on this page to review the division algorithm. Lined paper turned sideways helps the disorganized child keep track of place value. Every digit can be lined up in its proper column.

Explain to pupils that to check whether a number less than 100 is prime, they need test only for 2, 3, 5, and 7. The reason may be a little too sophisticated at this level. (The square of 10 is 100; therefore only primes less than 10 need be tested.) They already know a quick test for 2 and 5, so they need only to divide by 3 and by 7.

Problem 1 is independent work. Talk about the results. Problem 2 is strictly for discussion—and imaginations!

When last seen, the number 83 had just been tagged as a prime number.
What number comes next in the never ending search for prime suspects?

Number 84 is in the lineup.
But it's not even a suspect.
84 is an even number.
You know 2 is a factor of any even number.

85 can also be dismissed.
The last digit is 5,
so 5 must be a factor.

86? No luck.
Here is another even number.
But 87 looks suspicious.
Is 87 a composite number?
Use division to find out.

$$\begin{array}{r l} 29 & \\ \underline{9} & \\ 20 & \\ 3\overline{)87} & \\ \underline{60} & 20\times 3 \\ 27 & \\ \underline{27} & 9\times 3 \\ 0 & \end{array}$$

It is a composite number.
3 is a factor.

89 is the next number to check.

$$\begin{array}{r l} 29 & \text{R2} \\ \underline{9} & \\ 20 & \\ 3\overline{)89} & \\ \underline{60} & 20\times 3 \\ 29 & \\ \underline{27} & 9\times 3 \\ 2 & \end{array}$$

The remainder tells you
3 is not a factor.

So find out if 7 is a factor.

$$\begin{array}{r l} 12 & \text{R5} \\ \underline{2} & \\ 10 & \\ 7\overline{)89} & \\ \underline{70} & 10\times 7 \\ 19 & \\ \underline{14} & 2\times 7 \\ 5 & \end{array}$$

The remainder tells you
7 is not a factor.

89 must be a
PRIME NUMBER

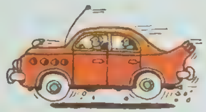
1. Use division to find the other primes between 90 and 100 (if there are any). 97

2. How many more primes do you think there are? They go on and on forever.

Remainders in regular computation are not a problem.

Remainders in actual division can be handled in different ways. Sometimes common sense tells you what to do.

1. 34 people want to go. Only 6 people could go in each car. How many cars have to go?



$$\begin{array}{r} 5\text{ R}4 \\ 6\overline{)34} \\ \underline{30} \\ 4 \end{array}$$

How many 6s in 34? $6 \times 5 = 30$
$6 \times 6 = 36$

6 cars have to go. Or how many people have to stay home? 4

2. There were 38 guppies. 8 people were going to divide them.

$$\begin{array}{r} 4\text{ R}6 \\ 8\overline{)38} \\ \underline{32} \\ 6 \end{array}$$

How many 8s in 38? $4 \times 8 = 32$
$5 \times 8 = 40$

What are you going to do with the remaining 6?
Answers will vary. Examples: Give them to someone else. Draw straws to see who gets them.

3. 28 watermelons were ripe. 3 families are going to divide them.

$$\begin{array}{r} 9\text{ R}1 \\ 3\overline{)28} \\ \underline{27} \\ 1 \end{array}$$

How many 3s in 28? $9 \times 3 = 27$
$10 \times 3 = 30$

What can be done with the remaining watermelon?
If it were divided, how much would each get?
Answers may vary. Example: Split it into thirds; $\frac{1}{3}$ (plus 9 whole watermelons)

4. The candy was 2 for 15¢.
How much will 1 bar cost?

$$\begin{array}{r} 7\text{ R}1 \\ 2\overline{)15} \\ \underline{14} \\ 1 \end{array}$$

You can't pay $7\frac{1}{2}$ cents.
What will you pay? 8¢

41

lesson Pages 41, 42

goal Examining remainders in real-world situations

page 41 A discussion of this page could suggest some imaginative alternatives. Remainders are frequently a part of practical, real-world division. Problem 4 may raise a question. Where does the $\frac{1}{2}$ cent come from? One cent divided in two equals how much? Extend the idea by deciding why the consumer pays the extra $\frac{1}{2}$ cent. Consider the situation in which a store forgets about the $\frac{1}{2}$ cent in selling one item but adds it in selling 10, 100, 1000, 1,000,000 items. How much money would be lost in each situation? How much would be made? Under what circumstances will the fractional part of a cent result? What can you as a consumer do to avoid the problem?

goal Practice in division with remainders

page 42 Focus on estimating a range for the quotient in the example. The exact answer should be more than ? but less than ?. This estimate will indicate whether the computed answer is reasonable. To check for sure, however, use multiplication. Discuss when a **sure** check is necessary.

Have the youngsters check by multiplication any four problems in rows 1 through 3. This method of checking is not too popular since they have to do two problems – divide **and** multiply. It is hoped the necessity for a correct answer in a situation will provide incentive.

$5\overline{)91}$

How many 5s in 91?
There are more than 10, because $10 \times 5 = 50$.
There are less than 20, because $20 \times 5 = 100$.

$$\begin{array}{rrl} & 18 & \text{R1} \\ & 8 & \\ & 10 & \\ 5) & 91 & \\ & \underline{50} & 10 \times 5 \\ & 41 & \\ & \underline{40} & 8 \times 5 \\ & 1 & \end{array}$$

You know the answer is reasonable. The estimate was a number more than 10 and less than 20.

Try These

	a	b	c	d
1.	$2\overline{)62}$ 31	$4\overline{)79}$ 19 R3	$3\overline{)78}$ 26	$6\overline{)68}$ 11 R2
2.	$7\overline{)88}$ 12 R4	$3\overline{)51}$ 17	$5\overline{)64}$ 12 R4	$4\overline{)41}$ 10 R1
3.	$2\overline{)89}$ 44 R1	$6\overline{)84}$ 14	$9\overline{)96}$ 10 R6	$5\overline{)78}$ 15 R3

If you want to make sure your answers are reasonable *and* correct, you can check your division.

$3\overline{)76}$

There are more than 20: $20 \times 3 = 60$
There are less than 30: $30 \times 3 = 90$

$$\begin{array}{rrl} & 25 & \text{R1} \\ & 5 & \\ & 20 & \\ 3) & 76 & \\ & \underline{60} & 20 \times 3 \\ & 16 & \\ & \underline{15} & 5 \times 3 \\ & 1 & \end{array}$$

Multiply to check division.

$$\begin{array}{r} 25 \\ \times \;\; 3 \\ \hline 75 \\ + \;\; 1 \\ \hline 76 \end{array}$$

Add the remainder.

Choose one of the three estimates given.
Answers will vary. Examples with reasons are shown.

1. How many 9s in 160? a 10 nines b 20 nines c 30 nines

   Why did you pick the estimate that you did? 20 nines = 180, which is more than 160.

2. How many 7s in 480? a 50 sevens b 60 sevens c 70 sevens
   70 sevens = 490, which is more than 480.
3. How many 5s in 390? a 60 fives b 70 fives c 80 fives
   80 fives = 400, which is more than 390.
4. How many 6s in 900? a 90 sixes b 100 sixes c 200 sixes
   200 sixes = 1200, which is more than 900.
5. How many 3s in 420? a 90 threes b 100 threes c 200 threes
   200 threes = 600, which is more than 420.
6. How many 4s in 952? a 100 fours b 200 fours c 300 fours
   300 fours = 1200, which is more than 952.
7. How many 2s in 875? a 300 twos b 400 twos c 500 twos
   500 twos = 1000, which is more than 875.

Put your estimating skills to work.

```
   87 R4
    7
   80
6)526
  480   80 × 6
   46
   42   7 × 6
    4
```

You could have estimated 60. It would be O.K.
Or you could have estimated 70.
What would have happened if you had tried 90? It would have been larger than 526.

Try These

	a	b	c	d	e
8.	6)126 (21)	7)154 (22)	8)431 (53 R7)	4)565 (141 R1)	7)623 (89)
9.	9)375 (41 R6)	5)785 (157)	7)201 (28 R5)	5)433 (86 R3)	6)732 (122)
10.	8)597 (74 R5)	9)700 (77 R7)	9)452 (50 R2)	6)541 (90 R1)	3)906 (302)

43

lesson Pages 43, 44, 45, 46

goal Practice with estimation skills and in computing 2-digit quotients

page 43 Practice in rows 1 through 7 will check a pupil's ability to estimate **and** his skill with basic facts. Success is essential. This skill is prerequisite to dividing with accuracy and efficiency.

You are the best judge of how to handle rows 8 through 10. No pupil needs to be practiced to death—killing all enthusiasm—but **skill** is necessary. Adjust the assignment to the needs of the individual learner. Provide additional help for those who need it; send capable learners on to page 44.

goal Practice with estimation skills and in computing 3-digit quotients

page 44 Everyone should review the example. If estimation is understood, 3-digit quotients should be no big thing. Reinforce checking by multiplication, but do not require this step for every problem. Please remember – checking doubles the number of problems. Adjust the assignment to meet the needs of individual pupils.

Study this example. 5)644

How many 5s in 644?
$10 \times 5 = 50$ Oops!
$100 \times 5 = 500$ That's about right.
$200 \times 5 = 1000$ That's too many!
There are between 100 and 200 fives in 644.

```
  128 R4
    8
   20
  100
5)644
  500    100 × 5
  144
  100    20 × 5
   44
   40    8 × 5
    4
```

The answer is reasonable. Is it right? Check to make sure.
Multiply 128×5. Then add the remainder of 4.
Is the answer correct? Yes

```
  128
×   5
  640
+   4
  644
```

Your turn. Try these.

	a	b	c	d	e
1.	4)565 141 R1	3)824 274 R2	7)741 105 R6	5)918 183 R3	8)896 112
2.	3)755 251 R2	4)911 227 R3	5)840 168	7)762 108 R6	6)963 160 R3

Why not start thinking really big!

How many 6s in 1200? 20? 200? 2000?
If you answered 20, you were right.
But there are a LOT more than 20.
If you answered 200, you were really right.
$200 \times 6 = 1200$
If you answered 2000, you were thinking too BIG!
$2000 \times 6 = 12{,}000$

Estimate. How many 8s in 6500? More than 800 How many 4s in 2500? More than 600

How many 5s in 4100? More than 800 How many 7s in 2900? More than 400

How many 9s in 2750? More than 300 How many 6s in 4830? More than 800

How many 2s in 1652? More than 800 How many 3s in 2563? More than 800

One problem like this takes up a lot of paper. **Are you ready?**

$3\overline{)8227}$

How many 3s in 8227?
$1000 \times 3 = 3000$ Not enough.
$2000 \times 3 = 6000$ That's better.
$3000 \times 3 = 9000$ Too many!
The answer will be between 2000 and 3000.

"WOW!" That's a lot of threes.

```
  2742 R1
     2
    40
   700
  2000
3)8227
  6000   2000 × 3
  2227
  2100   700 × 3
   127
   120   40 × 3
     7
     6   2 × 3
     1
```

These problems are for very brave people only. Have a lot of paper ready.

1. $4\overline{)4558}$ 1139 R2
2. $6\overline{)7821}$ 1303 R3
3. $5\overline{)9426}$ 1885 R1
4. $2\overline{)5632}$ 2816
5. $2\overline{)1089}$ 544 R1

If you got those done, you are a hero. Want to try these?

6. $8\overline{)4347}$ 543 R3
7. $7\overline{)4927}$ 703 R6
8. $2\overline{)8641}$ 4320 R1
9. $3\overline{)3003}$ 1001
10. $4\overline{)9416}$ 2354

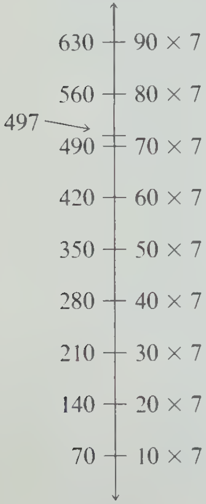
goal Practice with estimation skills and in computing 3- and 4-digit quotients

page 45 The ability to estimate is extremely important for these problems. Learners who are having difficulty should focus only on developing estimation skill. Do not expect mastery in computing quotients at this time. Praise pupils who are willing to try computing these problems. You may wish to lead the uncertain group through a few examples. Lined paper turned sideways will prove helpful on these ridiculously long problems.

goal **Progress Check** – division skills with a 1-digit divisor

page 46 Criteria for additional help are stated on the pupil page. No additional practice is necessary for errors made in group 5.

Check the estimation skill of all youngsters who make errors. Number lines of multiples may prove helpful.



$$\begin{array}{r} 71 \\ 1 \\ 70 \\ 7\overline{)497} \\ \underline{490} \\ 7 \\ \underline{7} \\ 0 \end{array}$$

Locate 497 on the number line. The quotient is greater than 70 but less than 80.

Check also for errors in subtraction and for errors in addition of partial quotients.

You have come a long way in division. Below you will find an example of each type of division that you have done. Use these problems to check your progress.

PROGRESS CHECK

Skill		a	b	c	d
Skill: Division facts	**Group 1**	7)63 (9)	9)81 (9)	6)54 (9)	8)48 (6)
Skill: Dividing 2-digit by 1-digit number	**Group 2**	6)76 (12 R4)	5)63 (12 R3)	4)81 (20 R1)	3)45 (15)
Skill: Dividing 3-digit by 1-digit number	**Group 3**	7)497 (71)	6)612 (102)	9)990 (110)	8)648 (81)
	Group 4	5)623 (124 R3)	7)715 (102 R1)	8)968 (121)	9)949 (105 R4)

Skill: Dividing 4-digit by 1-digit number

And maybe you tackled these too.

	a	b	c	d
Group 5	8)2048 (256)	6)6720 (1120)	5)6267 (1253 R2)	4)4820 (1205)

If you only missed one or two, you should feel very proud.

If you missed more than one in group ① or group ②, you'd better get someone to help you. You need to get problems like these right or you'll be getting very discouraged.

If you missed more than one in group ③ or group ④, go back and look at your work. Can you find your mistakes? Get someone to help you if you can't find any mistakes.

46

STOP See activity 5, page 52b.

See activity 6, page 52b.

1. How many 4s in 40? in 400? in 4000?
10; 100; 1000
2. How many 8s in 160? in 1600? in 16,000?
20; 200; 2000
3. How many 2s in 40? in 400? 20; 200
4. How many 20s in 40? in 400? 2; 20

LOOK OUT!

5. How many 3s in 6? in 60? in 600?
2; 20; 200
6. How many 30s in 60? in 600? 2; 20
7. How many 12s in 24? in 240? in 2400?
2; 20; 200
8. How many 9s in 360? in 3600? 40; 400
9. Are there a hundred 20s in 2794? Yes
Think $100 \times 20 = 2000$
10. Are there a hundred 30s in 2416? No
Think $100 \times 30 = 3000$
11. Tell if the answer will be more than or less than the estimate given.

a	$1722 \div 40$	more than / less than	100	b	$4562 \div 40$	more than / less than	100
c	$1953 \div 30$	more than / less than	100	d	$6345 \div 30$	more than / less than	100

Pick the right symbol.

e $9342 \div 90$ $>$ or $<$ 100 f $2178 \div 20$ $>$ or $<$ 100
g $1586 \div 10$ $>$ or $<$ 100 h $6051 \div 30$ $>$ or $<$ 100

lesson Pages 47, 48, 49

goal Practice in estimating quotients involving multiples of 10 and 100

page 47 This page represents the key to success for dividing by a multiple of 10. The estimation skill practiced is a prerequisite. Provide additional practice for any pupils who are shaky.

goal Introduction to dividing by a multiple of 10

page 48 You really need to have a buzz session here. There are three main steps to each problem—

- Estimate the greatest multiple of 10 in the quotient.
- Compute the quotient.
- Check the quotient by multiplication.

Go right on to page 49.

EXAMPLE A

30)810 The quotient is less than 100, because $100 \times 30 = 3000$.
The quotient is greater than 10, because $10 \times 30 = 300$.
So how many 30s?

THINK $20 \times 30 = 600$
$30 \times 30 = 900$

The quotient is between 20 and 30.

How many 30s in 810?
THINK $20 \times 30 = 600$
How many 30s in 210?
THINK $7 \times 30 = 210$

```
     27
      7
     20
30)810
   600
   210
   210
     0
```

CHECK

```
  30
× 27
 210
 600
 810
```

EXAMPLE B

20)1380 Is the quotient between 10 and 100? Yes
How many 20s? Estimate.
$40 \times 20 = 80$
$50 \times 20 = 1000$
$60 \times 20 = 1200$
$70 \times 20 = 1400$ Too big!
The quotient is between 60 and 70.
Copy the problem and finish it.

How many 20s in 1380?
THINK $60 \times 20 = 1200$
How many 20s in 180?
THINK $9 \times 20 = 180$

```
      ▮▮   69
       ▮   9
      60
20)1380
   1200
    180
    ▮▮▮   180
      ▮   0
```

Check your answer with multiplication.

48

EXAMPLE C

$70\overline{)3426}$ Is the quotient between 10 and 100?

THINK How many 70s in 3426?

$20 \times 70 = 1400$ Way too small.

$40 \times 70 = 2800$ That's better.

$50 \times 70 = 3500$ Too big.

The quotient is between 40 and 50.

How many 70s in 3426?

THINK $40 \times 70 = 2800$

How many 70s in 626?

THINK $8 \times 70 = 560$

How many more 70s?

What is the remainder?

```
     48 R66
      8
     40
70)3426
   2800
    626
    560
     66
```

CHECK

```
    70
  × 48
   560
  2800
  3360
+   66
  3426
```

Your turn. Divide and check.

1. $30\overline{)510}$
2. $60\overline{)3660}$
3. $40\overline{)2640}$
4. $80\overline{)3920}$
5. $70\overline{)4970}$
6. $50\overline{)5050}$
7. $20\overline{)2000}$
8. $40\overline{)8080}$
9. $80\overline{)6488}$
10. $30\overline{)3933}$
11. $60\overline{)1206}$
12. $90\overline{)2709}$

Here is another time when remainders in division problems have to be treated with common sense.

A bus could hold 40 passengers.

80 wanted to go. How many buses?
90 wanted to go. How many buses?
160 wanted to go. How many buses?
200 wanted to go. How many buses?

goal Practice in dividing by a multiple of 10

page 49 Continue from page 48 through example C. Make sure that pupils understand how to check a quotient having a remainder.

Have pupils work in teams of three to do at least some of the problems. One team member does the estimating, the second does the actual computation, and the third completes the check. Change roles for the next problem, and so on. You will find that some good peer tutoring results. Everyone needs to establish confidence now so that mastery can come later.

There is another real-world situation on the page where the remainders pose a special problem. Talk about it.

lesson Pages 50, 51

goal Applying multiplication and division skills

page 50 The page is self-directive and can be used whenever time permits or additional practice is needed. Remember it for capable students while you are involved with those who need additional help. You may wish to make your answer key available so that these pupils can correct the page independently.

Be an operator.
Copy and complete this chart.

	×3	×5	×8	×9	×10	×100
4	? 12	? 20	? 32	? 36	? 40	? 400
6	? 18	? 30	? 48	? 54	? 60	? 600
7	? 21	? 35	? 56	? 63	? 70	? 700
9	? 27	? 45	? 72	? 81	? 90	? 900

There are 7 days in 1 week. Copy and complete these charts too.

Number of days	42	63	84	105	126	147	182	365
Number of weeks	? 6	? 9	? 12	? 15	? 18	? 21	? 26	? 52 R1

There are 3 feet in 1 yard.

Number of feet	3	6	12	20	50	100	500	1000
Number of yards	? 1	? 2	? 4	? 6 R2	? 16 R2	? 33 R1	? 166 R2	? 333 R1

There are 6 ounces of milk in one carton.

Number of cartons	? 1	? 2	? 4	? 8	? 20	100	150	260
Number of ounces	6	12	24	48	120	? 600	? 900	? 1560

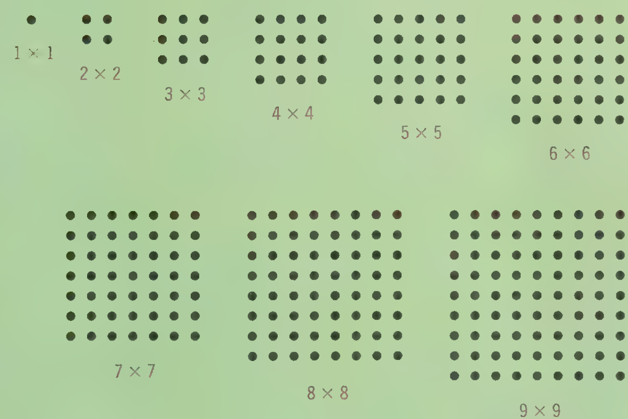
There are 12 doughnuts in 1 dozen.

Number of donuts	12	24	? 60	120	? 180	? 240	300	600
Number of dozen	? 1	? 2	5	? 10	15	20	? 25	? 50

50

Just for fun

The diagonal on your multiplication table contains the products 1, 4, 9, 16, 25, 36, 49, 64, and 81. What pairs of factors give these results? These products are called *square numbers*. What might be the reason they are called square numbers? Do the following dot patterns help you to tell why? The arrays are squares.



What would be the next three square numbers? 100 (10 × 10) 121 (11 × 11) 144 (12 × 12)

51

goal Examining arrays for square numbers

page 51 Some pupils may need to refer to the multiplication chart developed on page 25 to help them understand how the SQUARE NUMBERS form a diagonal.

This is a page that all pupils can enjoy.

goal **Checkout**—multiplication and division skills developed in the chapter

page 52 Rows 1 through 4 should be completed independently. Ask your pupils to jot down answers for exercise 5, then discuss these ideas as a group.

Examine errors in an attempt to identify the pupil's trouble. Look for:

- Ability to round numbers
- Ability to estimate
- Basic facts
- Sequence of steps in an algorithm
- Addition and subtraction errors when they are steps used in the multiplication or division algorithm

An invaluable diagnostic technique is to ask the pupil to explain the steps to you. Listen for faulty thinking.

CHECKOUT

Write an estimate. Then find the exact answer. Estimated ans in parenthese

Skill: Multiplying 3-digit by 1-digit number

	a	b	c	d
1.	420 × 5	107 × 9	596 × 6	125 × 19
	(2000) 2100	(900) 963	(3600) 3576	(2000) 2375

Skill: Multiplying 3-digit by 2-digit number

	a	b	c	d
2.	424 × 48	213 × 94	178 × 82	890 × 71
	(20,000) 20,352	(18,000) 20,022	(16,000) 14,596	(63,000) 63,190

Skill: Dividing 3-digit by 1-digit number

	a	b	c	d
	*(> 90) 99	*(> 100) 102	*(> 100) 112 R4	*(> 100) 101
3.	9)891	7)714	8)900	6)606

Skill: Dividing 4-digit by 1-digit number — Dividing by multiple of ten

	a	b	c	d
	*(> 2000) 2082	*(> 400) 403	*(> 30) 32 R5	*(> 100) 103
4.	4)8328	3)1209	20)645	70)7210

Skill: Applying estimation to actual situations

5. Think of some good answers for these.

a If you buy a package of nails or a package of pins, one size will say "Approximately 100." What does that mean? Why is it printed on the package? Do you think there will be more or less than 100? How many more or less? Accept re answers.

b Cookies are many times sold by weight. Do you have any way of knowing how many might be in 1 pound? If you want to make sure you have 24 cookies, what could you do? Accept reasonab answers.

*Note: > or < symbols in division estimates are optional.

See activity 7, page 52c.

See activity 8, page 52c.

RESOURCES

another form of evaluation

for progress check–page 34

Estimate these products. Do not compute the exact product. You'll have to round the 2-digit number so that you can get an estimate without using a paper and pencil.

1. 7 × 45	350	2. 8 × 26	240
3. 1 × 14	10	4. 5 × 89	450
5. 4 × 63	240	6. 2 × 49	100
7. 4 × 51	200	8. 7 × 36	280
9. 3 × 16	60	10. 6 × 68	420
11. 5 × 38	200	12. 3 × 23	60

Round the 3-digit number to the nearest hundred, and estimate the product. Do not compute the exact product.

13. 2 × 655	1400	14. 8 × 682	5600
15. 7 × 301	2100	16. 6 × 159	1200
17. 1 × 769	800	18. 3 × 209	600
19. 7 × 891	6300	20. 8 × 786	6400
21. 4 × 287	1200	22. 4 × 854	3600
23. 5 × 170	1000	24. 3 × 927	2700

Find the exact products.

25. 990 × 5	26. 891 × 2	27. 436 × 3	28. 634 × 7
4950	1782	1308	4438
29. 147 × 4	30. 238 × 6	31. 523 × 9	32. 786 × 8
588	1428	4707	6288

for progress check–page 38

Estimate the answers. Complete the multiplication. Compare your estimates with the exact answers. Estimates in parentheses

1. 76 × 6	2. 43 × 4	3. 278 × 7
(480)456	(160)172	(2100)1946
4. 492 × 2	5. 618 × 20	6. 94 × 10
(1000)984	(12 000)12 360	(900)940
7. 145 × 40	8. 906 × 70	9. 67 × 53
(4000)5800	(63 000)63 420	(3500)3551
10. 80 × 64	11. 35 × 37	12. 59 × 28
(4800)5120	(1600)1295	(1800)1652

for progress check–page 46

	(a)	(b)	(c)	(d)
Group 1.	5)35 7	8)64 8	7)42 6	9)72 8
Group 2.	3)44 14 R2	4)25 6 R1	5)61 12 R1	6)38 6 R2
Group 3.	7)147 21	8)488 61	6)324 54	9)945 105
Group 4.	6)545 90 R5	8)264 33	9)353 39 R2	7)788 112 R4
Group 5.	4)4048 1012	8)6488 811	9)3215 357 R2	7)1267 181

for checkout–page 52

Write an estimate. Then find the exact answer.
Estimates in parentheses

1. (a) 326 × 7 (2100) 2282 (b) 750 × 4 (3200) 3000
   (c) 203 × 8 (1600) 1624 (d) 256 × 16 (6000) 4096

2. (a) 549 × 23 (10 000) 12 627 (b) 681 × 45 (35 000) 30 645
   (c) 904 × 52 (45 000) 47 008 (d) 735 × 67 (49 000) 49 245

3. (a) (>90) 92 4)368 (b) (>100) 104 6)624
   (c) (>90) 97 R3 9)876 (d) (>90) 96 R3 5)483

4. (a) (>300) 307 7)2149 (b) (>2000) 2141 3)6423
   (c) (>20) 23 40)920 (d) (>100) 104 60)6240

5. There may be a sign on a bridge:
   Load limit 2500 kg
   What does this mean?
   Things with a total mass of 2500 kg can be on the bridge at the same time.
   What will happen if there are 2510 kg on the bridge at one time?
   The bridge is designed to bear a reasonable overage in mass.

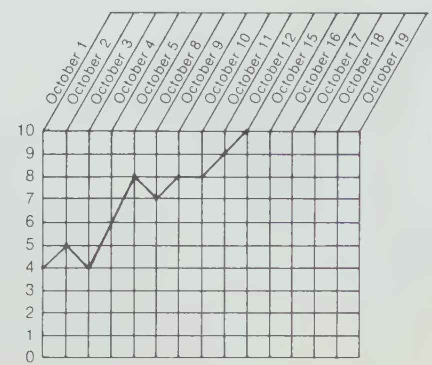
activities

1. things spirit master; graph paper

Make up 10 or 12 sets of 10 problems each. Code each set according to level of difficulty. Three possible sets are included. Use a maximum of 1 set of problems a day. Have the pupil maintain a graph of his progress.

GRAPH OF MY PROGRESS

NAME:____________________



Set A

36	57	99	40	72
× 3	× 6	× 9	× 5	× 6
108	342	891	200	432
(120)	(360)	(900)	(200)	(420)
123	800	333	321	704
× 4	× 2	× 3	× 2	× 3
492	1600	999	642	2112
(400)	(1600)	(900)	(600)	(2100)

Set B

356	572	666	821	729
× 20	× 40	× 30	× 10	× 40
7120	22 880	19 980	8210	29 160
(8000)	(24 000)	(21 000)	(8000)	(28 000)
46	54	22	37	76
× 23	× 45	× 22	× 45	× 30
1058	2430	484	1665	2280
(1000)	(2500)	(400)	(2000)	(2400)

Set C

243	342	432	765	567
× 8	× 8	× 8	× 4	× 4
1944	2736	3456	3060	2268
(1600)	(2400)	(3200)	(3200)	(2400)
231	562	921	163	628
× 12	× 12	× 24	× 6	× 14
2772	6744	22 104	978	8792
(2000)	(6000)	(18 000)	(1200)	(6000)

2. Individual activity (Provide the pupil with the following directions.)

Chance of a lifetime.
Buy your own business for $2500.
Various investment plans available.
Write Box 456.

You decide to answer the newspaper ad shown above. These three investment plans are presented to you.

Plan A – Invest $475 for 5 years
Plan B – Invest $325 for 8 years
Plan C – Invest $750 for 3 years

Which plan is the best one to use if you want to buy a business? Can you think of a better investment plan? What is it?

3. things pictures showing a large number of objects such as people in a big crowd, cars on a busy freeway, a group of many animals

Display the pictures and challenge your students to write an estimate of how many there are in each picture on a slip of paper. Labeling the pictures in some way will help keep the record of the estimates straight.

Use the sample method to find a more accurate estimate and to help decide whose estimate is the closest. Have someone mark off a small square on the picture, then count the number of whatever is shown in the square. Next estimate how many squares are in the picture. Finally multiply the number of squares by the number of objects in the sample square.

The size of the picture and the density of the objects will determine the multiplication skill to be practiced.

4. Individual projects (Provide the pupil with the following directions.)

things grass lawn

Grass study. Count the number of grass plants in one square unit – a square inch or a square foot – of the school's lawn. Determine the number of square units in the entire lawn. Estimate the number of grass plants in the entire lawn.

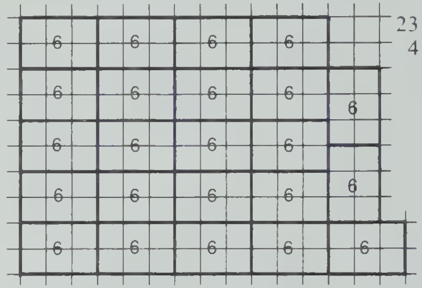
things water faucet

Drippy faucet study. Observe the water dripping from a faucet. Count the number of drops that drip in one minute. Estimate how many drops will fall in one hour, one day, and one week.

5. things graph paper

Use this activity with pupils who seem to lack understanding of what is happening in division. Be careful to select reasonable problems. On graph paper, have the pupil first make a border around sets of squares, each set containing as many squares as the divisor. Finally have him record the answer.

For example: $6\overline{)142}$



23 sixes
4 left over

6. Individual projects (Provide the pupil with the following directions.)

things milk carton or metal can; box; weight scale; water

Water-weight study. Find a rectangular container such as a milk carton or metal can (empty duplicating-fluid can) and a large box. Decide how many of these containers could fit inside the box. Weigh the container. Fill the container with water and weigh it again. How much does just the water weigh? How much would a "box full" of water weigh? Now find the weight of a "cupboard full" of water.

things books; weight scale

Words-per-book study. Choose a book. Count the number of words in 10 lines of the book. Count the number of lines on the page. Use this information to estimate the number of words on each page of the book. Then estimate the number of words in the entire book. Now weigh the book. Estimate the number of words in a pound of the book.

7. things spirit master

Prepare a spirit master as shown.

A MESSAGE TO THE BABYSITTER

Problems	$\begin{array}{r}49\\ \times\ 7\\ \hline\end{array}$	$\begin{array}{r}188\\ \times\ 5\\ \hline\end{array}$	$\begin{array}{r}4987\\ \times\ 2\\ \hline\end{array}$			
Answers	3 4 3	9 4 0	9 9 7 4			
Message	D O _ T _ A T T H E _ O L D F I S _					
Answers				6 4 1	3 5 8	8 7
Problems				$3\overline{)1923}$	$4\overline{)1432}$	$12\overline{)1044}$

A MESSAGE TO KIDS WITH MUDDY BOOTS

Problems		$\begin{array}{r}182\\ \times\ 5\\ \hline\end{array}$	$\begin{array}{r}26\\ \times 4\\ \hline\end{array}$	$\begin{array}{r}1987\\ \times\ 2\\ \hline\end{array}$		
Answers		9 2 0	1 0 4 3	9 7 4		
Message	_ O N _ WA _ K O N T _ E _ A L _ S					
Answers	3 4 3				2 0	1 1 8
Problems	$24\overline{)8232}$				$8\overline{)160}$	$16\overline{)1908}$

Pupils compute the problems, writing the answers where indicated. Each digit corresponds to a letter from the letter selector. (Some letters have already been picked.) They choose letters to complete the message. Each digit provides 2 or 3 choices of letters, one of which will make sense. Then the letters have to be split to form words.

LETTER SELECTOR

0	1	2	3	4	5	6	7	8	9
A	B	C	D	E	F	G	H	I	J
K	L	M	N	O	P	Q	R	S	T
U	V	W	X	Y	Z				

8. things spirit master

Duplicate the letter selector given in activity 7. Challenge the youngsters to make their own messages to decode, using the following steps:

- Write a message.
- Find the digits from the letter selector that correspond to the letters in the message.
- Make up problems that produce these digits.
- Erase some of the letters in the message.

Caution the pupils to begin with simple problems and work their way up. The messages can be exchanged for classmates to solve, or reproduced on an index card and put in a problem box to be used at another time.

additional learning aids

concept—chapter objective 1

SRA products

Computapes, SRA (1972)
  Module 4, Lesson: MD 32
Mathematics Involvement Program, SRA (1971)
  Card: 145
Skill through Patterns, level 5, SRA (1974)
  Spirit masters: 48, 49, 50, 51, 52

operation—chapter objectives 2, 3, 4, 5, 6

SRA products

Mathematics Learning System, Activity Masters, level B, SRA (1974)
  Spirit masters: W-5, 6, 7, 9, 13, 14, 19, 20, 25, 26, 30; P-3, 6, 10

Arithmetic Fact Kit, SRA (1969)
  Multiplication cards: 2–29
  Division cards: 2–27
Computapes, SRA (1972)
  Module 3, Lesson: MD 11
  Module 4, Lessons: MD 21, 22, 24, 29, 34
Computational Skills Development Kit, SRA (1965)
  Diagnostic Tests: 3, 4
  Multiplication cards: 14, 15, 16, 22, 23
  Division cards: 1–8
Cross-Number Puzzles (Whole Numbers), SRA (1966)
  Multiplication cards: 14–16
  Division cards: 1–9
diagnosis: an instructional aid—Mathematics Level B, SRA (1972)
  Probe: M-3
Math Applications Kit, SRA (1971)
  Appeteasers cards: 1,2
  Science cards: 7,8
  Everyday Things cards: 15,20
Mathematics Involvement Program, SRA (1971)
  Card: 126
Skill through Patterns, level 5, SRA (1974)
  Spirit masters: 23, 24, 28, 29, 31, 32, 36, 37, 41, 45, 46, 47

other learning aids (described on page 72j)—Dividing Machine, Good Time Mathematics, Napier's Rods, Numble*, Orbiting the Earth (multiplication and division), Prime-O, Rally with Remainders, Veri-Tech Senior (multiplication book), Winning Touch

*Trademark of Sigma Scientific, Inc.

3 GEOMETRY

before this chapter the learner has—

1. Traced 3-dimensional shapes to form 2-dimensional figures
2. Made a movable model of a figure by tracing and tested whether a second figure is congruent
3. Used a right-angle model to test the corners of a figure

in chapter 3 the learner is—

1. Identifying line segments, lines, and rays
2. Identifying curves, closed curves, simple closed curves, and polygons
3. Exploring the concepts of parallel and perpendicular lines
4. Testing for congruent line segments and angles
5. Mastering the identification of various polygons by the number of sides

in later chapters the learner will—

1. Identify congruent figures by tracing and matching
2. Find line(s) of symmetry by folding

Notes & Things

Our world is three dimensional. It provides the reference for the study of shape, size, and space. Up to this point, the learner has been bombarded with experiences that were designed to develop an intuitive understanding of shape and size. The vocabulary words that have been introduced were based on an obvious need to communicate ideas about shape and size to other people. Geometry is a joy when the study is not based on the memorization of formal definitions. But the time has come in this level to piece all the geometric ideas together and get organized. Therefore there will be more specialized words than usual.

The pupil should already know the way a box, a can, and a ball feel in his hand. He should know how those shapes are alike and how they are different. He should know that a flat face can be found on the surface of a box or on the end of a can. He should also know that a flat surface can represent a plane figure, and that a plane figure may have straight or curved sides.

So far, the child's experiences have been centered on manipulating and sorting solid shapes, tracing their flat surfaces, and putting them into categories too. The only notion of point that the learner knows about is that of a sharp corner where three or more edges meet. The only line that he has experienced is the mark that has resulted from tracing.

The labels that are advanced in this chapter are much more precisely defined in comparison to all the child's earlier experiences, but the definitions need to be kept in perspective. Your role as an instructional leader is very important.

The new words introduced in this chapter will help us communicate with one another. But please help the child know that the world will still go around if a description or some appropriate hand waving is used rather than the precise geometric label.

The goal of this chapter is to explore, to examine similarities and differences, and to classify. Using all the right words will be a bonus.

Please relate the discussion to the child's world whenever possible. (And don't get discouraged if you can't convince everyone that a line goes on forever without end because real-world examples for the concept of a line are hard to find.)

You may want to take time out to see how the pupils use a ruler or a straightedge. The task of using a ruler is many times necessary, but some pupils have such a rough time with the mechanics that they forget the concept they are working with. There usually are problems in just drawing a straight line. In a group of 30 you should get at least 3 thumbprints on the first line ( ). Most pupils can't draw 5 parallel lines one inch apart without at least one going astray ( ).

There will be a great many tracing activities. This also requires a certain skill that the youngsters may or may not have. If the work is too sloppy, the child cannot benefit from the activity. Ridiculous as it may seem, you may have to plan for some tracing practice.

things

set of geometric solids or boxes, cans, and a ball
poster paint or ink pad
rods, straws, or construction toy
ball of string
tracing paper
geopaper or graph paper (see page 72g)
dictionaries
cardboard strips and paper fasteners
flat geometric shapes

For the extra activities you will want to have these things available:

- spirit masters
- geoboard and rubber bands
- flour paste
- heavy paper or cardboard
- pegboard and pegs
- flat straight-sided geometric figures (see page 72h and 72i)

prebook activities 1 or 2 days

goal Survey – pupils' background in geometry

things set of geometric solids or boxes, cans, and a ball
poster paint or ink pad

first activity Use solid geometric shapes or real-world objects. Hold up one of the shapes and ask for its name. Write responses on the board. Most responses are correct and should be listed. For example, if you hold up a cube, you might get the following responses: cube, block, square, rectangle, box, and so on. All responses, even though not technically correct, are close enough for your purpose now. If, however, circle is a response for cube, it in no way describes the cube and has to be rejected.

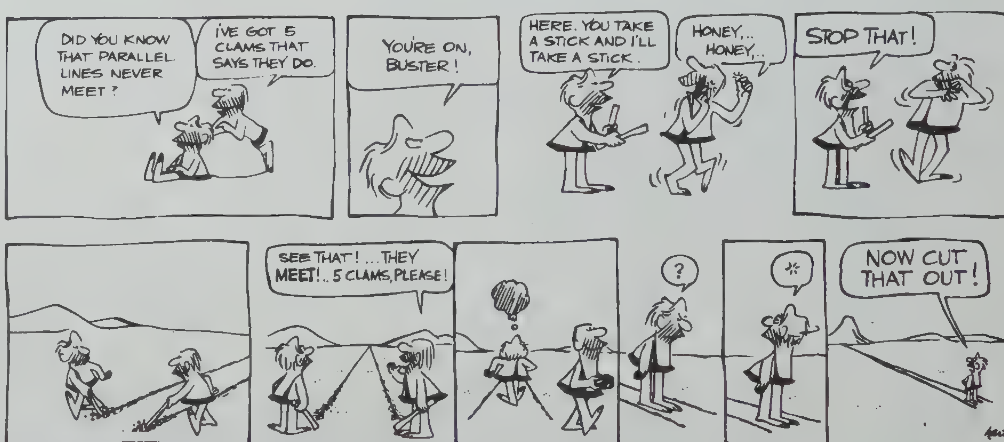
After all the names have been collected, underline the correct name for each object and explain that this is the name agreed on for this particular object.

second activity Now, ask for a list of the parts of the object. In the example of the cube, you will get responses such as: side, corner, edge, front, back, top, bottom, and so on. At this point have the pupils pretend to have or, better yet, actually have a large ink pad or pan of poster paint. Holding up a solid, ask what image would be stamped on a sheet of paper if you dipped a part of the solid (point to a surface) into paint and laid it on the sheet of paper. (Don't hesitate to ask the same question about the image the curved surface of a sphere would leave on the paper.) Then point to an edge on a prism and ask again what image the edge would leave on the paper.

Finally, point to a corner (vertex) on the prism and repeat the question. Let the pupils volunteer their own names for the images on paper, but be sure to summarize by saying that the following specific labels for those images have been agreed upon:

- Flat surfaces and/or curved surfaces are found on any geometric solid.
- Flat surfaces are also called faces (the ink blot pictures a plane figure).
- A straight edge is formed when two faces meet (the ink blot pictures a line segment).
- A vertex is formed when three edges meet (the ink blot pictures a point).

Encourage the use of correct labels but don't insist upon them at this stage of learning.



By permission of John Hart and Field Enterprises, Inc.

goal Think about and explore ideas through a picture clue

page 53 A steel superstructure is an everyday sight. Geometric ideas were used to build magnificient structures in ancient Egypt, Mexico, and Greece. The ideas are still used today. The purpose of this photograph is twofold – to explore the geometric ideas used in the superstructure that is pictured, and through exploration to increase awareness of the geometric ideas used in buildings being constructed.

Get the youngsters thinking with questions such as these. No matter what kind of building is being built, what is the first part constructed? Why is a foundation necessary? If the building is to have a rectangular shape, what construction comes after the foundation is prepared? What would happen if the upright (vertical) frame were not straight? How does the construction worker make sure it is straight? Are any two vertical supports the same distance apart? Are the horizontal pieces the same distance apart? What would happen if they weren't? What shapes do you see in the superstructure in the photograph? Are all tall buildings built the same way?

Now is the time to start each person on a personal research assignment. Let individuals choose whether to do a sketchbook filled with buildings from the community or a collection of pictures from magazines and newspapers of different types of buildings. This will prove to be a rewarding assignment and you will have a source of real things that apply the geometry to be presented in the chapter.

lesson Pages 54, 55

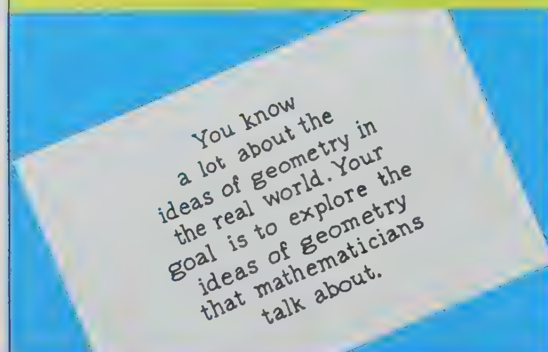
goal Introduction to the concepts of point and line segment

page 54 The first two of lots of new words and ideas are introduced on this page. The text is complete enough for the independent reader to go through without any trouble. But sharing thoughts and talk about this page is a rewarding experience.

Suppose you dipped a sponge ball in ink. If you touched it to a piece of paper, what kind of mark would it make? Point (dot)

If you rolled the ball a little way, what kind of mark would it make? Line segment

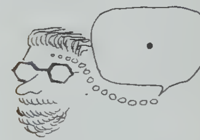
A



54

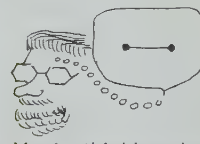
Imagine a ball such as that rolling straight, never starting or stopping, and never running out of ink. What kind of mark would it make? Line

Can you imagine nothing? It's pretty hard, isn't it?



Now try to think of just one point in space. That's easy, isn't it?

Think of another point near the first one. (Dots can be used to show points.)



Now think of these two points and a path going straight from one to the other. You're thinking about a figure called a *line segment*. The points at its ends are called *endpoints*.

To help us talk about a line segment, we usually give letter names to its endpoints—*A* and *B*, for example.

A *B*

Then we name the line segment by naming its endpoints. The diagram shows *line segment AB*.

Do you know how many points are in a line segment?

A B

Point C is halfway between point A and point B.

A E D C B

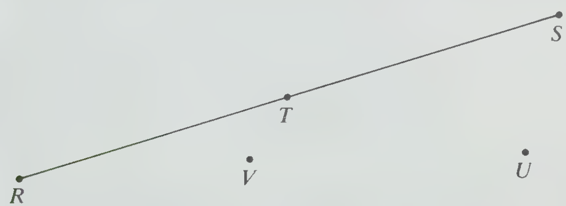
1. Copy and label points A, B, and C.

2. Now mark a point halfway between point A and point C. Label this point D. Mark a new point halfway between point A and point D. Label it E. See if you can mark another point halfway between point A and point E.

3. If you had a very sharp pencil and a microscope, could you keep on marking these halfway points? You can also mark halfway points between C and B, and so on. How many points do you think there are in a line segment? Could you mark all the points with dots?

Yes, for a while; infinite number; no

4. A line segment consists of two points and all the points between them.



a) Do you think point T is on line segment RS? Yes
b) Is point U? No
c) Is point V on line segment RS? No

55

goal Examining some characteristics of a line segment

page 55 Some of the ideas presented here could be too abstract for learners whose previous experience has been only with concrete objects in real-world settings. Not being able to see or count the points on a line segment bothers some children (and some adults too). If this happens, extend problem 3 by having the youngsters continue to mark the point halfway between two points. Soon the entire segment will fill in and the points will no longer be countable.

The emphasis of this lesson is that a line segment is made up of two endpoints and an uncountable number of points **between** them.

Have pupils name examples of line segments and their endpoints that are in the classroom — the edges of the chalkboard, the door frame and window frames, the edges of a desk or table, and so on.

See activity 1, page 72a.

See activity 2, page 72a.

lesson Pages 56, 57, 58

goal Introduction to the concept of ray

things ball of string

warm-up Have each pupil hold a pencil between his index fingers. (Colored rods or sticks would make better models than a pencil if any of them are available.) We've been talking about line segments. Can the pencil represent a line segment? (Fingers are the endpoints.)

Now take a ball of string. Have one pupil hold the end of the string and stand somewhere in line with the door. Have another pupil take the ball of string and start walking—right out of the classroom. Talk about how far this youngster can go, where he may have stopped, whether a wall could be in the way, and whether the ball of string shows a line segment.

page 56 You will want to talk about this page together. Ask the youngsters whether they can think of something other than that ball of string that starts at one point and heads in a straight line forever. This will be difficult for them and may require some hints. There are just a few examples of rays in our universe, and they probably have endpoints. We just want pupils to visualize the concept of rays. Possible examples are: a beam of light (flashlight, lighthouse), a laser beam, a radio wave, the path of a rocket ship shot into space.

1

Look at line segment AB and point C.
Point B is between point A and point C.

a Can you extend the line segment to get a line segment whose endpoints are A and C? Yes

b Of course you can. Now suppose you mark point D so that point C is between point B and point D.

c Could you extend the line segment again so that the endpoints are point A and point D? Do you think that Yes you could keep on extending the line segment in this way? Yes

2

In the diagram above, the arrowhead shows that the figure extends without end. This figure is called a *ray*.

a Look at the following ray: 

This ray starts at point J, passes through point K, and continues without end. Point J is thought of as the "front end" point of the ray. But it is simply called the *endpoint*.
How many endpoints does a ray have? 1

b What is the endpoint of this ray? S

The ray shown at left is called *ray ST*. We always say the name of the endpoint first.

56

What is the endpoint of ray AB? A Ray AB begins at A, passes through B, and continues without end.

B is also the endpoint of a ray that passes through point A and continues without end.

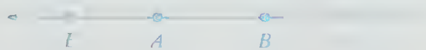
Now consider the following diagram.

Point B can be thought of as the endpoint of two rays that go in opposite directions.
We call this set of points a *line*.

Here's another way to think of a line.

Start with line segment AB.
Now extend it so that its new endpoints are point C and point E.

Could you keep on extending the line segment to new endpoints? Could the line segment be Yes extended in both directions without end? Yes



We name a line after any two points on it. For example, this diagram shows line RS or line SR or line ST.
The order of the letters doesn't matter in naming a line.

We have seen several pictures of lines.
Is it possible to draw an entire line?
Not really—because a line goes on without end.
We indicate this idea with arrowheads.

57

goal Introduction to the concept of line

things ball of string or construction toy

warm-up A line can be simulated by rolling string so that there is a ball at each end. ●——● Select two pupils. Have them walk in opposite directions. Another idea is to fit together rods and connectors from a construction-toy set. Remember to put arrowheads at each end. ←●—●—●→ Can anyone describe this model? After the discussion, refer the group to page 57 to discover more about the model just illustrated.

page 57 Emphasize that ray AB and ray BA are **not** the same ray—they go in opposite directions. The endpoint of a ray tells where the ray begins. The second point determines the direction. The order in which the letters are given is very important when naming a ray. (This is not true in naming a line.) Some good discussion is possible with this page.

goal Practice with points, lines, line segments, and rays

page 58 Problems 1 through 10 should be completed individually. The page has some good questions for the youngsters. Have an overhead projector ready so that the pupils can share the ways they resolved questions 8 through 10. Encourage more than one pupil to show his solution and talk about how he tackled the problem.

Points you may wish to summarize from the discussion are—

- Many lines can go through one point; only one line can pass through two points.
- At least two points must be identified on a ray—the endpoint and one point to give the direction.
- One line segment can be part of another line segment (problem 9).
- Two rays can have the same endpoint (problem 10).

We draw a picture of a line to express the *idea* of a line. In the same way, we draw pictures of points, line segments, and rays to express the ideas of points, line segments, and rays.

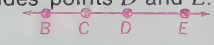
1. Name this line in six different ways.

Line *UV*, line *UW*, line *VU*, line *VW*, line *WU*, line *WV*

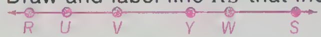
Problems 2–10: Accept other appropriate drawings.

2. Draw and label line *XZ* that includes point *R*.

3. Draw and label line *BC* that includes points *D* and *E*.

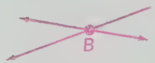


4. Draw and label line *RS* that includes points *U*, *V*, *Y*, and *W*.

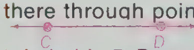


5. Draw two lines that include the same point.

6. Mark a point. Label it *B*. Draw a line through *B*. Draw another line through point *B*. Is there a limit to the number of lines that pass through point *B*? No

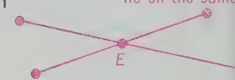


7. Mark two points. Label them *C* and *D*. Draw a line through point *C* and point *D*. If you can, draw another line through points *C* and *D*. How many lines are there through points *C* and *D*? 1



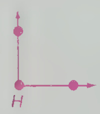
8. Mark a point. Label it *E*. Draw two line segments that pass through point *E*. Draw a line that contains both line segments, if you can.

You can't unless the two line seg lie on the same



9. Mark and label two points, *F* and *G*. Draw two line segments that pass through both point *F* and point *G*. Draw a line that contains both line segments, if you can.

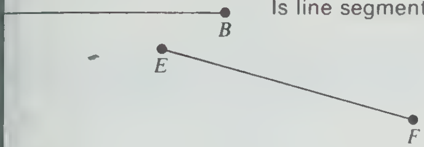
10. Mark and label a point *H*. Draw two rays that have *H* as the endpoint. Draw a line that contains both rays, if you can. (This is the only answer.)



58

See activity 3, page 72a.

See activity 4, page 72a.

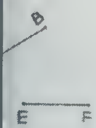


Is line segment AB congruent to line segment EF?

???What does **"CONGRUENT"** mean???

When we say two segments are congruent,
we mean that if you place one segment on the other,
the segments will match exactly.

/ do you do that?



. 1

ure 1 shows
line segments.

Fig. 2

One way to tell if they are congruent is to place a piece of paper over them.

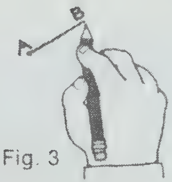


Fig. 3

Trace one of the line segments.

Fig. 4

Then turn the paper so that the line segment you traced is directly over the other.

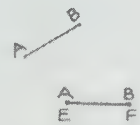
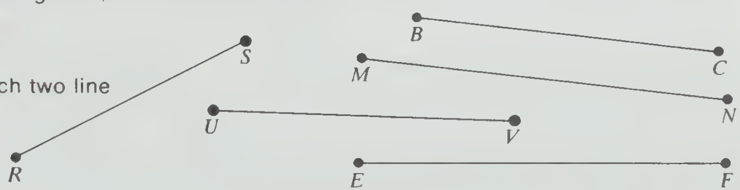


Fig. 5

This way you can easily see if the two line segments are congruent.

If two line segments are congruent, are they the same length? Yes

Determine by tracing which two line segments are congruent.
ine segments EF and MN
ine segments BC and UV



59

lesson Pages 59, 60

goal Introduction to the concept of congruent line segments

things tracing paper
rods of the same and different lengths

warm-up Distribute several pairs of rods to different pupils—some pairs with rods the same length and some with rods of different lengths. Ask how to tell whether the rods are of the same length. You should get responses such as "Measure them." or "Lay them side by side."

Draw two nonparallel line segments of similar length on the board. Ask how to tell whether these two line segments are the same length. Someone will probably reply, "Measure them." Is there another way? Continue challenging until someone thinks of **tracing.**

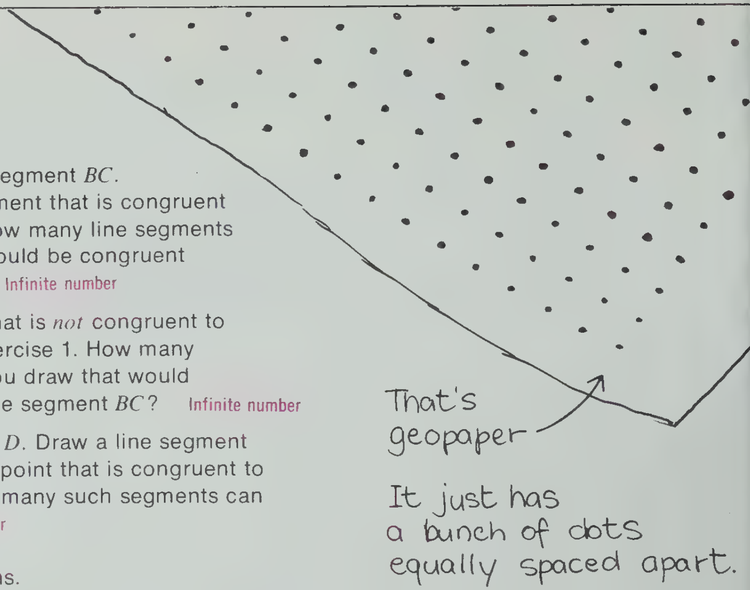
page 59 Discuss the meaning of **congruent.** You'll want to review also some hints for tracing—use soft pencils and be as accurate as possible or the matching may not be a valid test at all.

goal Experiences with congruent line segments, including applications

things geopaper or graph paper

page 60 The idea of an uncountable or infinite number of line segments is very sophisticated and abstract. The youngster who counts possible answers to questions 1 through 3 needs help in thinking beyond the space limitations of his paper.

Answers to problems 4 through 8 just have to be shared. The emphasis is on **thinking** and examining how and why people have made use of the concept of congruent line segments.



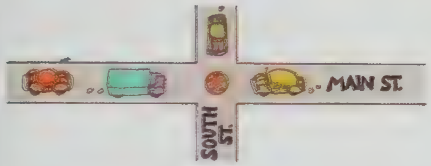
The following exercises can be done on geopaper.

1. Draw and label a line segment *BC*. Draw another line segment that is congruent to line segment *BC*. How many line segments could you draw that would be congruent to line segment *BC*? Infinite number
2. Draw a line segment that is *not* congruent to line segment *BC* of exercise 1. How many line segments could you draw that would *not* be congruent to line segment *BC*? Infinite number
3. Mark and label a point *D*. Draw a line segment with point *D* as an endpoint that is congruent to line segment *BC*. How many such segments can you draw? Infinite number

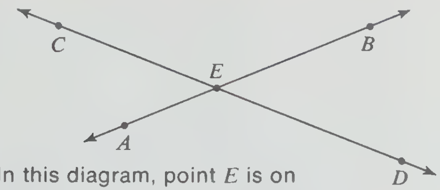
Think about these questions.

4. What would a table look like if opposite edges were not congruent? Something like this →
5. What kinds of problems would you have raising or lowering a venetian blind if opposite sides were not congruent? It could not be raised or lowered evenly.
6. Are the opposite edges of a book congruent? What makes you think so? Yes
7. What would happen if the four legs of a table were not congruent? It would tilt.
8. Could you slip the lid of a box down over the bottom of a box if the lid and the bottom were congruent? No

60



In the picture a manhole cover is in the middle of an intersection. Which street does it lie on? On both streets



In this diagram, point E is on how many lines? 2

These two lines are said to intersect.

E is the *point of intersection*.

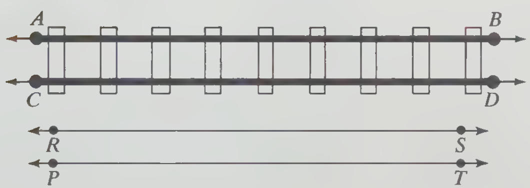
What can you think of that reminds you of two intersecting lines? Answers will vary. Examples: Railroad crossing sign, diagonal crosswalks at an intersection, piece of paper folded in fourths, etc.

Do the rails of a straight track appear to intersect? Yes

If you walked along the tracks, could you find the intersection? No

Do the rails really intersect? No

If you looked down on a railroad track from above, you might see something like this.



The rails represent two lines that do not intersect.

Line RS and line PT do not intersect.

Such lines are said to be *parallel* to each other.

61

See activity 5, page 72a.

lesson Pages 61, 62, 63, 64, 65

goal Introduction to the concepts of intersecting and parallel lines

warm-up Have the pupils draw two line segments on a piece of scratch paper. Encourage them to draw the line segments in any direction they want. Pick one example each of three different types: parallel lines, intersecting lines, and nonintersecting-nonparallel lines. Display these examples or duplicate them on a transparency. Ask whether anyone can describe how these pairs of line segments differ and how they are alike.

page 61 Here are some more new words—talk about them. Who can think of other examples of lines that intersect? Continue the discussion of parallel lines right on to page 62.

goal Examining applications of various types of line segments

page 62 Youngsters can work effectively in groups to get this page started. Have each group make a list of examples of parallel line segments and a list of intersecting line segments. After a given time, have the groups exchange lists and look for additional ideas.

Questions 1 and 2 should be completed independently. There are just too many letters to keep track of in oral work, but questions 3 through 5 are great for sharing ideas again.

Parallel lines are lines that do not intersect and are always the same distance apart.

Find things in a room that suggest two parallel lines. Answers will vary.
Examples: Frames at the top and bottom of chalkboard, edges where ceiling and wall and floor and wall join, opposite sides of this book,

Two line segments or rays are parallel if they are contained in two parallel lines. Find parallel line segments in a room. Same as above plus windows, tabletops, floor tile, etc.

1. Which line segments look parallel?
Line segments *CD* and *JK*

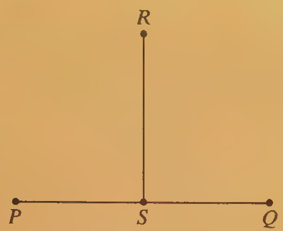
2. Which line segments intersect? Line segments *CD* and *BA*
Line segments *JK* and *BA*
Line segments *JK* and *EF*

3. Why are opposite sides of most boxes parallel?
The most efficient shape to manufacture, store, and ship

4. Can you think of a building that does not suggest at least one pair of parallel lines? Yes—a tepee, for example. And even thou pyramid or an igloo may be made of piec have parallel sides, the building itself do parallel lines.

5. Think of things of nature. What kind of lines do you see most often? parallel? straight? curved?
Leaves, flowers, pieces of bark, sun, fruits, and vegetables have curved sides.

62



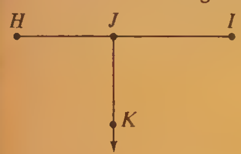
Line segments PQ and RS intersect.
What do you notice about them?

The two line segments form square corners.
These square corners are called *right angles*.

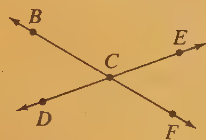
wo lines, line segments, or rays that form right angles are said o be *perpendicular* to each other. Above, line segment RS s perpendicular to line segment PQ and line segment PQ is perpendicular to line segment RS.

Vhich of the following look perpendicular?

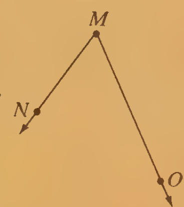
1.



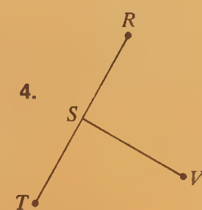
2.



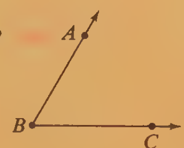
3.

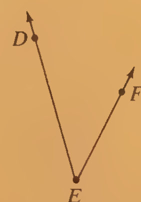


4.



5. Do angle ABC and angle DEF look like right angles?
6. Do they look congruent?





Not all angles are right angles. Some are larger than a right angle, some are smaller.

We can determine if any two angles are congruent by tracing.

7. Trace to tell whether angle ABC is congruent to angle DEF.

See activity 6, page 72a.

goal Introduction to the concept of perpendicular lines; developing a way to test angles for congruency

things tracing paper

page 63 Ask whether anyone can describe a RIGHT ANGLE. Is there such a thing as a left angle? Some pupils may already know how to form a right-angle model by folding a piece of paper once and then folding it a second time so that the first fold lies on itself. Trace a right-angle model on the board. Flip the model and continue tracing. The result should be the same as the first illustration. What kinds of angles are these?

Emphasize the word PERPENDICULAR. Even though pupils have a tough time saying perpendicular, it is a good descriptive word for ordinary use. When someone tells you that Main Street is perpendicular to Central Avenue, for example, you have a clear notion of the relationship of the two roads.

When comparing angles, stress that the opening of an angle, not the length of the sides, is what determines its size. Discuss how angles are named. What angles are formed by perpendicular line segments RS and PQ? (Angle PSR and angle RSQ) Which letter is always in the middle? (The common endpoint) The pupils will need tracing paper for some of these problems. Have them jot down answers and then exchange papers to correct one another's work. Their judgments will help reinforce their own knowledge of the ideas. If there is an argument, you might ask other pupils to help resolve it.

goal Finding congruent angles

things tracing paper

page 64 Alphabet soup again. Look at the example together. You may want the youngsters to name the various angles shown in the illustration. Doing this will help them when they must name specific angles in problem 1.

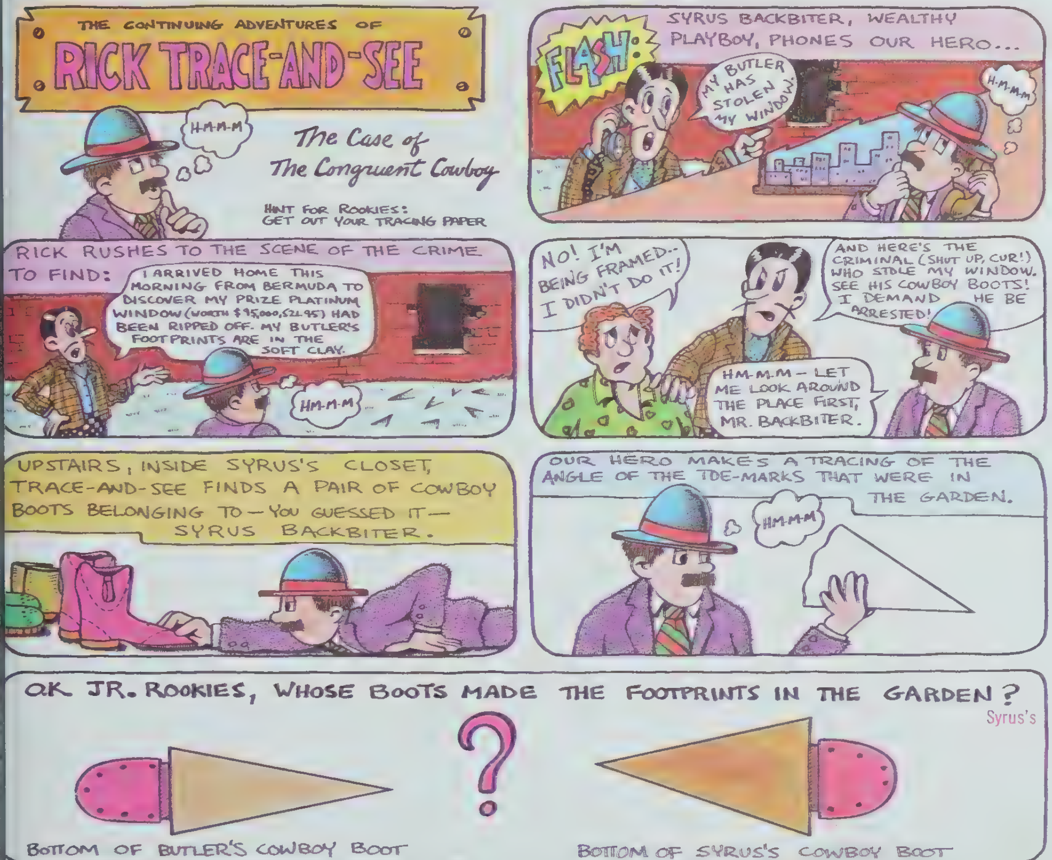
Line AB is parallel to line CD.
Line EF is parallel to line HI.
How many angles are formed by these four intersecting lines? 16

1. Trace angle AOF. Which other angles are congruent to angle AOF? Determine by tracing if there are any other congruent angles. Be sure to check all the angles. Angles *LON, OLI, MLB, CNO, ENM, NML, HMD*

2. Trace the figures. Get two pencils of different colors. With one color shade a little of the inside of angle AOF and of all the angles congruent to angle AOF. With the other color shade a little of the inside of angle FOL and of all the angles congruent to angle FOL.

   You should not need more than two colored pencils. Do you see a pattern? Try to describe the pattern.
   Angles opposite each other at an intersection are congruent.

See activity 7, page 72b.



65

goal Application of congruent angles

page 65 Strictly independent work.

Enjoy!

things pegboard and pegs or geoboard; rubber bands

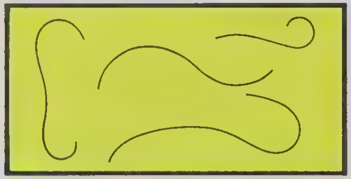
Have pupils construct models such as congruent line segments, right angles, congruent angles, parallel line segments, perpendicular line segments, and so on.

lesson Pages 66, 67, 68, 69, 70, 71

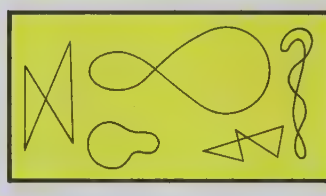
goal Introduction to the concepts of curve, closed curve, simple closed curve, and polygon

page 66 Examine sets A, B, and C. How are these sets alike? How are they different? Does everyone agree that the names for the figures contained in each set make sense? Everyone should be on his own for the rest of the page, but you may want to reinforce the definition of polygons.

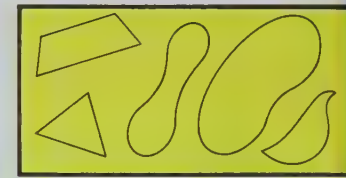
Go right on to page 67.



These sets of points are called *curves*.

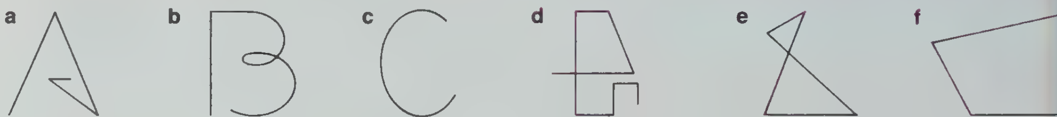


These sets of points are called *closed curves*. Why do you think they are called "closed"?
No beginning or end (no openings)



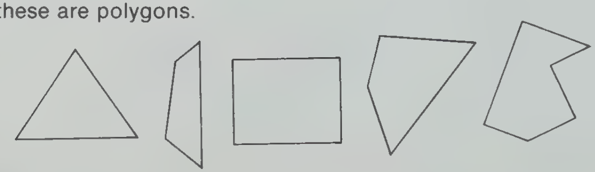
These sets of points are called *simple closed curves*. What do you think is the difference between a simple closed curve and a closed curve?
Simple closed curves do not intersect each other as do some closed curves

1. Which of the following are curves? a, b, c, d, e f

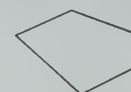


2. A closed curve is a curve that doesn't stop or start at any particular point. If you were tracing a closed curve, you'd eventually end up where you began.
Which of the curves above are closed curves? e, f

Simple closed curves made up of line segments are *polygons*. All these are polygons.



This is not a polygon.

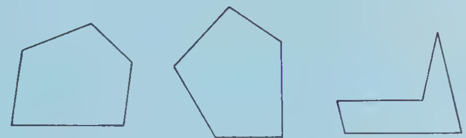


66

See activity 8, page 72b.

See activity 9, page 72b.

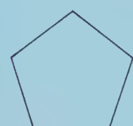
'ou might think of polygons as being a family. The ids of the polygon family are all made up of line egments. If polygons could talk and someone said Hey, polygon,'' all of them would answer. If you vanted a specific one, you would have to call it by s first name. (The first names generally give a hint bout the number of sides.)



olygons with five sides are called *pentagons*.

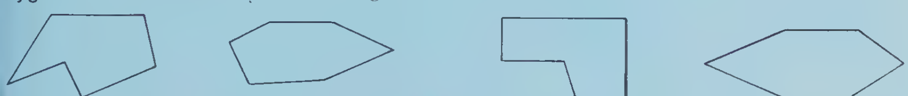
. Which of the following are pentagons? a, e

A *regular pentagon* is a pride and joy of the polygon family. It looks like this: Why do you think it is called regular?
ll sides and angles are congruent.

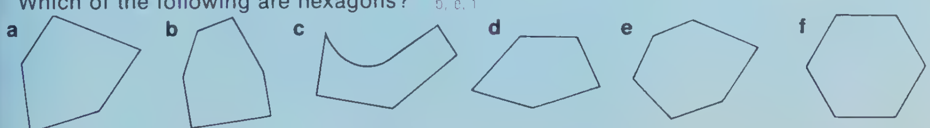


2. Determine by tracing.
Are all sides congruent? Yes
Are all angles congruent? Yes

olygons with 6 sides are called *hexagons*.



3. Which of the following are hexagons? b, e, f



67

goal Examining the characteristics of pentagons and hexagons

memo Treat the remainder of this chapter lightly. The single objective is to let the learner examine a set of simple closed curves made up of line segments—the commonly used polygons. The emphasis should be on similarities and differences, not on the memorization of names for the polygons. The child should recognize that the name for a given subset of polygons results from a common characteristic—the number of straight sides.

things dictionaries
tracing paper

page 67 Write the word PENTAGON on the board. Underline the letters **penta** and ask the youngsters to use their dictionaries and find out how many words start with those letters. What number seems to be in every definition? Write the numeral 5 beside the word **pentagon** on the board. Discuss the page through problem 2. Can anyone think of a building shaped like a regular pentagon? (The Pentagon Building in Washington, D.C.)

Back to the dictionaries. Write the word HEXAGON on the board. Underline **hexa** and use the same procedure that you used with pentagon. Then complete the discussion of the page.

things flat straight-sided geometric figures

See pages 72h and 72i.
Each pupil will need a set of figures including triangles, rectangles, and squares. Project: Try to form a pentagon by tracing some of the geometric figures. This may not be possible.

A regular pentagon will be possible only if one of the figures has a 108° angle.

Next challenge the youngsters to try forming a hexagon. A regular hexagon will be possible only if one of the figures is an equilateral triangle.

goal Introduction to quadrilaterals and parallelograms

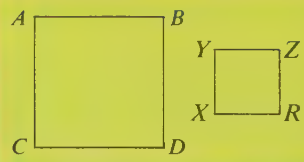
memo Do pages 68 and 69 together.

things geopaper

page 68 Review the term VERTEX and discuss how this word will help in naming polygons and angles. For problem 1 everyone will need geopaper. (This can be duplicated with a spirit master.) This problem is a good one but a hard one. Be patient.

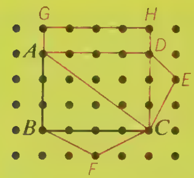
You will want to help with that word QUADRILATERAL. How are these figures alike? (All have four sides.) How are they different? You might even use the dictionaries again. Make sure that the learners understand that rectangles and parallelograms are both quadrilaterals; **but** a rectangle must have square corners, a parallelogram need not.

Go right on to page 69.



Does a square belong to the family of polygons? How do you know?
Yes; simple closed curve made of line segments

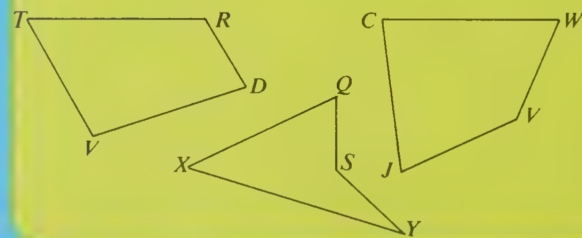
In order to talk about polygons more easily, we usually label each corner point with a capital letter. The corner point is called a *vertex* of the polygon. Now when we say that square $ABCD$ is large[r] than square $XYZR$, we know which squares we're talking about.



Do the following exercises on geopaper.

1. Start with figure ABC. Complete it so that you have each of the following:
   Accept other appropriate drawings.
   a triangle *ABC* **b** rectangle *ABCD* **c** pentagon *ABCED*
   d hexagon *ABFCED* **e** polygon *a, b, c, d, or f* **f** square *GBCH*

All four-sided polygons are called *quadrilaterals.*



2. Draw a rectangle. Is a rectangle a quadrilateral? Yes

3. Draw a square. Is the square a rectangle? Is it also a quadrilateral? Yes; yes

4. Draw a pentagon. Is the pentagon a quadrilateral? Why? No; it has 5 sides.

5. This is a quadrilateral. It is a parallelogram. Is this quadrilateral a rectangle? Why?
   No; it doesn't have square corners.

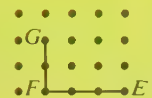
A parallelogram is a quadrilateral whose opposite sides are parallel.

Which side is opposite side BC? Side AD
Which side is opposite side BA? Side CD
Determine which sides are congruent by tracing. Sides BC and AD Sides BA and CD

6. Is this rectangle a parallelogram? Yes
Which side is opposite side KL? Side JM
Which side is opposite side KJ? Side LM
Determine which sides are congruent by tracing. Sides KL and JM Sides KJ and LM

7. Answer the same set of questions about a square.
Then try to tell how all parallelograms are alike. Square: All sides congruent, opposite sides parallel, square corners Parallelogram: Opposite sides congruent, opposite sides parallel

8. You will be using the figure on the right four times. See answers below.



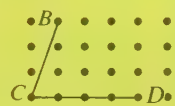
a Copy GFE. Try to draw a quadrilateral that is *not* a parallelogram.

b Copy GFE. Try to draw a parallelogram that is *not* a rectangle. Can't be done

c Copy GFE. Try to draw a rectangle that is *not* a square.

d Copy GFE. Try to draw a square.

9. You will be using the figure on the right four times also. See answers below.

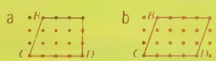


a Copy BCD. Try to draw a quadrilateral that is *not* a parallelogram.

b Copy BCD. Try to draw a parallelogram that is *not* a rectangle.

c Copy BCD. Try to draw a rectangle that is *not* a square. Can't be done

d Copy BCD. Try to draw a square. Can't be done



goal Examining some characteristics of parallelograms, rectangles, and squares

memo Please remember—the focus is on likenesses and differences, not on memorization of terminology.

things geopaper

page 69 You may want to review the definition for **parallel lines** (page 62) before having the youngsters start to work independently. The work on this page will take some good thinking. The activities will be fun if there is plenty of time for exploration and for trial and error.

goal Applications of geometric shapes

memo Do pages 70 and 71 together.

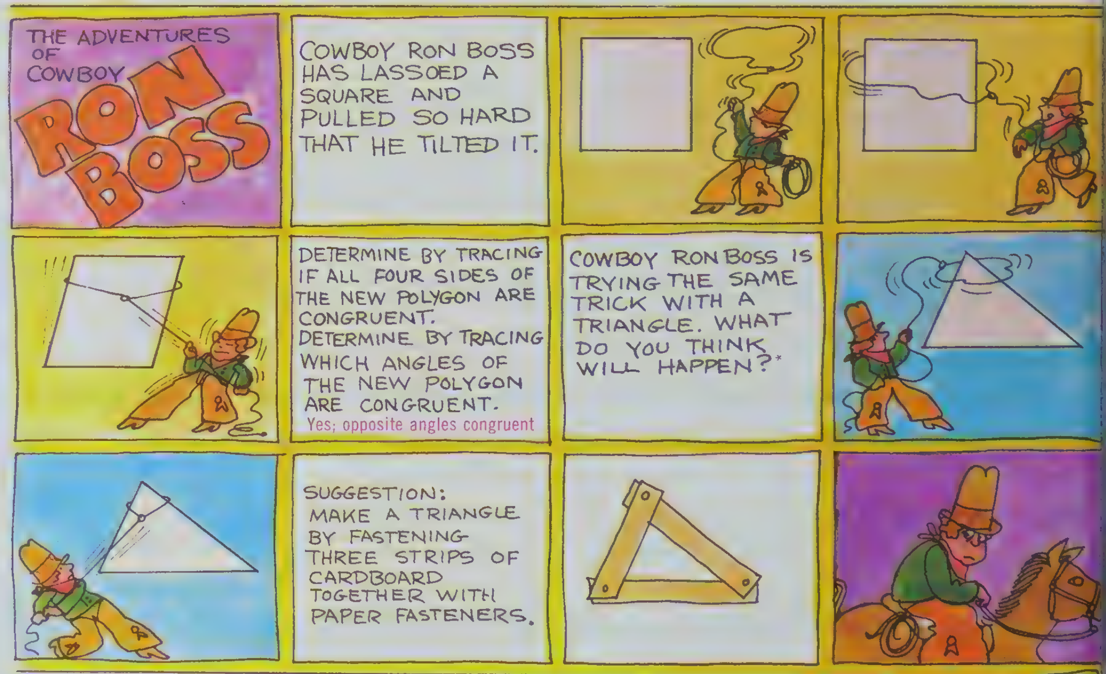
things strips of cardboard
paper fasteners

page 70 A page to enjoy! For the curious like Cowboy Ron Boss, a tilted square does have a special name—RHOMBUS.

Have pupils actually construct a triangle as directed. A square made in a similar way will quickly illustrate how easy our cowboy's job was when he tilted the square. Why are triangles used in construction work? (To make the structure rigid)

Discuss the remainder of the page. Be aware of the changes in traffic signs.

Go right on to page 71.



* Nothing will happen. It will keep its shape. (A triangle is a rigid figure.)

Look around you.

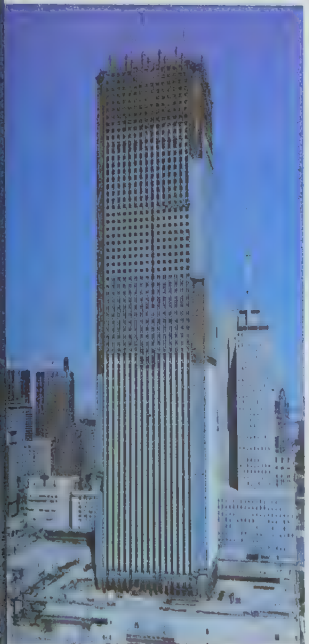
- What is the shape of most dishes? Circular
- What shape do you see most often inside a room? Rectangles (squares)
- What shape are the sides of most boxes? Rectangular
- What shapes are traffic signs? Circle, triangle, rectangle, octagon, square

70

What shapes can you find in these pictures?

Rectangles, squares, circles

Rectangles, squares, triangles



Rectangles, squares

Rectangles, squares

Rectangles, circles, triangles

71

goal Applications of geometric shapes

page 71 Have pupils sketch the geometric shapes they see in each picture first; then take time to talk about people's use of geometric shapes. Why were these shapes chosen? What are their advantages?

lesson Page 72

goal **Checkout**—identifying various polygons

page 72 Everyone on his own.

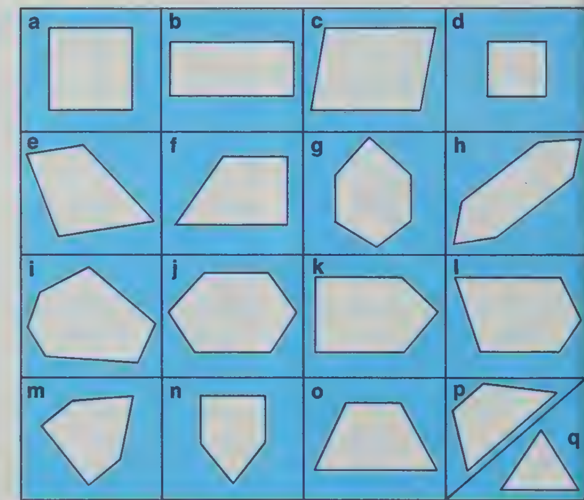
If some pupils have trouble remembering the characteristics of the different polygons, have them make a chart showing a model and the name of each type. Models can be made by flattening paper straws (hard to find sometimes) and pasting them on the chart. Let them look up the various names—the task will be more meaningful if this is **their** chart.

CHECKOUT

Skill: Identifying various polygons

Look at the figures below.

1. Which are quadrilaterals? a, b, c, d, e, f, o, p
2. Which are parallelograms? a, b, c, d
3. Which are rectangles? a, b, d
4. Which are squares? a, d
5. Which are pentagons? k, l, m, n
6. Which are hexagons? g, h, i, j
7. Which are triangles? q
8. Which are regular polygons? a, d



72

See activity 10, page 72b.

See activity 11, page 72b.

RESOURCES

another form of evaluation

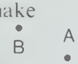
for checkout–page 72

Draw a—
1. Quadrilateral
2. Parallelogram
3. Rectangle
4. Square
5. Triangle
6. Pentagon
7. Hexagon
8. Is the figure you drew in problem 4 also a quadrilateral? also a parallelogram? also a rectangle?
9. How many polygons did you draw?

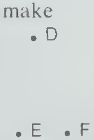
activities

1. things spirit master or activity cards

Independent activity (Duplicate with a spirit master or write on activity cards.)
1. How many line segments can you make whose endpoints are the 2 points shown? Name these line segments.



2. How many line segments can you make whose endpoints are the three points shown?
   Is point D on segment EF?
   Is point F on segment DE?
   Is point E on segment DF?



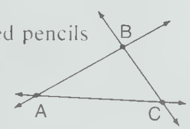
3. Make 4 points. Be sure to spread them around. How many line segments can you make?
4. Make 5 points—all spread around. Guess how many line segments you could make. Now make the segments to check your guess.

Variation: Repeat the activity, but this time make all the points in a row. Did you get the same answers?

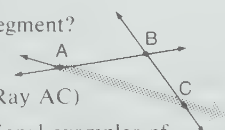
2. Extend activity 1 for capable pupils.
1. Can you predict the number of line segments for 6 points? for 10 points? for 1 point?
2. Complete this chart.

Number of points	Number of line segments
	(answers)
1	(0)
2	(1)
3	(3)
4	(6)
5	(10)
6	(15)
7	(21)
8	(28)
9	(36)
10	(45)

3. things spirit master; colored pencils



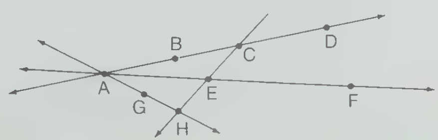
1. Shade line segment AB green.
   Shade ray BC red.
   Shade line AC orange.
   Shade ray CB blue
2. What is shaded—a line segment?
   —a line?
   —a ray?
   Name the shaded part. (Ray AC)



You'll want to include additional examples of each type.

4. things spirit master

Provide each individual or group with a copy of the figure shown or a similar figure.



1. How many different line segments can you find? What are the names of these line segments?
2. How many different rays can you find? How many rays can you find that have two names? Give both names for these rays.
3. How many different lines can you find? What are the names of these lines?

5. Individual activity (Provide the pupil with these directions.)

things centimetre rulers

1. On a piece of paper, draw a line segment.
2. Locate several points, each 2 cm from your line segment. (Some pupils will locate the points on one side only; some will use both sides. Either is correct.)
3. If you locate 100 more points, each 2 cm from your line segment, what would the figure look like? (Pupils who locate the points on only one side of the line segment will form a parallel line segment; those who use both sides will form two parallel line segments.)

6. Individual activity (Provide the pupil with these directions and questions.)

things for each pupil: 2 paper straws

1. Flatten your 2 straws.
2. Arrange your straws in as many ways as you can. On paper, sketch the ways you found. (Here are some possible arrangements. You may want to include these for the pupil.)

3. Arrange your straws to look like a capital T. Do the straws look perpendicular? What kind of angles are formed by the straws?
4. Arrange your straws to look like opposite sides of a street. Describe how the straws look. (Parallel)

5. Use the words parallel, intersecting, or perpendicular to describe these pictures of straws. You may use more than one of the words or none of them.

6. Arrange your straws in the air. Can you describe these arrangements? Can you draw them?

7. Individual activity (Provide the pupil with these directions and questions.)

things for each pupil: 3 pieces of waxed paper

1. Make 1 fold across your paper. Open the paper. What does this fold represent? (Line segment)
2. Make another fold anywhere on the paper so that this fold crosses the first fold. Open the paper. What do the 2 folds make? (4 angles) Are any of the angles congruent? Test them.
3. Use another piece of paper. Make a fold. Without unfolding the paper, make another fold so that the first fold lies on itself. Open the paper. What do the folds make? Do these angles have a special name? What about the line segments?
4. Draw an angle on another piece of paper. Fold the paper so that one side of the angle lies on top of the other side. Open the paper. What do you see? What does the fold line do to the angle?

8. things spirit master; colored pencils

Prepare a spirit master of simple closed curves similar to those shown.

Challenge the youngsters to determine whether the point shown is inside or outside the simple closed curve. Answers can be verified by shading the interior regions with a colored pencil.

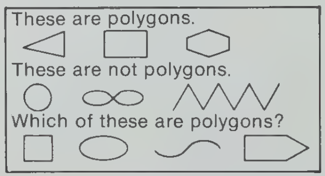
9. things geoboard; rubber bands

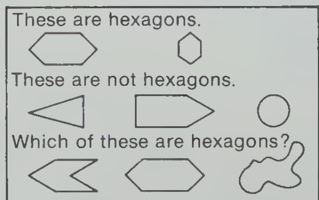
Individual activity (Provide the pupil with these directions.)

1. With a geoboard, try to make a 3-sided figure that is closed but not simple. Could you do it?
2. With a geoboard, try to make a 4-sided figure that is closed but not simple. Do you think it can be done?
3. On paper, make a 3-sided figure that is simple but not closed.
4. On paper, make a curve that is not simple and is not closed.

10. things string; paper straws; flour paste; heavy paper or cardboard

Paper straws can be flattened to form line segments. Use string dipped in paste to form curves. Have pupils work in small groups to prepare a set of cards (or a bulletin-board display).





Exchange the completed products and challenge another group to identify the appropriate figure.

11. things small cards; markers; small box

Each group needs a set of these names:

curve	polygon
closed curve	regular polygon
simple closed curve	pentagon
line	hexagon
line segment	rectangle
ray	triangle
congruent lines	square
intersecting lines	right angle
parallel lines	congruent angles
perpendicular lines	parallelogram
angle	quadrilateral

Each player folds a sheet of paper into 16 boxes and draws a figure to fit one of the descriptions given above, one figure in each box.

Mix cards. Players alternate drawing a card and reading it aloud. Anyone having an appropriate figure covers it on his individual game board with a marker. The first player to cover a row, column, or diagonal wins.

additional learning aids

geometry – chapter objectives 1, 2, 3, 4, 5

SRA products

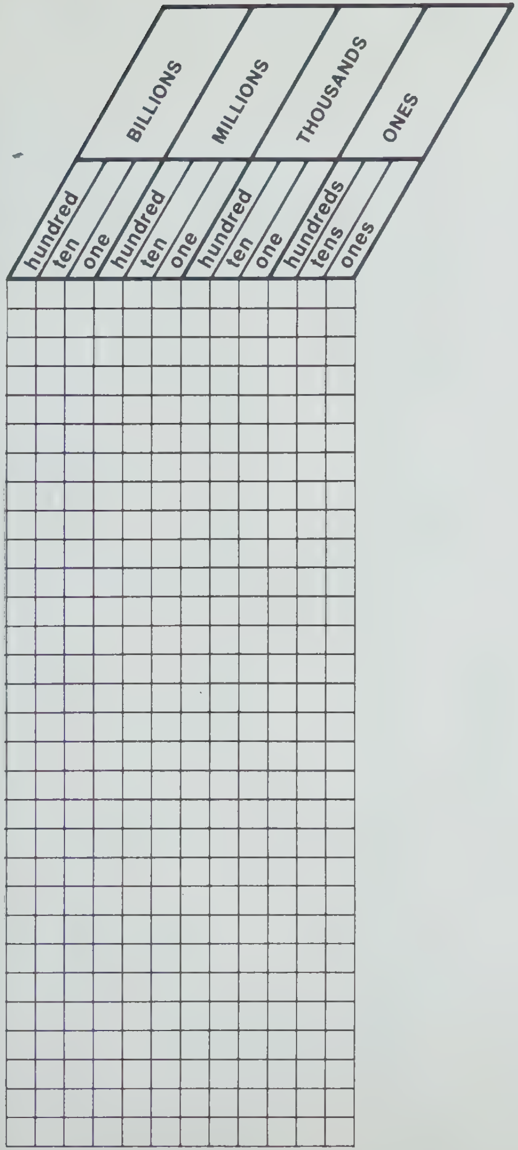
Mathematics Learning System, Activity Masters, level B, SRA (1974)
Spirit masters: G-1, 2, 3

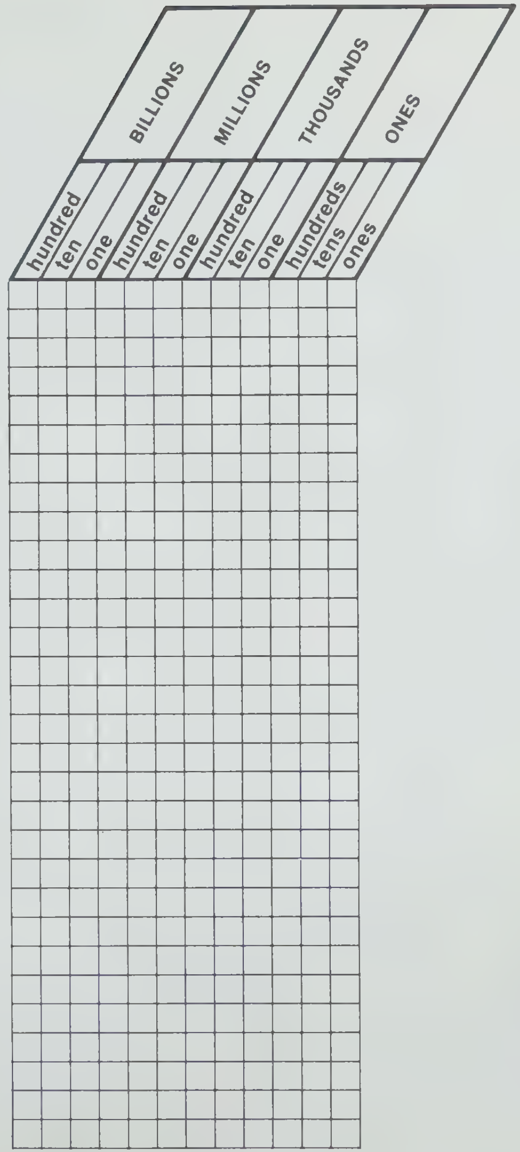
Math Applications Kit, SRA (1971)
Appeteasers card: 15
Sports and Games cards: 9, 28
Occupations card: 3

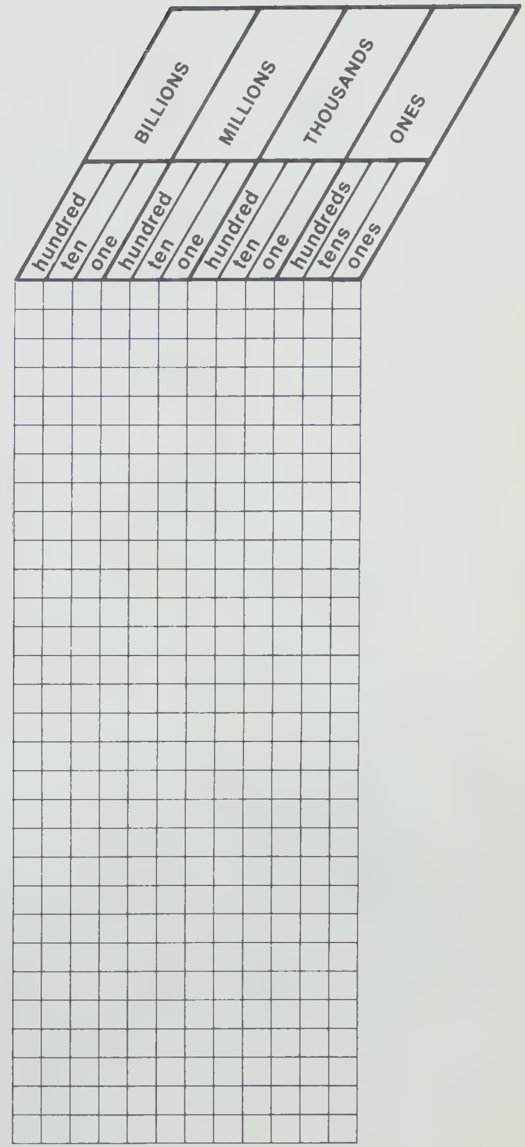
Mathematics Involvement Program, SRA (1971)
Cards: 44, 106

Skill through Patterns, level 5, SRA (1974)
Spirit masters: 6, 20

other learning aids (described on page 72j) – Geoboard Activity Cards (intermediate set), Geoboards and Motion Geometry, Good Time Mathematics, Mira, Mira Math for Elementary School, Shape Tracers


BILLIONS
MILLIONS
THOUSANDS
ONES
hundred
ten
one
hundred
ten
one
hundred
ten
one
hundreds
tens
ones


BILLIONS
MILLIONS
THOUSANDS
ONES
hundred
ten
one
hundred
ten
one
hundred
ten
one
hundreds
tens
ones


BILLIONS
MILLIONS
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No. ______ ______ 19___

Pay to the
order of ______________________

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Bogus Bank ______

No. ______ ______ 19___

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order of ______________________

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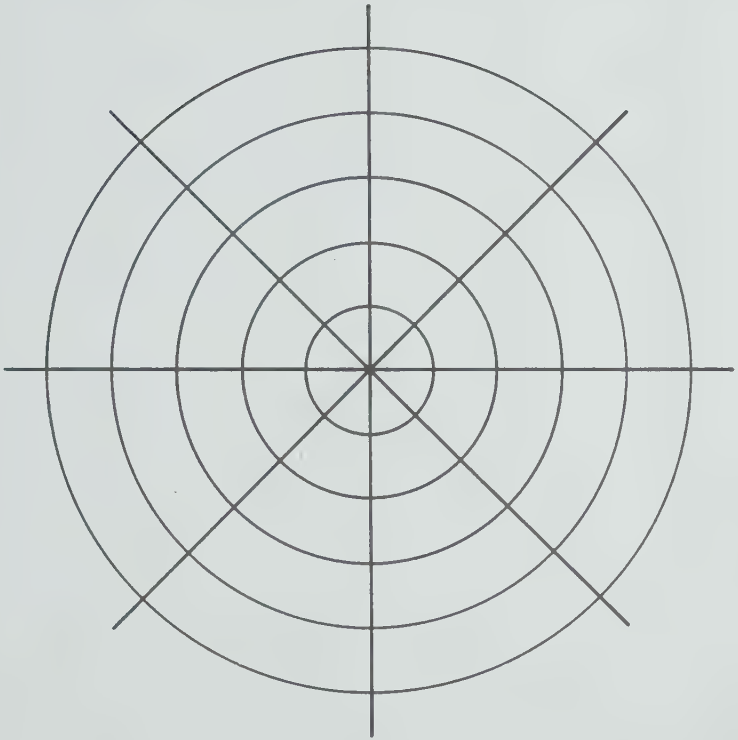
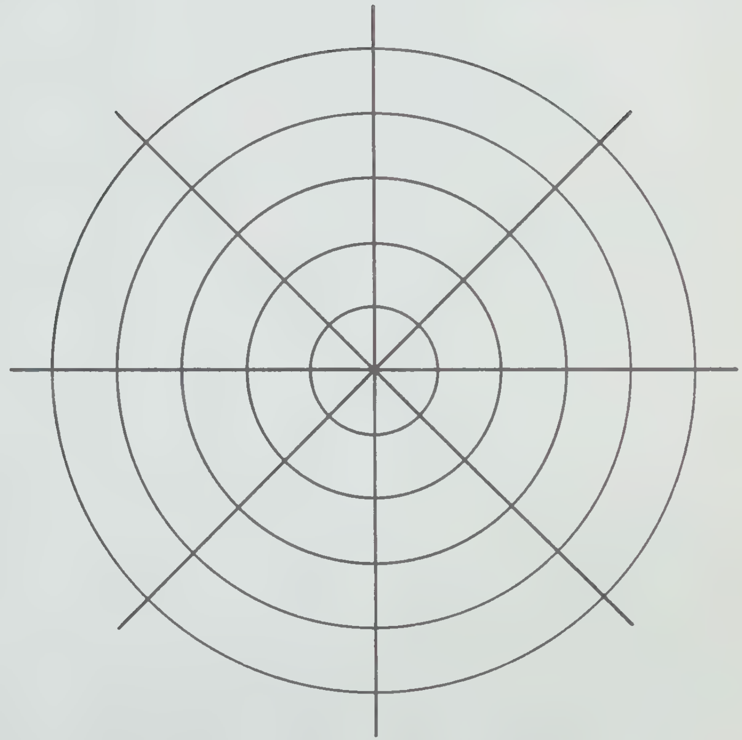
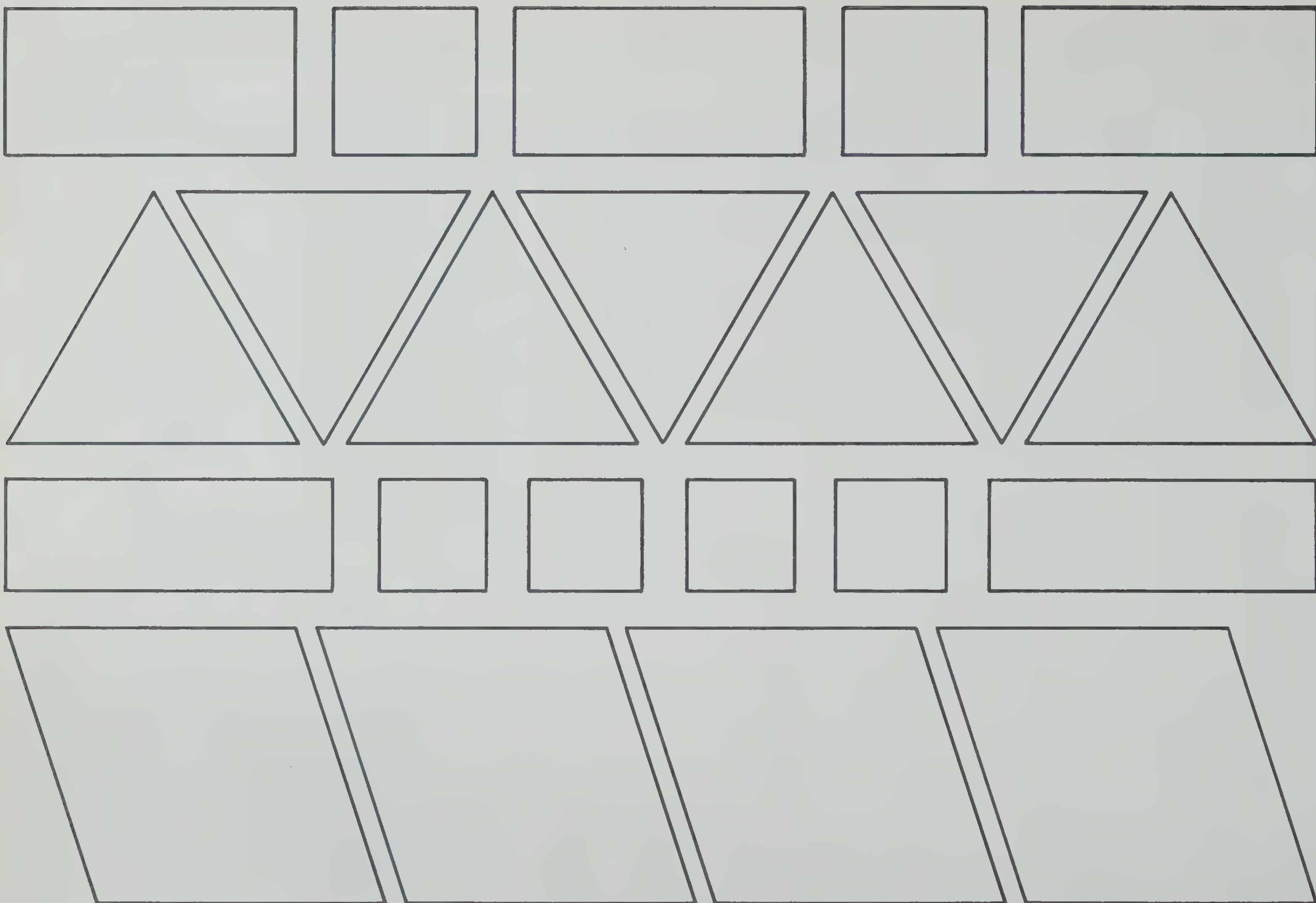
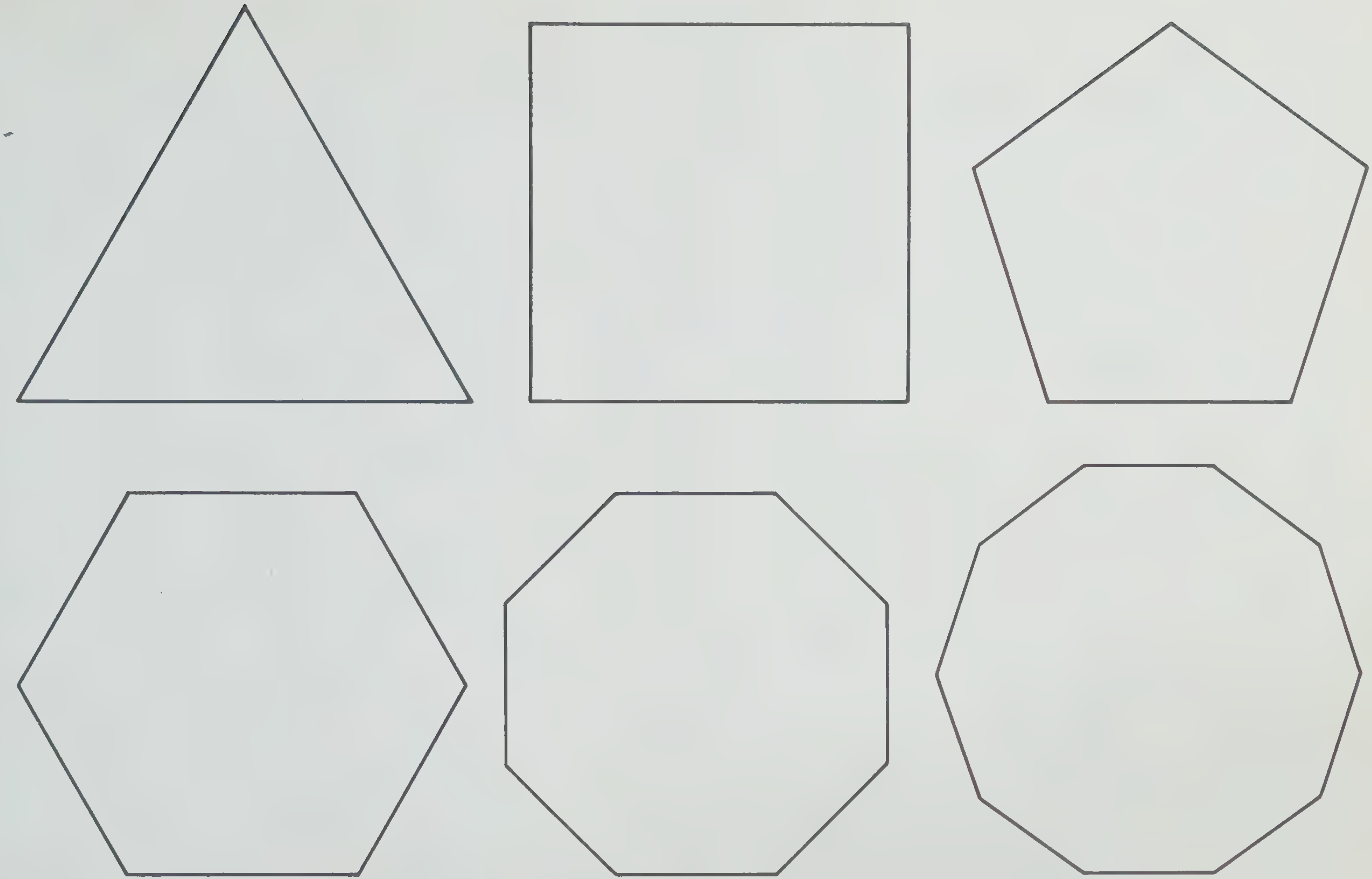




name

Build a table.





Other Learning Aids

whole-number notation

Abacus board (Creative Publications) Counting board useful for teaching place value
Chip Trading (Scott Scientific) Game to develop an understanding of place value
Place Value I and II (Creative Publications) Self-correcting cards to provide practice in reading numbers through hundred millions

whole-number operations

Dial-A-Matic Adding Machine (Sigma Scientific) A simple calculator for practice in addition and subtraction
Dividing Machine (Developmental Learning Materials) Self-checking machine to be used for practice with the division facts
Good Time Mathematics (Holt, Rinehart & Winston) A multimedia program to give activity-based learning experiences
I Win (Scott, Foresman) [sets 1, 2, and 3] Card game for practice in basic operations
Japanese Abacus (Creative Publications) An abacus for place value and basic operations
Mathfacts Games (Milton Bradley) Self-checking games for multiplication and division facts
Napier's Rods (Sigma Scientific) Rods for practice in multiplication
Numble (Sigma Scientific) Crossword-type number game for basic operations
Orbiting the Earth (Scott, Foresman) Game to provide practice in all four operations
Prime-O (Creative Publications) Card game to provide practice in prime factorization
Rally with Remainders (Math Shop) A self-correcting game providing division practice
Sequence (Math Shop) Puzzle-type game that uses addition combinations to find patterns
Ting (SEE) A jigsaw puzzle for multiplication
Triscore (Creative Publications) Games for practice in the basic operations
Veri-Tech Senior (ETA) [addition, subtraction, and multiplication books] A self-checking device that provides practice with operations
Winning Touch (Ideal) Game for reinforcement of multiplication facts

fractional-number notation

Decimal Fraction Dominoes (Mind/Matter Corp.) Game for practice in recognizing relationships of fractions and decimals
Decimal/Fraction Matching Cards (SEE) Cards to aid in learning about decimals and fractions
Experiments in Fractions (Math Shop) Activities for notation and operations
The Fat Fraction Game (ETA) Card game for drill in simplifying fractions
Fraction Bars Student Activity Book (Creative Publications) Games and activities to teach fractions and their operations
Fraction Dominoes (SEE) Game involving matching a fractional numeral with its model
Fraction Line Set (Sigma Scientific) Activity to help visualize operations by computing with fraction strips
Fraction Tally (Math Shop) Game for practice in addition and subtraction
Fractional Number Cards (Math Shop) Cards used with a geoboard for finding equivalent fractions
Tripletts (Math Shop) A rummy-type card game for identifying equivalent fractions

fractional-number operations

Action Fraction Games (Constructive Playthings) Game to develop concepts and skills
Fraction Multifax (Math Shop) Game to reinforce multiplication of fractional numbers
Mathimagination (Math Shop) [book D—Fractions] Puzzles to reinforce operations

geometry

Geoboard Activity Cards (Creative Publications) [intermediate set] Activities for the geoboard
Geoboard Kit (Cuisenaire) Plastic geoboards and activity cards that show geometric concepts
Geoboards and Motion Geometry (Scott, Foresman) Resource book dealing with congruence, coordinates, transformations, and area
Great Shapes (Cuisenaire) Game to develop principles of patterns, symmetry, and so on
Learn to Fold—Fold to Learn (Lyons & Carnahan) Workbook of paper-folding activities to demonstrate geometric figures and symmetry
Mira (Creative Publications) An aid for investigating properties of plane geometry
Mira Math for Elementary School (Creative Publications) Activities for the Mira
Mirror Magic (Lyons & Carnahan) Workbook activities for exploring the concept of symmetry
Paper and Pencil Geometry (Lyons & Carnahan) Activities to develop geometric concepts
Polygons (Math Shop) Cards for understanding of basic properties of geometric figures
Rotation, Translation and Reflection Kit (Invicta) Activity cards dealing with motion geometry
Shape Tracers (Math Shop) Set of basic geometric shapes
Tangrams (Creative Publications) Puzzle to aid in the discovery of geometric properties
Tangramath (Creative Publications) Book of tangram shapes to assist in learning concepts of shapes, congruence, similarity, and area

measurement

The Fatal Foot (Math Shop) Game for drill in addition of linear measures
Geometric Ruler (Math Shop) Folding rule to demonstrate perimeter-area relationships
The Ghastly Gallon (Math Shop) Game for practice in operations with liquid measures
Introducing the Metric System with Activities (Math Shop) Activities to develop basic understanding of the metric system
Learning about Measurement (Lyons & Carnahan) A workbook of activities using the metric and customary systems
Metric Place Value Chart (Ideal) Chart for meaning of metric system of measures
The Perilous Pound (Math Shop) Game for practice in regrouping weight measures

statistics and probability

Block Graph (ESA) Demonstration set for introducing bar graphs, averages, and so on
Histogram Board (ESA) Board for making bar diagrams—to be used with Stern Unit Cubes
Making and Using Graphs and Nomographs (Lyons & Carnahan) A workbook for skills in making and reading graphs
Probability Maze (ESA) A board to illustrate probability and statistics
Probability Set (Invicta) Apparatus to demonstrate simple probability
Towards Probability (Cuisenaire) Book of easy experiments dealing with probability

problem solving and applications

Foo (Cuisenaire) Card game for order of operation in open math sentences
Heads Up (Creative Publications) Game providing practice with equations
True or False (ESA) Game for deciding true or false statements

4 FRACTIONS

before this chapter the learner has—

1. Mastered naming a fractional part of a region, set, or number line
2. Mastered the comparison of two fractions with like denominators
3. Mastered the addition and subtraction of two fractions with like denominators
4. Renamed fractions

in chapter 4 the learner is—

1. Reviewing identification of a fractional part of a region, set, or number line
2. Comparing fractions with like denominators or like numerators
3. Renaming mixed numbers as fractions and fractions as mixed numbers
4. Comparing mixed numbers
5. Reviewing and practicing the renaming of fractions
6. Identifying fractions that are not written in simplest form
7. Renaming sums and differences in simplest form
8. Applying fractions to real-world problems

in later chapters the learner will—

1. Master the renaming of fractions
2. Use renaming in the addition and subtraction of fractions with unlike denominators

Notes & Things

The concept of fractions is not new to a level-5 student. The primary emphasis of this chapter is a comprehensive review of the concept itself. Addition and subtraction of fractions with like denominators serve to reinforce concept development.

The first major thrust in renaming fractions begins in this chapter. The work starts with equivalent fractions that come from matching parts of models. It continues to include the computational method of multiplying the numerator and the denominator by the same number to get an equivalent fraction. And it ends with dividing by a common factor of the numerator and denominator. Don't expect mastery of renaming just yet. There will be lots more practice to follow later.

Since this may be the first fractions chapter you have taught in this program, it will be wise to review some of the ground rules for this series of books. Vocabulary, for example, is always troublesome if you don't have some advanced warning, so let's start there.

The definition of a fraction and the related vocabulary are very difficult to handle if you include all the different meanings for a simple idea such as $\frac{1}{2}$. You have seen the symbol $\frac{1}{2}$ called a numeral (or fractional numeral, common fraction, and so on) that represents a fractional number (or rational number, quotient, and so on). Rather than making life difficult, we're simply going to call numbers such as $\frac{1}{2}$ *fractions.* And we won't make a big fuss about the difference between numbers and numerals.

At this level the numerator of a fraction is a number that tells how many same-size parts of a given region, set, or number line are being considered; the denominator is a number that tells how many same-size parts are in the whole thing; and the fraction (numerator and denominator together) tells *how much* of a region, set, or number line is being considered.

If you prefer to define a fraction or its parts in another way, it's O.K. The chapters have been written in a way that lets you do this. However, you should know that this series does not thoroughly investigate the fraction as a quotient until level 7.

You will find an emphasis placed on getting each pupil to see that fractions have order, just like whole numbers. If the denominators are the same, then the numerator tells the comparative size.

$$\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}, \frac{7}{7}, \ldots$$

If the numerators are the same, then the denominators tell the story.

$$\ldots, \frac{1}{8}, \frac{1}{7}, \frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{1}{1}$$

Knowledge of the order of fractions is as important as knowledge of the order of whole numbers. It is the basis for the common sense that is so frequently lacking in work with fractions.

Estimation also is as important for work with fractions as it is for work with whole numbers.

The adoption of the metric system of measure will eventually bring quite a different emphasis to the study of fractions. Decimal fractions will be much more important and will be studied earlier. But that time is not here yet. The present expectation that pupils will perform all operations with fractions is reinforced by standardized tests and by tradition. Yet the everyday use of operations with fractions is limited. This series therefore deliberately controls the set of fractions used in computational skill development and practice. Don't look for exotic fractions such as $\frac{4}{33}$. You won't find them.

You may be surprised to see so few real-world pictures used in the fractions chapters. There is a reason. The notion of same-size parts underlies the concept. Unfortunately, youngsters argue about who got the bigger half of the candy bar long before they get to the study of fractions. The region models therefore are drawn with precision to reinforce the notion of same-size parts.

things

set of cardboard fractional parts (optional) (see page 98d)

spirit master of number lines (optional) (see page 144b)

spirit master of 10-by-10 or 12-by-12 array charts (optional)

For the extra activities you will want to have these things available:

- geoboard and rubber bands
- geopaper
- tongue depressors

goal Think about and explore ideas through a picture clue

page 73 The grocery store is one place where fractions are used. Where else?

You can share ideas or you can use this simple question as another independent study topic. A notebook of personal experiences would be just fine. Or how about another collection of clippings (and labels)? Everyone may be surprised to find how often a fraction sneaks into our lives.

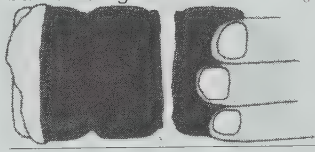
lesson Page 74

goal **Survey**—ability to name a fractional part, to rename fractions, and to compare fractions

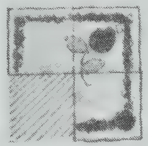
page 74 Through discussion you can identify those youngsters who are at ease in thinking and talking about fractional parts and those who are insecure.

Let this discussion be the signal of how best to handle the remainder of the page—as independent work, as further discussion, or simply to set the youngsters' learning goals for this chapter.

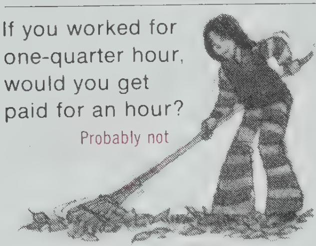
What part of the chocolate bar is being taken? $\frac{1}{3}$



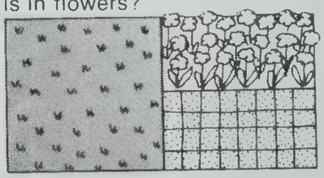
What part of the cake is gone? $\frac{1}{4}$



If you worked for one-quarter hour, would you get paid for an hour? Probably not



What part of the garden is in flowers? $\frac{1}{4}$



Fractions, fractions, fractions. You use them quite a bit. But you've got to be careful how you use them. By the time you finish this chapter, you will know the answers to these questions. Or do you know the answers now? Find out.

1. Which fraction is the the largest?

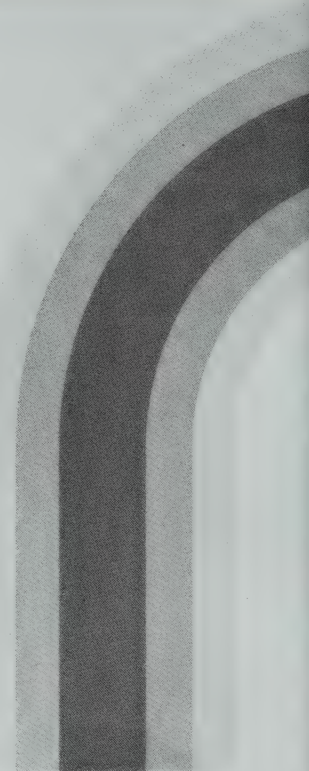
   $\frac{3}{5}$ $\frac{2}{3}$ $\underline{\frac{3}{4}}$

2. Which fraction is the same size as $\frac{3}{4}$?

   $\frac{3}{8}$ $\frac{4}{8}$ $\underline{\frac{6}{8}}$

3. What are two other fractions that name the same number as $\frac{8}{12}$? $\frac{2}{3}, \frac{4}{6}, \frac{16}{24}, \ldots$

You will learn much more than the answers to these questions.



Set your goal.
I'm going to learn about the idea of fractions.
I'm going to review how to add and subtract fractions, too.

74

You have to really think when you are working with fractions.
Pretend there are three pizzas. They are alike in every way.

You get $\frac{1}{2}$ of any one of them.
The half you take won't make any difference in the amount of food you get.

Here you get $\frac{1}{2}$.

Here you get $\frac{1}{2}$.
That is cut into two pieces.
You get $\frac{2}{4}$.

Here you get $\frac{1}{2}$.
That is cut into three pieces.
You get $\frac{3}{6}$.

BUT you had better take another look at when $\frac{1}{2} = \frac{1}{2}$.

Let's trade.
I'll trade you—

- $\frac{1}{2}$ of my allowance for $\frac{1}{2}$ of your allowance (How much is your allowance? More than mine?)
- $\frac{1}{2}$ of my lunch for $\frac{1}{2}$ of your lunch (But I don't like what's in your lunch.)
- $\frac{1}{2}$ of my dime for $\frac{1}{2}$ of your dollar (How many cents in the exchange?)

When does $\frac{1}{2} = \frac{2}{4}$ and $\frac{2}{4} = \frac{3}{6}$?
Let's trade again and find out.
I'll trade you—

- $\frac{1}{2}$ of my pint of ice cream for $\frac{2}{4}$ of your pint of ice cream (O.K., if the flavor is the same.)
- $\frac{2}{4}$ of my apple for $\frac{1}{2}$ of your apple (O.K., if the apple doesn't have a worm on one side.)
- $\frac{1}{2}$ of my 12 marbles for $\frac{3}{6}$ of your 12 marbles (Probably O.K., if yours aren't chipped.)
- $\frac{2}{4}$ of my 4 for $\frac{3}{6}$ of your 6 (How many of mine do you get? How many do I get?)

When you compare the fractional parts of the same thing, $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{3}{6}$ are equal. When you compare fractional parts of different things, then you had better be careful.

75

lesson Page 75

goal Comparing fractional parts

page 75 Fractions written on paper are abstract. Pupils learn to work with them but seldom think about what the fraction represents. The focus of this page is on **thinking**—thinking about the size, the quality, and the quantity of things that can be divided into fractional parts.

The page should provoke some interesting discussion. Keep asking your questions in a teasing manner to encourage the youngsters to think. The discussion should wind up with the generalization that when we deal only with numbers, number equality is all we have to think about. But when we compare a number of parts of a real thing, equality has a much broader meaning!

lesson Pages 76, 77

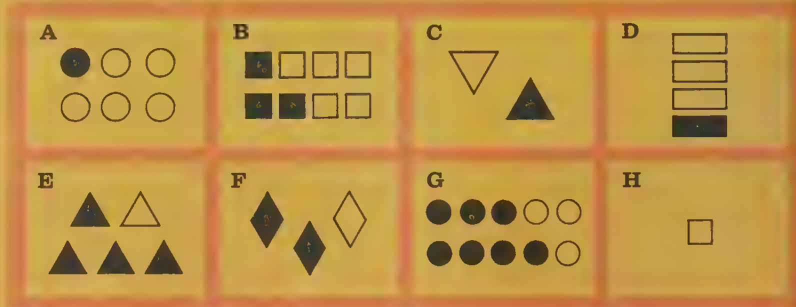
goal Naming fractional parts of a set

memo Three types of models—set, region, and number line—will be used throughout this work with fractions. The next three pages will in sequence check out the pupils' prior experiences with these models.

page 76 You may need to help with the directions, but this work should be completed independently.

This page will confirm your judgment about which individuals will need a great deal of help relating fractions to parts of a set. Any youngster who has difficulty should use counters of two different colors and explore possibilities. This child may need a great deal more practice with many different types of manipulatives.

Fractions can be shown by many different models. Here's one kind.



1 Copy and complete the chart to describe the sets.

SET	HOW MANY SHADED?	HOW MANY IN ALL?	WHAT FRACTION SHADED?
A	1	6	? $\frac{1}{6}$
B	? 3	8	? $\frac{3}{8}$
C	? 1	? 2	? $\frac{1}{2}$
D	? 1	? 4	? $\frac{1}{4}$
E	? 4	? 5	? $\frac{4}{5}$
F	? 2	? 3	? $\frac{2}{3}$
G	? 7	? 10	? $\frac{7}{10}$
H	0	? 1	? $\frac{0}{1}$

2 Draw sets of objects. Shade enough of the set to show each of the following fractions.

a $\frac{1}{3}$ **b** $\frac{3}{4}$ **c** $\frac{5}{6}$ **d** $\frac{5}{5}$

76

Get a sheet of paper. Fold it in half. Crease the fold. Open the paper. Lightly shade the left half of the paper.

Refold the paper. Fold it in half again by matching the folded edge to the open edges. Crease this fold.

a How many parts now? 4

b Draw some horizontal lines over the part you have shaded to show $\frac{1}{4}$ of the paper.

c What part of the paper is not shaded? $\frac{2}{4}$ or $\frac{1}{2}$

d What part of the paper does not have horizontal lines? $\frac{3}{4}$

e Which is more, $\frac{1}{4}$ or $\underline{\frac{2}{4}}$? $\frac{1}{4}$ or $\underline{\frac{3}{4}}$? $\frac{1}{2}$ or $\underline{\frac{3}{4}}$?

Open the paper.

Refold the paper one more time. Fold it in half again. Crease the fold. Then open the paper.

a How many parts now? 8

b Draw vertical lines over the horizontal lines to show $\frac{1}{8}$ of the paper. (It will look like a checkerboard.)

c What part of the paper is not checked? $\frac{7}{8}$

d What part of the shaded part is not checked? $\frac{3}{8}$

Turn the paper over.

a Mark $\frac{1}{2}$ on the fold line where $\frac{1}{2}$ stops and the next $\frac{1}{2}$ begins.

b Mark $\frac{1}{4}$ on the fold line where $\frac{1}{4}$ stops and the next $\frac{1}{4}$ starts. Mark $\frac{2}{4}$ where two $\frac{1}{4}$s stop and the next $\frac{1}{4}$ starts. Finally mark $\frac{3}{4}$ where three $\frac{1}{4}$s stop and the next $\frac{1}{4}$ starts. Is there a fold line to show $\frac{4}{4}$? No

c Mark eighths on the fold lines in the same way.

77

goal Developing a region model for fractions by paper folding

memo The activity suggested on the page serves two purposes—to develop a region model and readiness for showing fractions on a number line.

page 77 This is a good exploration activity for independent learners. With those for whom reading is a problem you may want to close the book and give the directions verbally as you demonstrate the folding steps. Emphasize that the parts must all be the same size before they can be named by a fraction. Watch for confusion of **horizontal** and **vertical** lines. Make sure everyone can read the fractions and name the shaded and the unshaded parts.

Save the paper model for page 78.

lesson Pages 78,79

goal Comparing fractions and naming fractions on a number line

page 78 Problem 1 should be completed independently. Encourage pupils to use their paper models rather than to guess if they are in doubt.

Fractions on a number line may be a new concept for some pupils. Focus on the units in the number line and the number of equal parts in the segment.

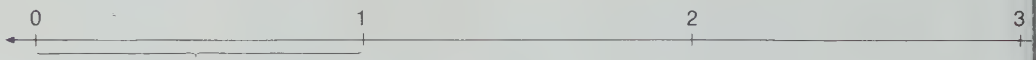
You will want to talk about fraction names for 1 and for 0. Challenge pupils to use their imaginations. Is $\frac{100}{100}$ a fraction name for 1? What about $\frac{1000}{1000}$? Can you think of more fraction names for 1? Repeat for fraction names for 0.

Your folded paper will help you answer these problems.

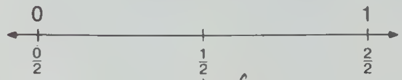
1. Which is more?

   a $\frac{1}{4}$ or $\underline{\frac{1}{2}}$ b $\underline{\frac{1}{4}}$ or $\frac{1}{8}$ c $\underline{\frac{1}{2}}$ or $\frac{1}{8}$ d $\underline{\frac{2}{4}}$ or $\frac{2}{8}$ e $\frac{1}{4}$ or $\underline{\frac{4}{8}}$ f $\frac{4}{8}$ or $\underline{\frac{3}{4}}$ g $\frac{3}{4}$ o

2. A number line can show fractions also.

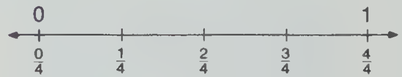


This is one unit.



The segment from 0 to 1 is now divided into 2 equal parts.

a Why is this point called $\frac{1}{2}$? Answers can vary.
It's the end of $\frac{1}{2}$ of a unit.

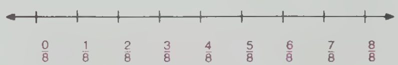


Now the segment from 0 to 1 is divided into 4 equal parts.

b Why is this point called $\frac{3}{4}$? Answers can vary.
It's the end of $\frac{3}{4}$ of a unit.

c Does $\frac{2}{4}$ name the same part as $\frac{1}{2}$? Yes

d Why can this point be named $\frac{0}{4}$? It's the starting point of a unit divided into fourths



e How many equal parts now? 8

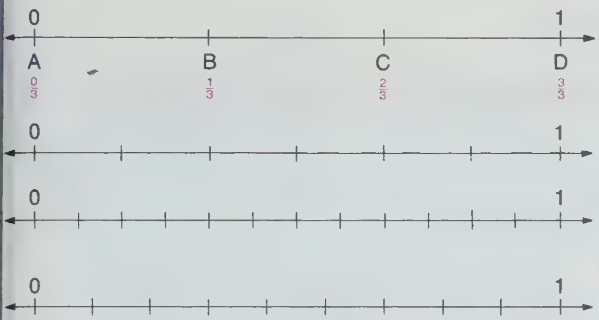
f Copy this number line. Write a fraction name for each point.

3. List at least four fraction names for the number 1. $\frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \ldots$

4. List at least four fraction names for the number 0. $\frac{0}{1}, \frac{0}{2}, \frac{0}{3}, \ldots$

78

Stick with the number line for a while. This time the segment has been divided into 3 equal parts. Write a fraction name for each lettered point.



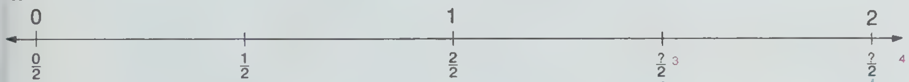
1. Now how many equal parts? 6

2. And now how many equal parts? 12

3. How many equal parts? 9

Why name only fractional parts of 1? Extend the number line.

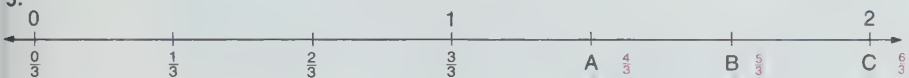
4.



a Name this point. $\frac{2}{2} + \frac{1}{2} =$

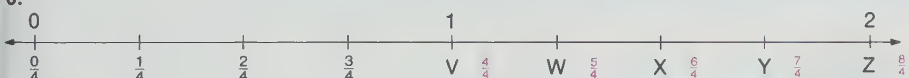
b Name this one also. $\frac{3}{2} + \frac{1}{2} =$

5.



Name them.

6.



Name them.

7. Could the number line be extended still more? How far? Yes; to an infinite length

79

goal Extension of the number line to more than one whole

page 79 Problems 1 through 3 should be completed independently. If additional practice with the number line is needed for some pupils, have them trace the number lines given and label each segment.

Problems 4 through 6 extend the number line beyond one whole. You'll have to decide how best to handle these problems, depending upon the ease with which your pupils are working. Confident pupils should be able to continue independently. Does everyone know what signals that a fraction is equal to 1? that a fraction is greater than 1? (The numerator is greater than the denominator.)

If interest is high, you might try extending the number line beyond 2—not for mastery, but simply for exploration.

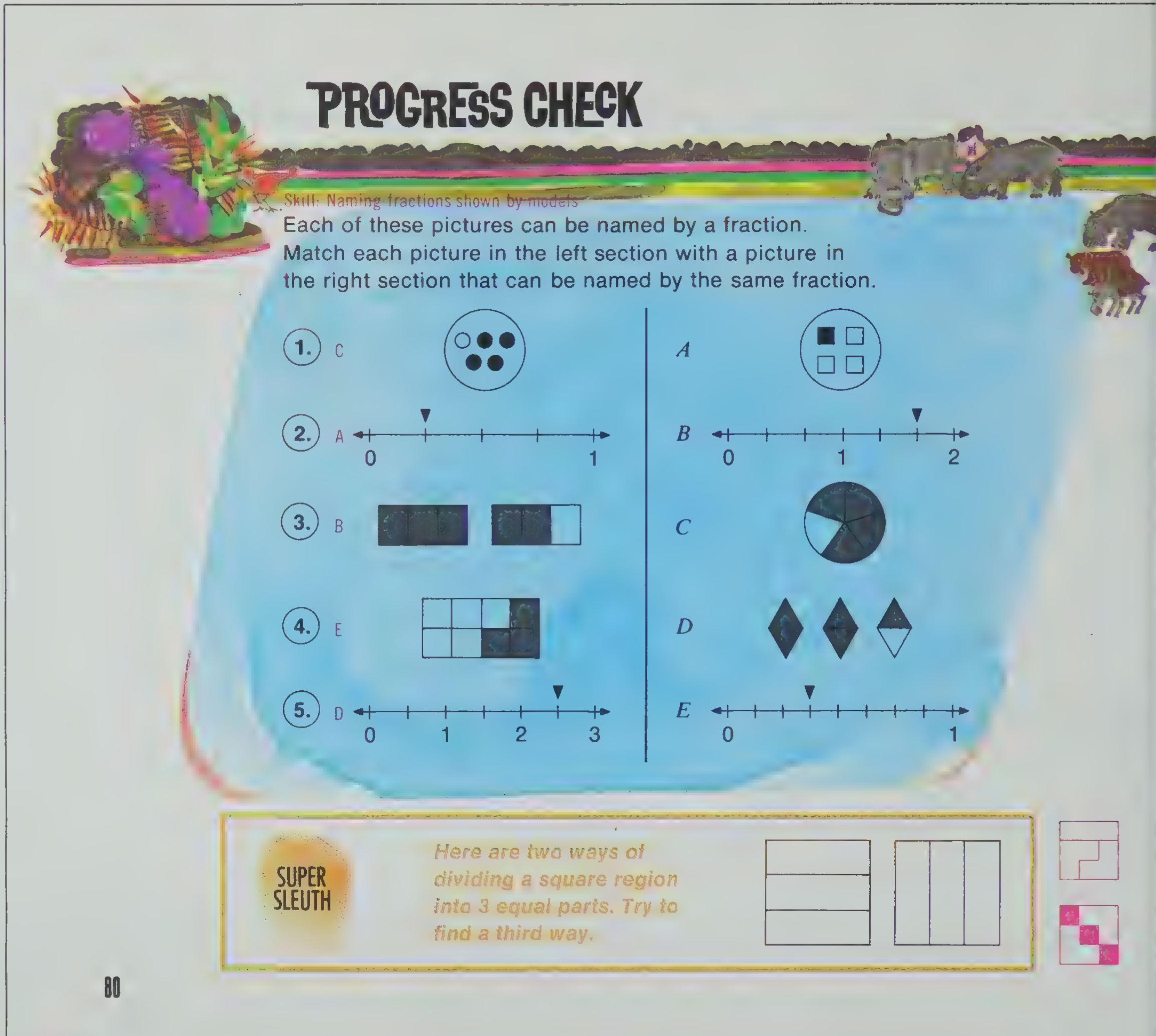
lesson Page 80

goal **Progress Check**—naming fractional parts of a region, a number line, or a set

page 80 This Progress Check may appear tricky at first glance. It really is not if directions are followed. The whole check simply amounts to naming each picture as a fraction and then matching like fractions.

Examine the errors of those pupils who need more help. Three basic models are used—set, number line, and region. Is the youngster unable to work with one specific type? Determine that type and provide additional practice.

The first Supersleuth challenge appears at the bottom of the page—there will be more. Encourage pupils who do not find the solution now to come back to the problem in their spare time. No fair giving away the answer once it is found! Encourage those who need a boost to make several models. Use scissors to cut the fractional parts and fit the parts on top of each other to check that they are the same size.


PROGRESS CHECK

Skill: Naming fractions shown by models

Each of these pictures can be named by a fraction. Match each picture in the left section with a picture in the right section that can be named by the same fraction.

Here are two ways of dividing a square region into 3 equal parts. Try to find a third way.

See activity 1, page 98a.

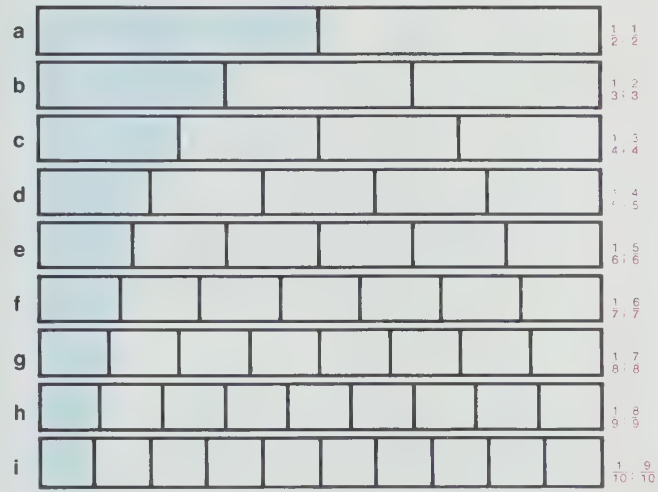
See activity 2, page 98b.

Start getting it all together.

Here's a bunch of number strips. Think of a number strip as sort of a combination of a region and a number line. They are arranged in an order. You are to do two things.

1. Tell what fraction of each strip is shaded.

2. Tell what fraction of each strip is NOT.



3. Now use the strips to help you pick the greater of each pair of fractions.

a $\frac{1}{2}$ or $\underline{\frac{2}{3}}$	**b** $\frac{2}{3}$ or $\underline{\frac{3}{4}}$	**c** $\frac{3}{4}$ or $\underline{\frac{4}{5}}$	**d** $\frac{4}{5}$ or $\underline{\frac{5}{6}}$
e $\underline{\frac{1}{2}}$ or $\frac{2}{10}$	**f** $\frac{5}{10}$ or $\underline{\frac{5}{8}}$	**g** $\frac{5}{8}$ or $\underline{\frac{2}{3}}$	**h** $\underline{\frac{2}{3}}$ or $\frac{3}{6}$
i $\frac{2}{4}$ or $\underline{\frac{3}{5}}$	**j** $\underline{\frac{3}{5}}$ or $\frac{1}{2}$	***k** $\frac{3}{6}$ or $\frac{4}{8}$ Both are the same amount.	***l** $\frac{5}{10}$ or $\frac{1}{2}$ Both are the same amount

81

goal Practice in naming fractional parts and in comparing two fractions, using a model

page 81 Make note of any pupils who have difficulty writing the fractions required for problems 1 and 2. You will want to work closely with them on the next page.

Some pupils may need actually to work with strips of paper to help in making the comparisons. They can trace the ones in the book and then cut them out. They can do any kind of labeling they need to help solve these problems.

goal Comparing fractions with like numerators or like denominators

memo This is a very important page. Make sure to monitor everyone's progress.

page 82 At last—our friend the pie model! Does anyone need more review of what the numerator and the denominator of a fraction indicate? This is the time to give special attention to those pupils who had trouble with problem 1 on page 81.

It is important to have pupils examine the answers to problems 1 and 2. Only then can the pattern of numbers in the numerator and denominator be found. As a region is divided into more parts, what happens to the size of the denominator? What happens to the size of these parts?

Problems 3 and 4 are independent work. Careful of the directions for problem 3—**largest to smallest.** Stress looking for like numerators or denominators with any pupils who are insecure. The most common error made is to use the rule for like numerators with like denominators and vice versa.

WHAT IS A FRACTION?

numerator ⟶ $\frac{1}{8}$ ⟵ the number of shaded parts
denominator ⟶ ⟵ the number of parts in all

$\frac{1}{8}$ $\frac{2}{8}$ $\frac{3}{8}$ $\frac{4}{8}$ $\frac{5}{8}$ $\frac{6}{8}$ $\frac{7}{8}$ $\frac{8}{8}$

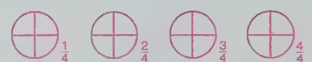
1. Look at the fractions that label the models above.

a Why are the denominators alike?
All models are divided into the same number of parts.

b Why are the numerators different?
All models have a different number of shaded parts.

c If the denominators are alike, the numerator tells you which fraction is the largest.

d Draw regions. Show fourths. Prove **c** is true.



$\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{4}{4}$

2. Look out for a change!

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{6}$ $\frac{1}{7}$ $\frac{1}{8}$

How do you know which fraction is the largest if the numerators are the same? By looking at the denominators. The smaller the denominator, the larger the fraction (or the larger the denominator, the smaller the fraction).

3. Put each set of fractions in order from largest to smallest.

a $\frac{2}{6}, \frac{2}{4}, \frac{2}{5}, \frac{2}{3}$ $\frac{2}{3}, \frac{2}{4}, \frac{2}{5}, \frac{2}{6}$ **b** $\frac{1}{9}, \frac{8}{9}, \frac{2}{9}, \frac{4}{9}$ $\frac{8}{9}, \frac{4}{9}, \frac{2}{9}, \frac{1}{9}$ **c** $\frac{7}{10}, \frac{3}{10}, \frac{9}{10}, \frac{0}{10}$ $\frac{9}{10}, \frac{7}{10}, \frac{3}{10}, \frac{0}{10}$ **d** $\frac{3}{5}, \frac{3}{7}, \frac{3}{4}, \frac{3}{11}$ $\frac{3}{4}, \frac{3}{5}, \frac{3}{7}, \frac{3}{11}$

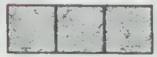
4. Use the symbol for "is greater than" (>) or the symbol for "is less than" (<) to make each sentence true.

a $\frac{5}{6} \overset{>}{?} \frac{5}{8}$ **b** $\frac{3}{4} \overset{>}{?} \frac{1}{4}$ **c** $\frac{2}{3} \overset{>}{?} \frac{2}{5}$ **d** $\frac{1}{9} \overset{<}{?} \frac{1}{8}$ **e** $\frac{1}{10} \overset{<}{?} \frac{3}{10}$ **f** $\frac{4}{9} \overset{<}{?} \frac{4}{7}$ **g** $\frac{4}{4} \overset{>}{?} \frac{1}{4}$ **h** $\frac{3}{5} \overset{<}{?} \frac{3}{3}$

1. How many thirds? How many shaded? 1 third
3 thirds

2. How many thirds? How many shaded? 2 thirds
3 thirds

3. How many thirds? How many shaded? 3 thirds
3 thirds
   a Name the fraction that names the parts shaded $\frac{3}{3}$
   b Is there another name that could be used? Yes, 1



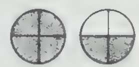
4. Now how many thirds? How many shaded?
6 thirds; 4 thirds
   a Is the amount shaded greater than 1? Yes
   b Name the fraction that names the parts shaded $\frac{4}{3}$
   c Does the mixed number $1\frac{1}{3}$ also name the parts of the regions shaded? Yes

5. Now how many thirds? How many shaded? 8 thirds
9 thirds
   a Name the fraction that names the parts shaded $\frac{8}{3}$
   b Name the mixed number that is another name for that fraction. $2\frac{2}{3}$

6. Write the fraction and the mixed number that name the parts shaded.

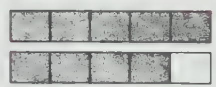
a  $\frac{5}{4}$, $1\frac{1}{4}$

b  $\frac{3}{2}$, $1\frac{1}{2}$

c  $\frac{5}{4}$, $1\frac{1}{4}$

d  $\frac{12}{6}$, 2

e  $\frac{11}{4}$, $2\frac{3}{4}$

f  $\frac{9}{5}$, $1\frac{4}{5}$

7. Which pairs show two names that could describe the same region? Draw the regions if you need help. a, b, e

a $1\frac{2}{3}, \frac{5}{3}$ b $3\frac{1}{2}, \frac{7}{2}$ c $1\frac{1}{6}, \frac{12}{6}$ d $2\frac{1}{4}, \frac{8}{4}$ e $5\frac{1}{2}, \frac{11}{2}$

83

lesson Pages 83, 84

goal Examining fractions greater than 1, using a region model

memo Pupils first met fractions greater than 1 on page 79, where a number-line model was used. This concept will now be used with region models. It may not be as easy for the children as it looks.

page 83 The name MIXED NUMBER may be new for some learners who have not been in the program before—and perhaps it has been forgotten by others. Why is **mixed number** a good name for numbers such as $1\frac{3}{5}$? What kinds of numbers are mixed together?

You decide how much discussion of this page is necessary. Let the pupils' level of confidence be your guide.

Problems 6 and 7 are independent work. The idea of renaming $1\frac{2}{4}$ as $1\frac{1}{2}$ **should not** be introduced now. It will come.

HAVE PATIENCE!

goal Comparing mixed numbers

page 84 In questions 1 to 4 it is tempting to ask "How many more?" or "How much more?" But please don't. It's too early to get tangled up with subtraction. Comparison is enough for now.

Problems 5 and 6 should be completed independently. Watch for pupils who have trouble with **5e**. They may need to be reminded that the fraction in each example must be renamed as a mixed number to make the comparison. When comparing two mixed numbers that have the same whole number, only the fractions need be compared.

1. Bill used $1\frac{1}{2}$ packages of nails.
Donna used $1\frac{1}{3}$ packages of nails.
Who used more nails? Bill, if the packages are the same si

2. Amy picked $2\frac{1}{2}$ baskets of berries.
Dennis picked $2\frac{1}{4}$ baskets of berries.
Who picked more berries?
Amy, if the baskets are the same size

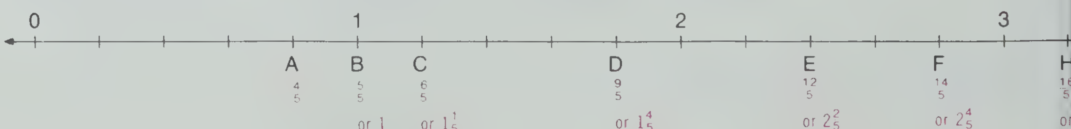
3. Roger collected $5\frac{1}{4}$ dozen eggs.
His brother collected $5\frac{1}{3}$ dozen eggs.
Who collected more eggs? His brother

4. Lee did homework for $1\frac{2}{4}$ hours.
Bill did homework for $1\frac{1}{2}$ hours.
Who did homework for a longer time?
They both worked the same time.

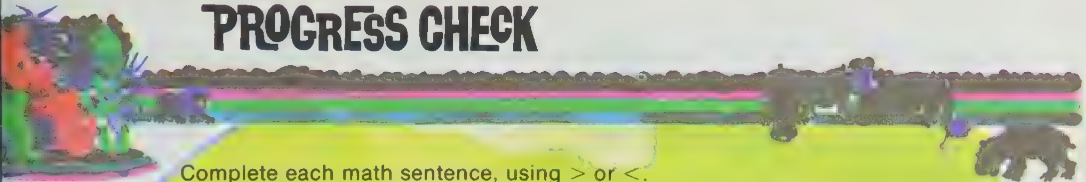
5. Use > (greater than) or < (less than) to make each true.

a $2\frac{1}{8}$? $2\frac{1}{3}$ (<) **b** $1\frac{2}{5}$? $1\frac{2}{7}$ **c** $3\frac{5}{6}$? $3\frac{5}{9}$ **d** $1\frac{7}{8}$? $1\frac{7}{9}$ (>) **e** $\frac{5}{2}$? 3 **f** 4 ? $\frac{4}{4}$ (>)

6. Name each point on the number line marked with a letter.



84



PROGRESS CHECK

Complete each math sentence, using > or <.
Remember > means "is greater than" and < means "is less than."

Skill: Comparing fractions

1. $\frac{2}{5}$? $\frac{4}{5}$ (<) 2. $\frac{3}{8}$? $\frac{5}{8}$ (<) 3. $\frac{3}{9}$? $\frac{2}{9}$ (>) 4. $\frac{1}{4}$? $\frac{3}{4}$ (<)

5. $\frac{3}{5}$? $\frac{4}{5}$ (<) 6. $\frac{1}{4}$? $\frac{1}{7}$ (>) 7. $\frac{2}{3}$? $\frac{2}{4}$ (>) 8. $\frac{7}{8}$? $\frac{3}{3}$ (<)

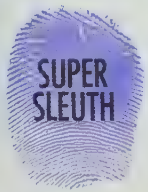
Skill: Comparing mixed numbers

9. $1\frac{1}{3}$? $1\frac{1}{2}$ (<) 10. $2\frac{1}{5}$? $2\frac{1}{8}$ (>) 11. $5\frac{1}{4}$? $5\frac{1}{6}$ (>) 12. $3\frac{1}{2}$? $3\frac{1}{9}$ (>)

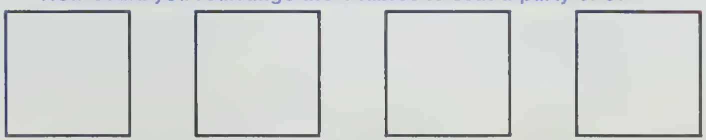
Skill: Comparing fractions with unlike numerators and denominators

Take time to think about these. They are harder.

13. $\frac{1}{3}$? $\frac{3}{4}$ (<) 14. $\frac{7}{8}$? $\frac{1}{5}$ (>) 15. $\frac{1}{6}$? $\frac{5}{8}$ (<) 16. $\frac{1}{10}$? $\frac{3}{4}$ (<)

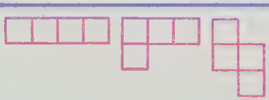


Below is a picture of 4 tables in a restaurant. Look for other arrangements.
Each table can seat 1 person on each side.
How could you rearrange the 4 tables to seat a party of 8?

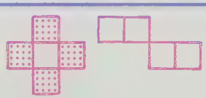


*Rearrange the tables to seat a party of 10.**
*This is a hard one. Try rearranging the tables to seat 12 people.***

*For a party of 10:



**For a party of 12:



lesson Page 85

goal **Progress Check**—comparing fractions and mixed numbers

page 85 Everyone on his own! No need to copy each statement—record the problem number and the symbol only. **But watch those symbols!**

Help the pupil who makes mistakes to focus on the signals and rules.

- Are the numerators alike? What is the rule? (Page 82)
- Are the denominators alike? What is the rule? (Page 82)
- Is one fraction equal, or almost equal, to 1? What about the other fraction? Think number line. Is the fraction closer to 0 or to 1?
- When the whole numbers of two mixed numbers are the same, what do you compare?

Here is another Supersleuth challenge. Once again, this is for fun. It's a puzzle that might take some youngsters a lot of time. You may want to discuss some of the completed table arrangements. Which arrangements are better to use? Why? Why not use some others?

See activity 3, page 98b.

See activity 4, page 98b.

lesson Pages 86, 87

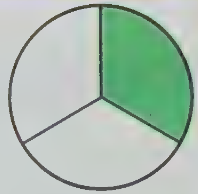
goal Exploration in renaming fractions; introduction to the concept of equivalent fractions

memo Renaming fractions by multiplication is introduced in level 4 of this program's materials. Be aware that this may be a new concept for some pupils, a review for others. Pace your lessons accordingly.

page 86 Careful of vocabulary. When we double the number of parts in a region, something also happens to the size of the parts. Pupils have a tendency to say "two times one-half," or whatever fraction. This statement means $\frac{1}{2} + \frac{1}{2} = \frac{2}{2}$, and that is not the emphasis now—one problem at a time. Do focus on multiplying the numerator and the denominator by the same number. Check the results with a region model.

The term EQUIVALENT FRACTIONS is new vocabulary. You'll want to make sure to discuss this part of the page with everyone.

Here's $\frac{1}{3}$



shaded parts → 1
parts in all → 3

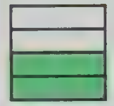
What happens if you double the number of parts in all?

The parts in all doubled → $\frac{1 \times ?}{3 \times 2}$ ← What about the shaded parts? 2

$\frac{1 \times 2}{3 \times 2} = \frac{2}{6}$ Hope you agree that $\frac{2}{6}$ is another name for $\frac{1}{3}$.

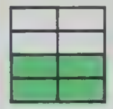
When you multiply the numerator and denominator by the same number, you get another name for the fraction.

Here's $\frac{1}{2}$



Now double the number of parts.
$\frac{1 \times 2}{2 \times 2} = \frac{2}{4}$ Right? Yes

Start with $\frac{1}{2}$ again.
Make three times as many parts.
$\frac{1 \times 3}{2 \times 3} = \frac{?}{6}$ 3



Start with $\frac{1}{2}$ again.
Make four times as many parts.
$\frac{1 \times 4}{2 \times 4} = \frac{?}{8}$ 4

$\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{4}{8}$ are all names for the same number. Try to name one more. $\frac{5}{10}$, $\frac{6}{12}$, $\frac{7}{14}$, . . .

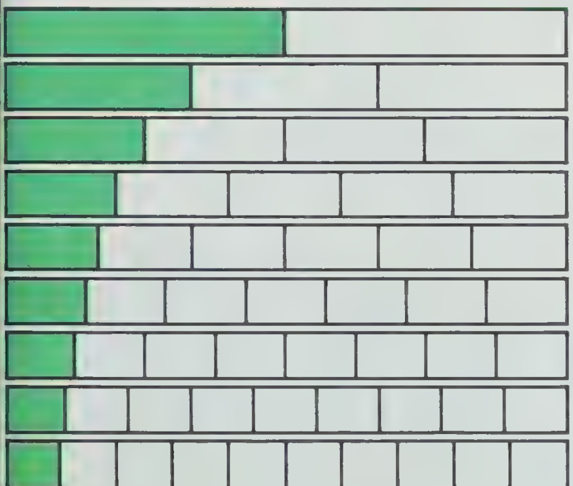
Fractions that name the same number are called *equivalent fractions*.

1. Name three equivalent fractions for $\frac{1}{4}$. $\frac{2}{8}$, $\frac{3}{12}$, $\frac{4}{16}$, . . .
2. Now try to name three equivalent fractions for $\frac{2}{3}$. $\frac{4}{6}$, $\frac{6}{9}$, $\frac{8}{12}$, . . .

86

EQUIVALENT FRACTIONS CAN BE SHOWN BY NUMBER STRIPS, TOO.

You have seen these strips before.



1 Use the number strips to find equivalent fractions.

a $\frac{1}{2}$ $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}$ **b** $\frac{2}{3}$ $\frac{4}{6}, \frac{6}{9}$

c $\frac{3}{4}$ $\frac{[illegible]}{8}$ **d** $\frac{4}{5}$ $\frac{8}{10}$

2 The number strips do not show an equivalent for $\frac{1}{7}$. Aren't there any? If you think there are, name one. Yes $\frac{2}{14}$

3 The number strips do not show an equivalent for $\frac{1}{9}$. There are some. Name at least one. $\frac{2}{18}, \frac{3}{27}, \frac{4}{36}$

4 Does $\frac{1}{10}$ name the same number as $\frac{2}{20}$? as $\frac{3}{30}$? as $\frac{10}{100}$? Yes; yes; yes
Each time the numerator and denominator are multiplied by the same number.

87

goal Practice in finding equivalent fractions

page 87 Use this page as independent work. Problem 1 will help you identify pupils who do not understand the concept of equivalent fractions. Get out those paper strips again or use a set of felt or cardboard fractional parts. Some pupils need to manipulate the pieces before they can find various groupings that will fit on top of each other.

Problems 2 through 4 will help you identify those pupils who are insecure in knowing the method for finding an equivalent fraction. Go back to more diagrams with emphasis on the multiplication step.

lesson Pages 88, 89

goal Exploration to find the simplest name for a fraction

memo Renaming fractions with their simplest name is also explored in level 4 of this program's materials. You will probably not want to make any assumptions, however, but to proceed as if this were a new concept.

page 88 You really need to guide the discussion on this page. If the words **common factor** cause a problem, take a minute to ask: $? \times 2 = 10$

$? \times 2 = 12$

This questioning will help pupils see that 2 is a factor of both 10 and 12 – it is a common factor.

Problems 1 and 2 should be completed independently. The results will help you identify those who have grasped the concept and those who need more help.

Pupils who have not yet mastered the multiplication facts need more practice now or they won't have a chance to succeed. Have them name all the ways they can to multiply two factors and get 12 as a product. $12 = 1 \times 12$

$= 2 \times 6$

$= 3 \times 4$

Repeat for other products less than 100.

Your job now is to find an equivalent fraction with the smallest numerator and denominator.

$\frac{5}{12} + \frac{5}{12} = \frac{5+5}{12} = \mathbf{\frac{10}{12}}$ ← There has to be a simpler name!

You have multiplied by the same number to get a larger numerator and denominator. You can do the reverse, too. Divide by the same number to get a smaller numerator and denominator.

There is only one rule to follow. The number you use to divide with must be a common factor of the numerator AND the denominator.

$\frac{10}{12}$ — Is 2 a common factor? Yes

$\frac{10 \div 2}{12 \div 2} = \mathbf{\frac{5}{6}}$ — Is there a common factor of 5 and 6? No
Don't count the common factor 1.
1 is a common factor of every number.
Looks like $\frac{5}{6}$ is the simplest name for $\frac{10}{12}$.

Here's a problem like the ones you will be doing.

$\frac{1}{10} + \frac{7}{10} = \frac{1+7}{10} = \mathbf{\frac{8}{10}}$ — Is there a common factor of 8 and 10? Yes

$\frac{8 \div 2}{10 \div 2} = \mathbf{\frac{4}{5}}$ — Is there another common factor? No
You have the simplest name.

1. Pick out the fractions that don't have their simplest name. a, b, c, e, f, h, i, j

a $\frac{2}{6}$ **b** $\frac{5}{10}$ **c** $\frac{6}{9}$ **d** $\frac{4}{5}$ **e** $\frac{4}{10}$ **f** $\frac{6}{8}$ **g** $\frac{7}{9}$ **h** $\frac{4}{8}$ **i** $\frac{6}{12}$ **j** $\frac{12}{16}$

2. Go back. Find the simplest name for any five of the fractions you picked.

a $\frac{1}{3}$ b $\frac{1}{2}$ c $\frac{2}{3}$ e $\frac{2}{5}$ f $\frac{3}{4}$ h $\frac{1}{2}$ i $\frac{1}{2}$ j $\frac{3}{4}$

88

Once in a while you will find a fraction like $\frac{8}{12}$.
No new rules—BUT you have to pay attention.

$\frac{8}{12}$ Is 2 a common factor? Yes

Sure it is. BUT 4 is also a common factor.
If you use the greatest common factor,
you will save yourself some work.

This is what happens when you use 2.

$\frac{8 \div 2}{12 \div 2} = \frac{4}{6}$ Ooops! You are not done.

$\frac{4 \div 2}{6 \div 2} = \frac{2}{3}$ No more common factors. You're done.

This is what happens when you use the greatest common factor.

$\frac{8 \div 4}{12 \div 4} = \frac{2}{3}$ You're done in only one step.

Don't worry if you take more than one step to find the simplest name.
You get it sooner or later. Be patient.

PROGRESS CHECK

Skill—Finding equivalent fractions

Complete.

1. $\frac{1}{2} = \frac{1 \times 8}{2 \times 8} = \frac{?}{?}$ $\frac{8}{16}$
2. $\frac{3}{4} = \frac{3 \times ?}{4 \times 2} = \frac{?}{8}$ 2 6
3. $\frac{2}{3} = \frac{2 \times ?}{3 \times 4} = \frac{?}{?}$ 4 $\frac{8}{12}$

Look for common factors. Then complete.

4. $\frac{4}{12} = \frac{4 \div ?}{12 \div ?} = \frac{1}{3}$ $\frac{4}{4}$
5. $\frac{4}{10} = \frac{4 \div ?}{10 \div 2} = \frac{?}{5}$ 2 2
6. $\frac{6}{16} = \frac{6 \div ?}{16 \div ?} = \frac{?}{8}$ $\frac{2}{2}$ 3

Complete.

7. $\frac{4}{5} = \frac{?}{15}$ 12
8. $\frac{7}{10} = \frac{?}{20}$ 14
9. $\frac{1}{6} = \frac{?}{24}$ 4
10. $\frac{3}{8} = \frac{?}{16}$ 6

Which pairs of fractions are equivalent? How do you know for sure? 11.

11. $\frac{1}{4}, \frac{5}{20}$
12. $\frac{3}{10}, \frac{3}{30}$
13. $\frac{5}{15}, \frac{2}{5}$
14. $\frac{2}{3}, \frac{4}{9}$ Numerator and denominator have been multiplied by same number to get the second fraction.

STOP See activity 5, page 98b.

See activity 6, page 98b.

goal More exploration to find the simplest name for a fraction; **Progress Check**—renaming fractions

page 89 Dividing by the greatest common factor of the numerator and denominator to find the simplest name for a fraction is not necessary, but it does cut down the number of steps. Mastery is not expected at this time—more practice is needed first.

The Progress Check should be completed independently. Watch for careless mistakes—generally the result of overconfidence or haste.

Pupils who miss problems 1 through 3 are in real trouble. Have them explain the problems to you. Try to determine their faulty thinking.

Use a missing-factor approach with pupils who have trouble with problems 4 through 10. For example:

$\frac{4}{12} = \frac{4 \div ?}{12 \div ?} = \frac{1}{3}$ **Think** $? \times 1 = 4$, $? \times 3 = 12$

$\frac{4}{5} = \frac{?}{15}$ $\frac{4 \times ?}{5 \times ?} = \frac{?}{15}$ **Think** $? \times 5 = 15$

To help with problems 11 through 14, encourage the pupil to rename one of the fractions so that both have the same denominator. Generally the one with the smaller denominator is renamed.

lesson Pages 90, 91, 92

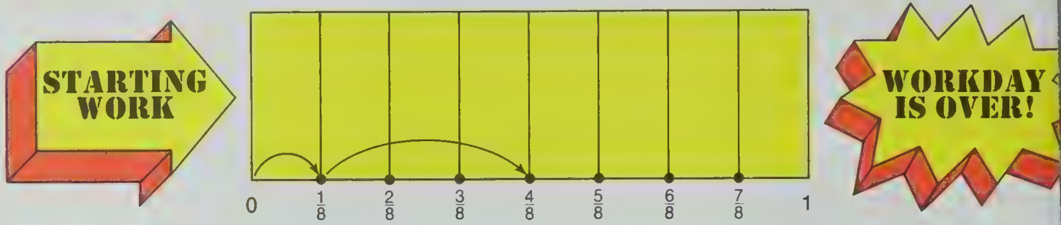
goal Review of adding fractions with like denominators

memo Addition of fractions with like denominators is introduced in level 3 and practiced in level 4 of this program's materials. This should be a review and practice page for pupils who have completed level 4—**but** make no assumptions.

page 90 You are starting with a completely different idea. It's a good time to get together with the pupils and at least discuss the example. Let the pupils tell you how to complete the problem. The confidence with which they speak will tell you whether this is review work or whether you should begin from scratch.

There is no need for confident pupils to spend time copying the sentences. Answers only are sufficient. You might even consider verbal answers for a quick drill. Identifying answers that can be renamed is sufficient for now, unless you have a bunch of sharpies who are ready and able to get the renaming done right now.

A man had to work for 8 hours. He looked at his watch and said to himself, only $\frac{1}{8}$ of my work is over. Time passed. He looked at his watch again. He thought, Another $\frac{3}{8}$ of my work is done. How much had he worked in all? Let the number line help.



hours being counted → hours in all → $\dfrac{1}{8} + \dfrac{3}{8} = \dfrac{4}{8} =$ fraction of the workday.

Were the numerators added together? Yes

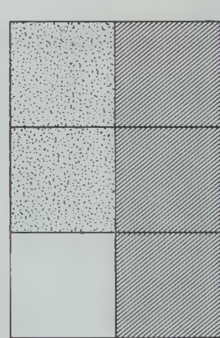
Were the denominators added together? No

Maybe you will want to show your work this way: $\frac{1}{8} + \frac{3}{8}$

ADD

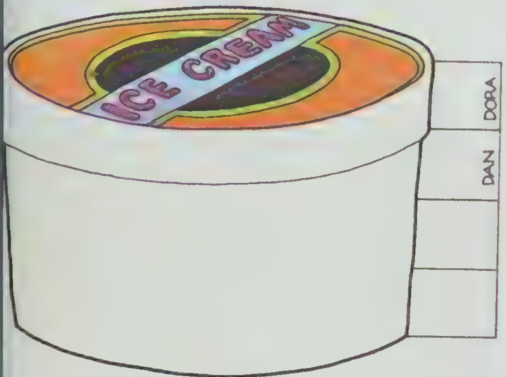
	a	b	c	d
1.	$\frac{1}{8}+\frac{1}{8}$ $\frac{2}{8}$	$\frac{1}{3}+\frac{1}{3}$ $\frac{2}{3}$	$\frac{1}{4}+\frac{1}{4}$ $\frac{2}{4}$	$\frac{1}{6}+\frac{1}{6}$ $\frac{2}{6}$
2.	$\frac{1}{8}+\frac{2}{8}$ $\frac{3}{8}$	$\frac{1}{3}+\frac{2}{3}$ $\frac{3}{3}$	$\frac{1}{4}+\frac{2}{4}$ $\frac{3}{4}$	$\frac{1}{6}+\frac{2}{6}$ $\frac{3}{6}$
3.	$\frac{1}{10}+\frac{1}{10}$ $\frac{2}{10}$	$\frac{1}{10}+\frac{3}{10}$ $\frac{4}{10}$	$\frac{5}{10}+\frac{1}{10}$ $\frac{6}{10}$	$\frac{5}{10}+\frac{4}{10}$ $\frac{9}{10}$
4.	$\frac{2}{5}+\frac{2}{5}$ $\frac{4}{5}$	$\frac{3}{8}+\frac{3}{8}$ $\frac{6}{8}$	$\frac{4}{9}+\frac{4}{9}$ $\frac{8}{9}$	$\frac{0}{2}+\frac{1}{2}$ $\frac{1}{2}$

5. Go back. Ring each sum that can be renamed.



6. Regions also can show addition of fractional parts. How many parts are dotted? 2 How many parts are lined? 3 How many parts in the whole region? 6 $\frac{2}{6} + \frac{3}{6} = \frac{2+3}{6} = \frac{?}{?}$ $\frac{5}{6}$

90



Dora opened a quart of ice cream. She ate about $\frac{1}{4}$. Dan ate about $\frac{1}{4}$. How much ice cream did Dora and Dan eat in all? You don't have a quart of ice cream for an experiment, so here's a picture. Now add: $\frac{1}{4}+\frac{1}{4}=\underline{?}$ $\frac{2}{4}$

When you add fractions such as $\frac{1}{4}$ and $\frac{1}{4}$, do you add the numerators? Yes
Do you add the denominators? No

Here's one way to show your work: $\frac{1}{4}+\frac{1}{4}=\frac{1+1}{4}=\frac{2}{4}$
What is the simplest name for $\frac{2}{4}$? $\frac{1}{2}$

ADD

Renamed answers are for problem 3.

	a	b	c	d	e	f	g	h
1.	$\frac{1}{5}+\frac{1}{5}$ $\frac{2}{5}$	$\frac{2}{5}+\frac{2}{5}$ $\frac{4}{5}$	$\frac{1}{7}+\frac{1}{7}$ $\frac{2}{7}$	$\frac{1}{10}+\frac{1}{10}$ $(\frac{2}{10})$ $\frac{1}{5}$	$\frac{3}{8}+\frac{2}{8}$ $\frac{5}{8}$	$\frac{1}{5}+\frac{2}{5}$ $\frac{3}{5}$	$\frac{1}{6}+\frac{2}{6}$ $(\frac{3}{6})$ $\frac{1}{2}$	$\frac{1}{7}+\frac{3}{7}$ $\frac{4}{7}$
2.	$\frac{1}{12}+\frac{5}{12}$ $(\frac{6}{12})$ $\frac{1}{2}$	$\frac{1}{8}+\frac{3}{8}$ $(\frac{4}{8})$ $\frac{1}{2}$	$\frac{1}{10}+\frac{7}{10}$ $(\frac{8}{10})$ $\frac{4}{5}$	$\frac{3}{4}+\frac{1}{4}$ $(\frac{4}{4})$ 1	$\frac{2}{7}+\frac{3}{7}$ $\frac{5}{7}$	$\frac{3}{9}+\frac{4}{9}$ $\frac{7}{9}$	$\frac{0}{2}+\frac{1}{2}$ $\frac{1}{2}$	$\frac{3}{10}+\frac{6}{10}$ $\frac{9}{10}$

3. Go back. Ring each sum that can be renamed.
Then rename as many as you can.

4. Try to rename each sum as a whole number or a mixed number.

a	b	c	d	e	f	g	h
$\frac{1}{3}+\frac{2}{3}$ $(\frac{3}{3})$ 1	$\frac{2}{5}+\frac{5}{5}$ $(\frac{7}{5})$ $1\frac{2}{5}$	$\frac{3}{4}+\frac{1}{4}$ $(\frac{4}{4})$ 1	$\frac{4}{9}+\frac{5}{9}$ $(\frac{9}{9})$ 1	$\frac{4}{8}+\frac{6}{8}$ $(\frac{10}{8})$ $1\frac{1}{4}$	$\frac{9}{10}+\frac{7}{10}$ $(\frac{16}{10})$ $1\frac{3}{5}$	$\frac{2}{6}+\frac{7}{6}$ $(\frac{9}{6})$ $1\frac{1}{2}$	$\frac{3}{8}+\frac{5}{8}$ $(\frac{8}{8})$ 1

91

goal Practice in adding fractions with like denominators and renaming the sums when possible

memo If adding fractions with like denominators is a new skill for your pupils, you may want to postpone any work with renaming sums so that you can concentrate on one new skill at a time. That's fine! But you will want to come back and practice the renaming skill later.

page 91 Answers only are sufficient for pupils who are working with confidence. Unnecessary copying of problems turns students off and cuts down speed tremendously. Encourage mental computation whenever possible. Acknowledge pupils who add correctly but have not yet mastered the renaming skill. If only renaming is causing trouble, practice verbally the naming of a common factor that will divide both the numerator and denominator.

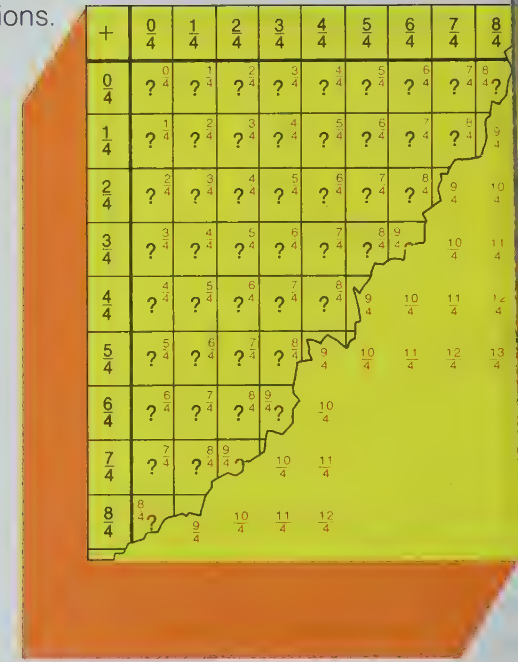
goal Practice in adding fractions with like denominators

page 92 Most pupils have experience using an addition chart with whole numbers. Fractions add a new dimension. Encourage them to look for patterns. If you have blank 10-by-10 or 12-by-12 array charts available, use them here. These can easily be duplicated with a spirit master.

This page will take more time than appears. There is really lots of work here and some rather wild patterns too.

Here is part of an addition table for fractions.

What's the whole-number name for $\frac{0}{4}$?
What's the whole-number name for $\frac{4}{4}$? for $\frac{8}{4}$?



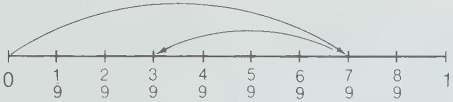
Make a chart like this. Complete it.

Here are some questions to help you find patterns on the chart.

1. Find the fraction combinations that equal $\frac{4}{4}$, or 1. List them.
   $\frac{0}{4}+\frac{4}{4}$, $\frac{1}{4}+\frac{3}{4}$, $\frac{2}{4}+\frac{2}{4}$, $\frac{3}{4}+\frac{1}{4}$, $\frac{4}{4}+\frac{0}{4}$
2. Find the fraction combinations that equal $\frac{8}{4}$, or 2. List them. $\frac{0}{4}+\frac{8}{4}$, $\frac{1}{4}+\frac{7}{4}$, $\frac{2}{4}+\frac{6}{4}$,
   $\frac{3}{4}+\frac{5}{4}$, $\frac{4}{4}+\frac{4}{4}$, $\frac{5}{4}+\frac{3}{4}$, $\frac{6}{4}+\frac{2}{4}$, $\frac{7}{4}+\frac{1}{4}$, $\frac{8}{4}+\frac{0}{4}$
3. What's another name for 3 on your chart? Is the pattern of fraction $\frac{12}{4}$ addition combinations for 3 the same as the pattern for 2? Yes
4. Does the order in which you add two fractions change the answer? No
5. Are there any other whole numbers named by fractions on the chart? 0, 1, 2

92

The wallpaper had 9 repeating patterns on each roll.
$\frac{7}{9}$ of one roll had not been used.
$\frac{4}{9}$ of the roll was then put on a wall.
How much of the roll remained?



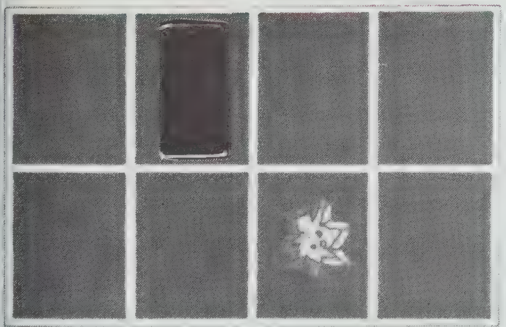
patterns being counted ⟶ 7 4 3
patterns in all ⟶ 9 9 9
fraction of one roll

You might want to show your work this way: $\frac{7}{9} - \frac{4}{9} = \frac{3}{9}$

SUBTRACT

	a	b	c	d
1.	$\frac{4}{5} - \frac{1}{5}$ $\frac{3}{5}$	$\frac{5}{6} - \frac{2}{6}$ $\frac{3}{6}$	$\frac{3}{8} - \frac{1}{8}$ $\frac{2}{8}$	$\frac{6}{9} - \frac{5}{9}$ $\frac{1}{9}$
2.	$\frac{7}{8} - \frac{2}{8}$ $\frac{5}{8}$	$\frac{9}{9} - \frac{2}{9}$ $\frac{7}{9}$	$\frac{7}{10} - \frac{4}{10}$ $\frac{3}{10}$	$\frac{6}{7} - \frac{4}{7}$ $\frac{2}{7}$
3.	$\frac{10}{10} - \frac{5}{10}$ $\frac{5}{10}$	$\frac{4}{5} - \frac{3}{5}$ $\frac{1}{5}$	$\frac{2}{4} - \frac{1}{4}$ $\frac{1}{4}$	$\frac{1}{4} - \frac{1}{4}$ $\frac{0}{4}$
4.	$\frac{8}{9} - \frac{5}{9}$ $\frac{3}{9}$	$\frac{9}{10} - \frac{3}{10}$ $\frac{6}{10}$	$\frac{5}{7} - \frac{3}{7}$ $\frac{2}{7}$	$\frac{1}{10} - \frac{0}{10}$ $\frac{1}{10}$

5. Go back. Ring each answer that can be given a simpler name.



6. Mr. Jones had a parking lot with spaces for 8 cars. He used one space himself, so $\frac{7}{8}$ of the spaces were left. There was a huge hole in $\frac{1}{8}$ of his lot. No cars could park there. How much of his lot was left for parking? $\frac{6}{8}$

93

lesson Pages 93, 94

goal Review of and practice in subtracting fractions with like denominators

memo The number line really isn't the best model in the world to show the operation of subtraction. The child must count rather than subtract to find the answer. It is a good model, however, for the problem in the example. That's why it is used here. But you won't see it used much in later work with the operation of subtraction.

page 93 You know by now who knows what about adding fractions with like denominators. Please proceed accordingly with subtraction.

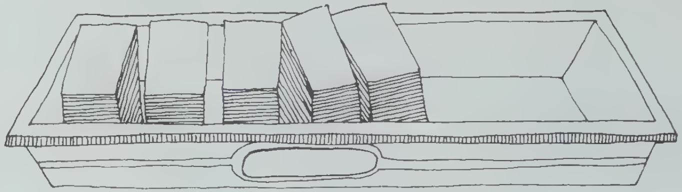
If possible, have the youngsters record answers only. Note that the directions ask the pupil to identify answers that can be renamed — doing the actual renaming is not necessary.

goal Practice in subtracting fractions with like denominators and renaming the answer when possible

page 94 Showing the subtraction of fractions with any kind of model is difficult. Although there are only $\frac{5}{8}$ to begin with, pupils must think of the entire stick of butter so that they can identify the parts as eighths. Someone must have used $\frac{3}{8}$ for a purpose, leaving $\frac{5}{8}$. Now $\frac{2}{8}$ of the $\frac{5}{8}$ are needed. What fraction of the stick of butter remains?

Encourage mental computation. With this page, place more emphasis on recognizing which answers can be renamed than on the actual renaming. These are two different skills. Recognizing when renaming is possible comes first.

There was $\frac{5}{8}$ of a stick of butter. $\frac{2}{8}$ of a stick was used in cooking. What fraction of a stick of butter was left? $\frac{5}{8} - \frac{2}{8} = ?$



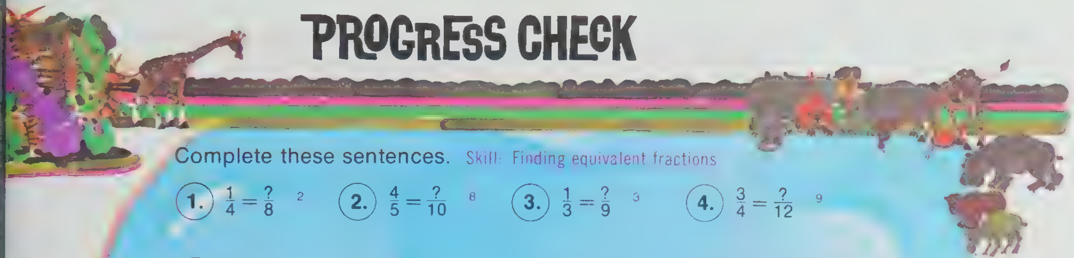
SUBTRACT

Renamed answers are for problem 5

	a	b	c	d	e	f
1.	$\frac{7}{8} - \frac{1}{8}$ $\frac{6}{8}$ $\frac{3}{4}$	$\frac{5}{6} - \frac{1}{6}$ $\frac{4}{6}$ $\frac{2}{3}$	$\frac{4}{5} - \frac{2}{5}$ $\frac{2}{5}$	$\frac{5}{8} - \frac{1}{8}$ $\frac{4}{8}$ $\frac{1}{2}$	$\frac{2}{7} - \frac{1}{7}$ $\frac{1}{7}$	$\frac{3}{10} - \frac{1}{10}$ $\frac{2}{10}$ $\frac{1}{5}$
2.	$\frac{5}{9} - \frac{2}{9}$ $\frac{3}{9}$ $\frac{1}{3}$	$\frac{6}{7} - \frac{5}{7}$ $\frac{1}{7}$	$\frac{7}{10} - \frac{3}{10}$ $\frac{4}{10}$ $\frac{2}{5}$	$\frac{3}{4} - \frac{1}{4}$ $\frac{2}{4}$ $\frac{1}{2}$	$\frac{5}{6} - \frac{1}{6}$ $\frac{4}{6}$ $\frac{2}{3}$	$\frac{3}{5} - \frac{2}{5}$ $\frac{1}{5}$
3.	$\frac{7}{9} - \frac{5}{9}$ $\frac{2}{9}$	$\frac{9}{10} - \frac{3}{10}$ $\frac{6}{10}$ $\frac{3}{5}$	$\frac{4}{5} - \frac{3}{5}$ $\frac{1}{5}$	$\frac{9}{10} - \frac{1}{10}$ $\frac{8}{10}$ $\frac{4}{5}$	$\frac{5}{8} - \frac{3}{8}$ $\frac{2}{8}$ $\frac{1}{4}$	$\frac{3}{7} - \frac{2}{7}$ $\frac{1}{7}$
4.	$\frac{8}{9} - \frac{2}{9}$ $\frac{6}{9}$ $\frac{2}{3}$	$\frac{5}{7} - \frac{2}{7}$ $\frac{3}{7}$	$\frac{1}{8} - \frac{1}{8}$ $\frac{0}{8}$ 0	$\frac{7}{10} - \frac{0}{10}$ $\frac{7}{10}$	$\frac{2}{5} - \frac{1}{5}$ $\frac{1}{5}$	$\frac{3}{8} - \frac{1}{8}$ $\frac{2}{8}$ $\frac{1}{4}$

5. Go back. Ring each answer that can be given a simpler name. Then rename as many as you can.

94



PROGRESS CHECK

Complete these sentences. Skill: Finding equivalent fractions

1. $\frac{1}{4} = \frac{?}{8}$ 2 2. $\frac{4}{5} = \frac{?}{10}$ 8 3. $\frac{1}{3} = \frac{?}{9}$ 3 4. $\frac{3}{4} = \frac{?}{12}$ 9

5. $\frac{2}{8} = \frac{?}{4}$ 1 6. $\frac{4}{6} = \frac{?}{3}$ 2 7. $\frac{3}{12} = \frac{?}{4}$ 1 8. $\frac{2}{10} = \frac{?}{?}$ $\frac{1}{5}$, $\frac{4}{20}$, $\frac{6}{30}$ ...

Add. Skill: Adding fractions with like denominators

9. $\frac{3}{7} + \frac{3}{7}$ $\frac{6}{7}$ 10. $\frac{1}{6} + \frac{5}{6}$ $\frac{6}{6}$ or 1 11. $\frac{1}{8} + \frac{3}{8}$ $\frac{4}{8}$ or $\frac{1}{2}$ 12. $\frac{5}{9} + \frac{4}{9}$ $\frac{9}{9}$ or 1

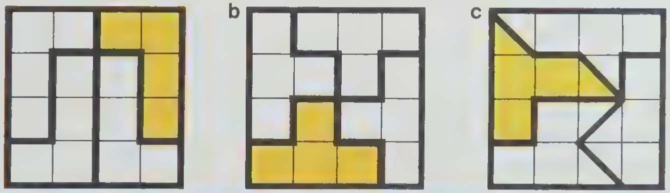
Subtract. Skill: Subtracting fractions with like denominators

13. $\frac{5}{6} - \frac{1}{6}$ $\frac{4}{6}$ or $\frac{2}{3}$ 14. $\frac{7}{8} - \frac{5}{8}$ $\frac{2}{8}$ or $\frac{1}{4}$ 15. $\frac{6}{7} - \frac{3}{7}$ $\frac{3}{7}$ 16. $\frac{2}{3} - \frac{2}{3}$ $\frac{0}{3}$ or 0

SUPER SLEUTH

Which regions show $\frac{1}{4}$ shaded? a, b, c

a b c



95

See activity 7, page 98b.

See activity 8, page 98b.

lesson Page 95

goal **Progress Check**—renaming fractions; adding and subtracting fractions with like denominators

page 95 Note that problems 1 through 4 require that the renaming be done by multiplication. Problems 5 through 8 require the use of division. Take time to check errors. Does the pupil make mistakes with only one type of renaming? To correct errors have the pupil write out the algorithm, using the thinking steps suggested on page 89 in your guide.

The pupil is not directed to rename the answers in problems 9 through 16. He may do this on his own. Great! Focus on the pupil's ability to add or subtract fractions with like denominators at this time. Renaming answers is another skill—it will be checked in the chapter Checkout. The child who makes addition or subtraction errors will need your help. Was he simply careless? Have him explain the problem to you. Listen for faulty thinking. Page 96 provides additional practice.

lesson Pages 96, 97

goal Practice in adding and subtracting fractions with like denominators

memo Page 96 is for extra practice, but page 97 is a real challenge.

page 96 You may choose to skip this page now and come back to it later, or to use it only with pupils who need more practice. Assure your pupils that the directions mean what they say. ". . . Compute only as many problems as you have to . . ." What do they think the prize in the middle may be?

Copy and compute only as many problems as you have to in order to get to the prize. Look out! Look for both addition and subtraction problems.

$\frac{1}{4}+\frac{3}{4}$	$\frac{1}{8}+\frac{1}{8}$	$\frac{3}{8}-\frac{1}{8}$	$\frac{6}{7}-\frac{2}{7}$	$\frac{7}{8}-\frac{3}{8}$	$\frac{5}{6}+$
$\frac{5}{6}-\frac{1}{6}$	$\frac{2}{5}+\frac{2}{5}$	$\frac{3}{5}+\frac{1}{5}$	$\frac{1}{2}+\frac{1}{2}$	$\frac{1}{6}+\frac{5}{6}$	$\frac{2}{3}-$
$\frac{1}{6}+\frac{5}{6}$	$\frac{3}{8}+\frac{5}{8}$	$\frac{3}{4}-\frac{1}{4}$	?	$\frac{5}{9}-\frac{2}{9}$	$\frac{6}{7}+$
$\frac{1}{3}+\frac{1}{3}$	$\frac{7}{14}+\frac{7}{14}$	$\frac{5}{7}-\frac{2}{7}$	$\frac{5}{9}+\frac{4}{9}$	$\frac{5}{8}-\frac{3}{8}$	$\frac{3}{5}+$
$\frac{2}{3}-\frac{1}{3}$	$\frac{7}{9}-\frac{2}{9}$	$\frac{1}{10}+\frac{9}{10}$	$\frac{1}{5}+\frac{1}{5}$	$\frac{2}{3}+\frac{0}{3}$	$\frac{1}{4}-$
$\frac{1}{2}+5$	$\frac{3}{4}-\frac{1}{4}$	$\frac{7}{8}-\frac{2}{8}$	$\frac{1}{8}+\frac{3}{8}$	$\frac{5}{12}-\frac{1}{12}$	$8-$

If you followed the shortest path, you found the answers to each addition problem were alike in some way. How?
They are all names for 1.

This set of problems is put into a 6-by-6 square grid. Then the heavy lines are drawn to form a maze. Draw just the grid. Then make heavy lines so that a different maze is formed.

96

ter lunch, the chef asked an
sistant to report on the supply of pies.
e assistant brought him a list.

peach	$\frac{1}{3}$ lemon cream
mince	$\frac{2}{3}$ banana cream
apple	$1\frac{1}{3}$ raspberry

e restaurant's usual serving is $\frac{1}{6}$ of a
e. If the chef did no more baking that
ay, how many people could have a
erving of:

peach pie? 6
mince pie? 3
apple pie? 5
lemon cream pie? 2
banana cream pie? 4
raspberry pie? 8

97

goal Application of fractions; practice in renaming fractions

page 97 Chefs and cooks certainly do need to know about fractions for calculating servings, as this challenging page will prove. You might want to have the pupils try to answer the questions independently or to solve the problems in small groups.

goal **Checkout**—comparing fractions; adding and subtracting fractions with like denominators; renaming fractions

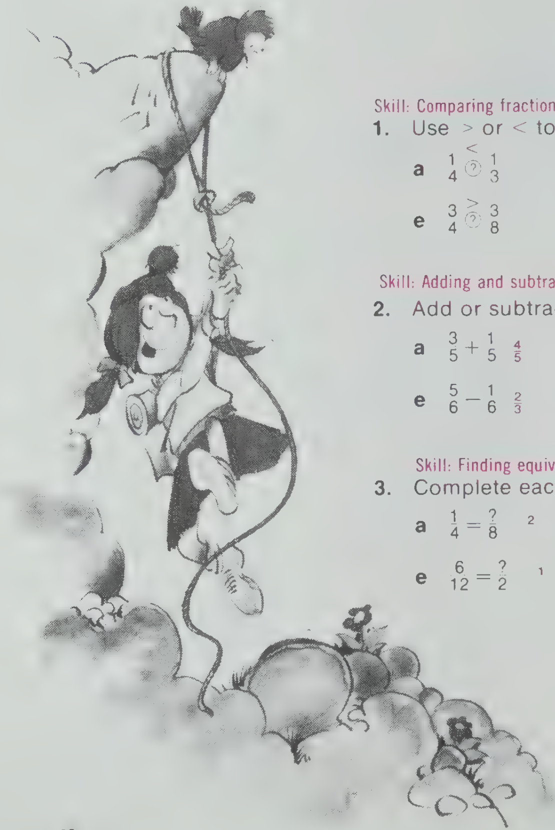
page 98 You want to check on skill, not on endurance—so require answers only, with the emphasis on mental computation.

Each pair of fractions in problem set 1 has either like numerators or like denominators. Check the pupil's errors. Does he make mistakes on only one type or on both? Refer to the guide on pages 82 and 85 for suggestions to help this pupil.

Check errors in problem set 2 carefully. Note that for problems **a** through **d** the answers cannot be renamed, but that for problems **e** through **h** all answers should be renamed. Did the youngster compute correctly but forget to rename? Have him go back and complete the answer.

Problem set 3 is also arranged to help you identify the type of error. Problems **a** through **d** require multiplication; problems **e** through **h** require division. Refer to the guide on pages 89 and 95 for hints to help pupils who make errors.

CHECKOUT



Skill: Comparing fractions

1. Use > or < to compare the following pairs of fractions.

a $\frac{1}{4} \overset{<}{?} \frac{1}{3}$ **b** $\frac{2}{5} \overset{<}{?} \frac{3}{5}$ **c** $\frac{3}{3} \overset{>}{?} \frac{3}{4}$ **d** $\frac{1}{6} \overset{>}{?} \frac{1}{8}$

e $\frac{3}{4} \overset{>}{?} \frac{3}{8}$ **f** $\frac{2}{3} \overset{<}{?} \frac{3}{3}$ **g** $\frac{1}{5} \overset{>}{?} \frac{1}{8}$ **h** $\frac{7}{12} \overset{<}{?} \frac{7}{2}$

Skill: Adding and subtracting fractions with like denominators. Only renamed answers are given.

2. Add or subtract. Make sure your answers have their simplest name

a $\frac{3}{5} + \frac{1}{5}$ $\frac{4}{5}$ **b** $\frac{5}{9} + \frac{2}{9}$ $\frac{7}{9}$ **c** $\frac{4}{9} - \frac{3}{9}$ $\frac{1}{9}$ **d** $\frac{3}{8} + \frac{2}{8}$ $\frac{5}{8}$

e $\frac{5}{6} - \frac{1}{6}$ $\frac{2}{3}$ **f** $\frac{2}{5} + \frac{3}{5}$ 1 **g** $\frac{5}{8} - \frac{1}{8}$ $\frac{1}{2}$ **h** $\frac{3}{12} + \frac{5}{12}$ $\frac{2}{3}$

Skill: Finding equivalent fractions

3. Complete each sentence.

a $\frac{1}{4} = \frac{?}{8}$ 2 **b** $\frac{2}{3} = \frac{?}{6}$ 4 **c** $\frac{3}{5} = \frac{?}{10}$ 6 **d** $\frac{3}{4} = \frac{?}{12}$ 9

e $\frac{6}{12} = \frac{?}{2}$ 1 **f** $\frac{12}{16} = \frac{?}{4}$ 3 **g** $\frac{9}{12} = \frac{?}{4}$ 3 **h** $\frac{8}{10} = \frac{?}{5}$ 4

98

STOP See activity 9, page 98c.

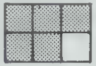
See activity 10, page 98c.

RESOURCES

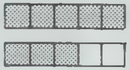
another form of evaluation

for progress check–page 80

Name the fraction shown by each model.

1.  $\frac{5}{6}$

2. 

3.  $\frac{8}{5}$

Draw a picture to show each of the following fractions.

4. $\frac{3}{8}$ 

5. $\frac{8}{3}$ 

6. $\frac{5}{2}$ 

for progress check–page 85

Complete each sentence, using > or <. Remember—> means "is greater than" and < means "is less than."

1. $\frac{2}{3} \overset{>}{?} \frac{1}{3}$ 2. $\frac{4}{8} \overset{<}{?} \frac{6}{8}$ 3. $\frac{3}{6} \overset{<}{?} \frac{5}{6}$ 4. $\frac{7}{9} \overset{>}{?} \frac{4}{9}$

5. $\frac{3}{5} \overset{>}{?} \frac{3}{8}$ 6. $\frac{4}{9} \overset{<}{?} \frac{4}{6}$ 7. $\frac{2}{3} \overset{>}{?} \frac{2}{5}$ 8. $\frac{5}{8} \overset{<}{?} \frac{4}{4}$

9. $3\frac{1}{3} \overset{>}{?} 3\frac{1}{6}$ 10. $1\frac{1}{8} \overset{<}{?} 1\frac{1}{5}$ 11. $4\frac{1}{4} \overset{>}{?} 4\frac{1}{7}$ 12. $2\frac{1}{2} \overset{>}{?} 2\frac{1}{3}$

THINK These are harder.

13. $\frac{1}{4} \overset{<}{?} \frac{3}{8}$ 14. $\frac{4}{5} \overset{>}{?} \frac{1}{2}$ 15. $\frac{1}{3} \overset{<}{?} \frac{3}{7}$ 16. $\frac{1}{5} \overset{<}{?} \frac{2}{8}$

for progress check–page 89

Complete.

1. $\frac{1}{4} = \frac{1 \times 2}{4 \times 2} = \frac{?}{?}$ $\frac{2}{8}$ 2. $\frac{2}{5} = \frac{2 \times ?^{4}}{5 \times 4} = \frac{?}{20}$ 8 3. $\frac{5}{6} = \frac{5 \times ?^{3}}{6 \times 3} = \frac{?}{?}$ $\frac{15}{18}$

Look for common factors. Then complete.

4. $\frac{3}{9} = \frac{3 \div ?^{3}}{9 \div 3} = \frac{1}{3}$ 5. $\frac{9}{12} = \frac{9 \div ?^{3}}{12 \div 3} = \frac{?}{4}$ 3 6. $\frac{8}{10} = \frac{8 \div ?^{2}}{10 \div ?_{2}} = \frac{?}{5}$ 4

Complete.

7. $\frac{3}{5} = \frac{?}{15}$ 9 8. $\frac{4}{7} = \frac{?}{14}$ 8 9. $\frac{1}{3} = \frac{?}{12}$ 4

Which pairs of fractions are equivalent?

10. $\frac{3}{10}, \frac{6}{20}$ 11. $\frac{2}{3}, \frac{9}{12}$ 12. $\frac{4}{8}, \frac{1}{2}$

for progress check–page 95

Complete these sentences.

1. $\frac{2}{3} = \frac{?}{12}$ 8 2. $\frac{3}{8} = \frac{?}{16}$ 6 3. $\frac{3}{5} = \frac{?}{15}$ 9 4. $\frac{1}{6} = \frac{?}{12}$ 2

5. $\frac{6}{8} = \frac{?}{4}$ 3 6. $\frac{3}{9} = \frac{?}{3}$ 1 7. $\frac{8}{18} = \frac{?}{9}$ 4 8. $\frac{5}{10} = \frac{?}{?}$ $\frac{1}{2}$

Add.

9. $\frac{2}{9} + \frac{5}{9}$ $\frac{7}{9}$ 10. $\frac{4}{5} + \frac{1}{5}$ $\frac{5}{5}$ or 1 11. $\frac{3}{10} + \frac{7}{10}$ $\frac{10}{10}$ or 1 12. $\frac{5}{8} + \frac{7}{8}$ $\frac{12}{8}$ or $1\frac{1}{2}$

Subtract.

13. $\frac{5}{7} - \frac{2}{7}$ $\frac{3}{7}$ 14. $\frac{7}{8} - \frac{3}{8}$ $\frac{4}{8}$ or $\frac{1}{2}$ 15. $\frac{5}{6} - \frac{5}{6}$ $\frac{0}{6}$ or 0 16. $\frac{4}{9} - \frac{1}{9}$ $\frac{3}{9}$ or $\frac{1}{3}$

for checkout–page 98

1. Use > or < to compare the following pairs of fractions.

a) $\frac{1}{5} \overset{>}{?} \frac{1}{6}$ b) $\frac{3}{4} \overset{>}{?} \frac{3}{6}$ c) $\frac{2}{5} ? \frac{4}{5}$ d) $\frac{2}{8} ? \frac{2}{5}$

e) $\frac{5}{6} \overset{>}{?} \frac{2}{6}$ f) $\frac{4}{4} \overset{>}{?} \frac{4}{8}$ g) $\frac{2}{3} \overset{<}{?} \frac{2}{2}$ h) $\frac{5}{12} \overset{<}{?} \frac{5}{9}$

2. Add or subtract. Make sure your answers have their simplest name.

a) $\frac{4}{9} + \frac{1}{9}$ $\frac{5}{9}$ b) $\frac{1}{8} + \frac{3}{8}$ $\frac{1}{2}$ c) $\frac{4}{5}$ $\frac{1}{5}$ $\frac{3}{5}$ d) $\frac{1}{3} + \frac{2}{3}$ 1

e) $\frac{9}{10} - \frac{5}{10}$ $\frac{2}{5}$ f) $\frac{7}{9} - \frac{4}{9}$ $\frac{1}{3}$ g) $\frac{1}{12}$, $\frac{5}{12}$ $\frac{1}{2}$ h) $\frac{5}{6} - \frac{1}{6}$ $\frac{2}{3}$

3. Complete each sentence.

a) $\frac{2}{3} = \frac{?}{9}$ 6 b) $\frac{1}{4} = \frac{?}{12}$ 3 c) $\frac{3}{5} = \frac{?}{15}$ 9 d) $\frac{5}{6} = \frac{?}{12}$ 10

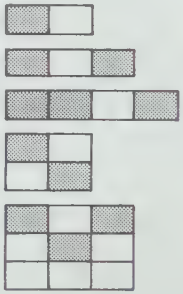
e) $\frac{14}{16} = \frac{?}{8}$ 7 f) $\frac{8}{12} = \frac{?}{3}$ 2 g) $\frac{15}{20} = \frac{?}{4}$ 3 h) $\frac{6}{10} = \frac{?}{5}$ 3

activities

1. **things** spirit master

Prepare a spirit master that includes the types of problems shown.

How much is shaded?

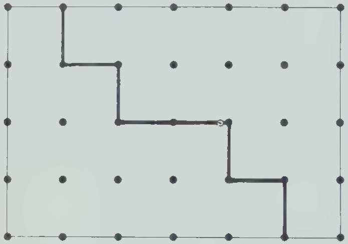


How much is not shaded?

2. things geoboard; rubber band; geopaper

CHALLENGE How many ways can you divide a 6-by-4 rectangle into two congruent parts? Show the ways you find on geopaper.

Challenge your pupils to use their imaginations. A number of solutions are possible.



3. things small cards

Prepare sets of cards by writing a fraction on each card. Each set must consist of fractions with either like numerators or like denominators. Vary the number of cards in each set. For example:

Set A: $\frac{2}{8}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}, \frac{9}{8}$

Set B: $\frac{2}{2}, \frac{2}{3}, \frac{2}{5}, \frac{2}{7}, \frac{2}{8}, \frac{2}{12}, \frac{2}{15}$

Mix up the cards in each set. The goal is to order a set of cards from least to greatest number. Make sure the youngster has experience with both types of sets.

4. things small cards

Prepare sets of 10 or 12 cards by writing a fraction on each card. Each set must consist of fractions with either like numerators or like denominators. The sets can be prepared by the youngsters.

Pupils work in pairs. The cards in a set are mixed and dealt facedown—each player receiving half the cards. These are placed in a stack facedown before the player. Each player turns over his top card. The player with the fraction that has the greater value takes both cards. Play continues until all the cards are used. The player with the most cards wins. Pairs of players can exchange sets of cards.

Variation: The holder of the card with the smaller value takes both cards.

5. things spirit master

Have your pupils jot down sets of 4 equivalent fractions. Use these sets to prepare a spirit master as shown.

Connect the equivalent fractions.

$\frac{6}{8}$	$\frac{6}{36}$	$\frac{1}{6}$	$\frac{9}{12}$
$\frac{3}{18}$	$\frac{5}{25}$	$\frac{3}{4}$	$\frac{4}{24}$
$\frac{7}{35}$	$\frac{12}{16}$	$\frac{9}{45}$	$\frac{1}{5}$

6. things game board; 2-inch squares of oaktag

For each pair of pupils, prepare a 6-by-6 game board consisting of 2-inch squares. On the 2-inch squares of oaktag write equivalent fraction names forming 18 pairs of cards that can be matched. For example:

$\boxed{\frac{1}{4}}$ $\boxed{\frac{4}{16}}$ $\boxed{\frac{3}{12}}$ $\boxed{\frac{6}{24}}$ $\boxed{\frac{1}{6}}$ $\boxed{\frac{2}{12}}$

Rules:
- Mix the cards and place them all facedown on the game board.
- The first player turns over any 2 of the cards. If the cards match (name the same fraction), the player removes these cards and takes another turn.
- If the 2 cards turned over do not match, the cards are again turned facedown and left on the board in play. The other player takes a turn.
- Play continues until all the cards have been removed from the game board.
- The player with more cards wins.
- Players predetermine a rule for what happens if an incorrect match is made.

7. things small cards

Have the pupils jot down sets of 3 equivalent fractions. These fractions are then written on the small cards, 1 per card, to form a deck of playing cards. The number of cards will depend on the size of the group of players.

The deck is shuffled. Players draw for dealer—greatest number indicates the dealer. Each player is dealt 5 cards. The remaining cards are placed facedown in a stack. The top card is turned faceup to form a discard stack. Play begins with the first player to the left of the dealer. He draws a card from either stack. If he has 3 cards that show equivalent fractions, he lays them down before him. To complete his turn, he discards a card on the faceup stack. The game continues with the next player on the left. The game ends when a player has laid down all his cards. Player with the most sets of 3 equivalent fractions wins.

8. things for each pupil: 9 tongue depressors

Multiple sticks make renaming with a common denominator easy. To make multiple sticks, divide each tongue depressor into 10 same-size sections. Write the first ten nonzero multiples of 1 in the sections on one stick, the first ten nonzero multiples of 2 on another. Continue until a stick has been made for the multiples of 1 through 9.

To add $\frac{2}{3} + \frac{5}{8}$ place the 2-stick above the 3-stick and the 5-stick above the 8-stick.

2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
5	10	15	20	25	30	35	40	45	50
8	16	24	32	40	48	56	64	72	80

Look only at the 3-stick and the 8-stick to find the first (least) common multiple of both 3 and 8 – and therefore the common denominator for these two fractions. On the $\frac{\text{2-stick}}{\text{3-stick}}$ 16 is above 24. so, $\frac{2}{3} = \frac{16}{24}$

On the $\frac{\text{5-stick}}{\text{8-stick}}$ 15 is above 24. So $\frac{5}{8} = \frac{15}{24}$.

The denominators are alike. Now simply add the numerators to find the sum.

9. things index cards

Prepare a deck of playing cards consisting of the following sets of fractions:

halves – from $\frac{0}{2}$ through $\frac{2}{2}$

thirds – from $\frac{0}{3}$ through $\frac{3}{3}$

and so on for sixths, eighths, twelfths, sixteenths, and twenty-fourths. Include a whole number card for 1, 2, 3, and 4.

The cards are shuffled. Seven cards are dealt facedown to each player. The remaining cards are stacked facedown in the center of the table. The top card is turned faceup to start the discard stack.

Play begins with the player on the left of the dealer discarding a card with the same denominator or with a fraction equivalent to the faceup card forming the discard pile. Whole-number cards are wild; the player may rename the whole number to a fraction of any denominator in order to continue playing.

Play continues until one player has discarded all his cards or until no player has an appropriate card to play. Players predetermine the number of points needed to win. The first player to discard all his cards earns 20 points. Players with cards left in their hands lose points as follows:

1 point for each fraction less than $\frac{1}{2}$
2 points for each fraction equal to $\frac{1}{2}$
3 points for each fraction greater than $\frac{1}{2}$
4 points for each whole number

10. things game boards; markers; small cards; small box

Each pupil can prepare his own game board consisting of a 4-by-4 array of squares. In each square he is to write a fraction not in simplest form. On each small card write a fraction in simplest form. The cards are mixed in the box. A pupil draws one card at a time and reads the fraction aloud. Any pupil who has another name for that fraction written on his game board covers it with a marker. Four squares covered in a row, column, or diagonal wins the game.

additional learning aids

notation – chapter objectives 1, 2, 3, 4, 5, 6, 8

SRA products

Mathematics Learning System, Activity Masters, level B, SRA (1974)
Spirit masters: F-1, 3, 4, 6, 7
Computapes, SRA (1972)
Module 5, Lessons: FR 3, 8, 9
diagnosis: an instructional aid – Mathematics Level B, SRA (1972)
Probe: M-15
Math Applications Kit, SRA (1971)
Sports and Games cards: 8, 15
Social Studies cards: 12, 19
Mathematics Involvement Program, SRA (1971)
Cards: 224, 234, 244, 265, 156
Skill through Patterns, level 5, SRA (1974)
Spirit masters: 61
Visual Approach to Mathematics (Rational Numbers), SRA (1967)
Visuals: 2, 3, 4, 5, 6, 7

other learning aids (described on page 144e) – Experiments in Fractions, Fraction Bars Student Activity Book, Fraction Dominoes, Fraction Line Set, Fractional Number Cards, Tripletts

operation – chapter objective 7

SRA products

Computapes, SRA (1972)
Module 5, Lesson: FR 14
Skill through Patterns, level 5, SRA (1974)
Spirit masters: 63, 66, 67, 70

5 DIVISION

before this chapter the learner has—

1. Mastered subtracting any 3-digit number from any 4-digit number
2. Mastered finding the product of any two 2-digit numbers
3. Mastered estimating and finding the quotient and remainder (if any) for any 3-digit number and any 1-digit number

in chapter 5 the learner is—

1. Mastering dividing any 4-digit number by a 1-digit divisor
2. Estimating the quotient and dividing a 4-digit number by a 2-digit number
3. Checking division computation by multiplication
4. Exploring dividing by a 3-digit number

in later chapters the learner will—

1. Master estimating the quotient and dividing any 4-digit number by any 2-digit number
2. Master checking a completed division computation by multiplication

Notes & Things

This chapter on long division picks up the thrust started in chapter 2. The emphasis on estimation continues. In fact, you may question the amount of time spent on estimating. The ability to round and to estimate are the KEYS to success in division. Whenever you divide—$15\overline{)145}$ for example—you say to yourself, "There are *about* 9 fifteens in 145." We estimate knowingly or unknowingly at each step of a long division problem. Rounding the divisor and then using a multiple of 10 will make estimation easier. The final step is to round both the divisor and the dividend. These steps are developed methodically throughout the chapter. Remainders are not treated as anything special, because they are a part of every step in the division process.

The shortened algorithm is introduced and should come naturally to the child as he increases the accuracy of his estimates. The continued emphasis on place value should allow him to know that when he writes $\begin{array}{r}1\\ 15\overline{)275}\end{array}$ the number 1 is in reality 10; and then the magic of $\begin{array}{r}18 \text{ R}3\\ 15\overline{)275}\phantom{\text{ R}3}\end{array}$ will not be magic at all but rather a "shorthand" for that process we must truthfully call long division.

When we take a hard look at division, we see that we expect a child not only to subtract but also to multiply and add while we adults say he is really dividing. We must have empathy for his learning problems in this area.

The topic of ecology is an important one and provides a good opportunity for pupils to apply what they have learned in math. Ecology is used as a theme for several pages. This may suggest an extension project for your class. Actually starting a project that involves salvaging materials for recycling is no small matter. There are many problems along with the rewards. You will have to check your community recycling plant or salvage company to determine what types of items your class could collect, have an approved location for storage, make all collection and transportation plans well ahead of the start of the project, and hope that enthusiasm holds up and that nobody gets hurt. If a project is not actually started, the pages will at least yield a good discussion.

things

No special materials are needed for this chapter.

For the extra activities you will want to have these things available:

- wood cubes
- spirit masters
- clock with second hand
- metrestick or yardstick

goal Think about and explore ideas through a picture clue

page 99 The very next page in the book explores some of the times the word or a form of the word **division** is used. The photograph sets the scene. But before going to that page, find out how many different situations the youngsters can think of where this word form is used.

Sometimes when the word **division** is used it has very little to do with an arithmetic operation. Take time to think about the reverse of these situations. Sometimes other words are used to signal division in word problems and in everyday problem-solving situations. Try some easy ones like this.

- I paid 36¢. I got 3. Let's see now—how much did each cost?

Try some hard ones too.

- I got paid $5 for the job. It took me 4 hours. Would I have gotten more money if I had charged $1.50 an hour?

Posing very real situations informally and talking about how to solve them will go a long way in increasing everyone's problem-solving ability. Finding out that sometimes there is a word clue and sometimes there is not a word clue will help your pupils approach problems in a different, more realistic manner. It will take a lot of practice though. Unfortunately there doesn't appear to be a secret to give children that will make them good problem solvers.

lesson Page 100

goal **Survey**—ability to divide by a 1- or 2-digit divisor

page 100 Discuss the purpose of this page. It is meant to find out how much the youngsters already know **and** to establish a learning goal for the chapter.

The errors shown are typical of those made by pupils. The ability to find one's own errors is an important skill. Emphasize the last two lines on the pupil page.

The top-stacking algorithm used in the text is designed to help students phase into the shorter algorithm with greater success.

This is a survey. Don't expect everyone to be able to do the work.

Your turn to be teacher. You have a bunch of papers to correct. There were only four problems assigned.

$7\overline{)918}$ $\quad$ $6\overline{)1844}$

$90\overline{)2005}$ $\quad$ $25\overline{)3030}$

But it's not your day. You can't find the answer key. You look at the first paper. You spot that the first one is wrong. Better get to work.

Find the errors. Show your check to make sure your answer is correct.

100

The word *divide* and the idea of division are part of our world. What does each of the following mean? Accept any good answers.

1. "United we stand, divided we fall." Not together as a whole
2. Divide the candy bar into two parts. Break a whole into parts
3. Divide the class into three groups. Break a set of objects into smaller sets
4. Divide the work among all the children. Break up a whole into parts
5. "Divide and conquer." Break a whole into parts
6. Divide the profits. Break a whole into parts
7. Second Army, Fourth Division A smaller set within a larger set
8. "A house divided against itself cannot stand." Not together as a whole
9. Division Street Breaking a whole into parts

Here are some situations that do not use the word *divide*. Is the idea of division used? Yes

10. My grandfather split kindling.
11. Let's choose up sides.
12. They formed eight teams by counting off.
13. He cut the cake.
14. He shared it with his friends.
15. The dividend check came in the mail.
16. Each of us can have three pieces of chicken.

Which ones could actually involve the arithmetic kind of division? What is the arithmetic kind of division? Involves numbers How much do you know about it?

2, 3, 4, maybe 5, 6, probably 7, 11, 12, 13, maybe 14, 15, maybe 16

101

lesson Pages 101, 102, 103, 104

goal Examining the many ways the word **divide** is used in real-world situations

page 101 Strictly for discussion—pupils should think and feel division. Encourage pupils to tell what idea comes to their minds for each of the statements given. You might have them guess what situations and dates prompted men to make the statements in quotes and also who originated them. They are as follows:

1. George P. Morris, "The Flag of Our Union," about 1850.
5. A military statement whose origin is not certain. (Adolf Hitler used the phrase as a tactical theory but fifth-graders are not old enough to understand this.)
8. Abraham Lincoln, Republican state convention speech, 1858, paraphrased from the New Testament, Mark 3:25: "If a house be divided against itself, that house cannot stand."

If your class says the pledge to the flag, you may want to include discussion on "one nation under God, indivisible . . ."

As you can see, the concept pupils are expected to grasp is that division means to partition or separate one thing or groups of things. This concept is something they should feel rather than be told.

goal Review of dividing by a 1-digit divisor

page 102 The directions in the example will enable some learners to review the algorithm and compute independently. The survey on page 100 gave you clues to help you identify those youngsters who will need your help. The thinking required and the steps for completing the algorithm are clearly given in the example.

Row 1 of the set of problems focuses on basic division facts and place value. Success with these is prerequisite to future achievement. Pupils should be able to compute these problems mentally, so you might want them to record answers only.

Row 2 leans heavily on basic facts. You will be able to identify early signs of trouble if the youngsters write out the division algorithm on these.

Pretend you and two others worked all summer. You formed a lawn-mowing company called the Sod Squad. You advertised in the local newspaper and got a lot of jobs. The profits were divided at the end of September. Your company had earned a total of $567. This was divided equally among the three of you. How much did each receive?

How many 3s in 567? More than 10? $10 \times 3 = 30$
More than 100? $100 \times 3 = 300$
More than 200? $200 \times 3 = 600$
There are between 100 and 200.
This is your estimate.

Now divide.

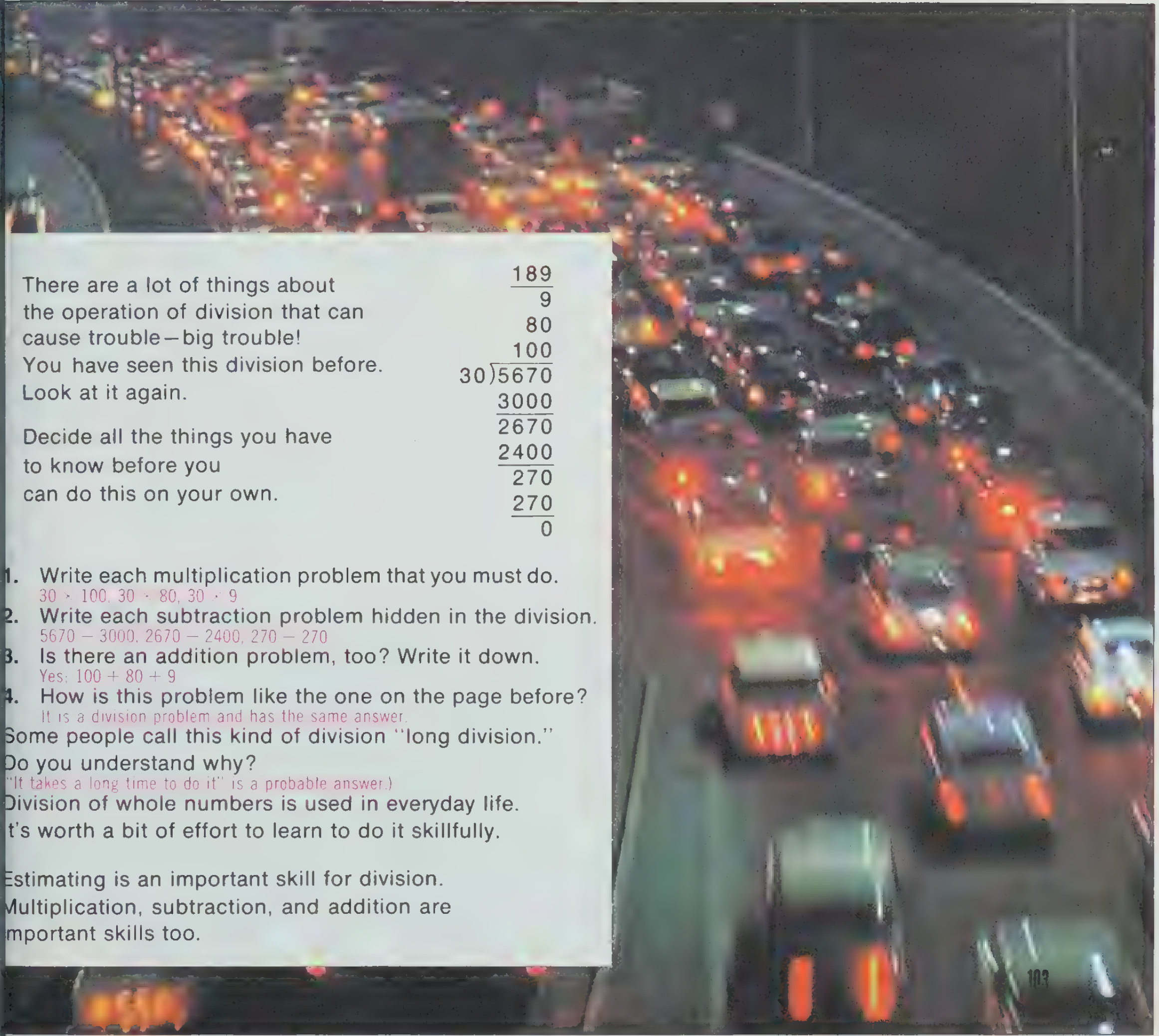
$$
\begin{array}{r l}
189 & \\
\hline
9 & \\
80 & \\
100 & \\
3\overline{)567} & \text{How many 3s in 567?} \\
\underline{300} & 100 \times 3 \\
267 & \text{How many 3s in 267?} \\
\underline{240} & 80 \times 3 \quad \text{Your estimate might have been different.} \\
27 & \text{How many 3s in 27?} \\
\underline{27} & 9 \times 3 \\
0 &
\end{array}
$$

Is the answer close to the estimate? Is the answer reasonable? Yes; yes

Do you remember examples like this from the last chapter on division? How much do you remember? Find out by doing these problems.

	a	b	c	d	e	f	g
1.	$3\overline{)6}$ 2	$3\overline{)60}$ 20	$3\overline{)600}$ 200	$3\overline{)6000}$ 2000	$30\overline{)60}$ 2	$30\overline{)600}$ 20	$30\overline{)6000}$ 200
2.	$6\overline{)300}$ 50	$3\overline{)690}$ 230	$6\overline{)126}$ 21	$7\overline{)714}$ 102	$8\overline{)640}$ 80	$9\overline{)990}$ 110	$4\overline{)368}$ 92

102



There are a lot of things about the operation of division that can cause trouble—big trouble! You have seen this division before. Look at it again.

$$\begin{array}{r} 189 \\ \underline{9} \\ 80 \\ 100 \\ 30\overline{)5670} \\ \underline{3000} \\ 2670 \\ \underline{2400} \\ 270 \\ \underline{270} \\ 0 \end{array}$$

Decide all the things you have to know before you can do this on your own.

1. Write each multiplication problem that you must do.
   30×100, 30×80, 30×9
2. Write each subtraction problem hidden in the division.
   $5670 - 3000$, $2670 - 2400$, $270 - 270$
3. Is there an addition problem, too? Write it down.
   Yes; $100 + 80 + 9$
4. How is this problem like the one on the page before?
   It is a division problem and has the same answer.

Some people call this kind of division "long division." Do you understand why?
("It takes a long time to do it" is a probable answer.)

Division of whole numbers is used in everyday life. It's worth a bit of effort to learn to do it skillfully.

Estimating is an important skill for division. Multiplication, subtraction, and addition are important skills too.

goal Review of dividing by a 2-digit multiple of 10

page 103 Pupils should work this page independently. If reading is a problem for some, do give help. You may want to share findings in a group discussion. The focus is on making the learner aware of the steps and skills necessary to divide accurately. Extend question 4 by asking also how the problems differ. (Both divisor and dividend have been multiplied by 10.)

goal Practice in prerequisite skills for division; **Progress Check**—estimating quotients when dividing by a multiple of 10

page 104 The prerequisite skills for division that are practiced in rows 1 through 8 must be mental exercises if they are to be useful for division. Provide additional help for anyone who has trouble **before** assigning the Progress Check.

Review rounding on a number line with pupils who made errors in rows 1 through 3.

Check those who have difficulty with rows 4 through 8 for mastery of multiplication facts and ability to work with placeholder zeros. Stress:

	Signal
ones × tens = tens	1 zero
ones × hundreds = hundreds	2 zeros
tens × tens = hundreds	2 zeros
tens × hundreds = thousands	3 zeros

Pupils who have no difficulty with rows 1 through 8 should go right on to the Progress Check. Estimated answers should be done **mentally**. Encourage recording answers only.

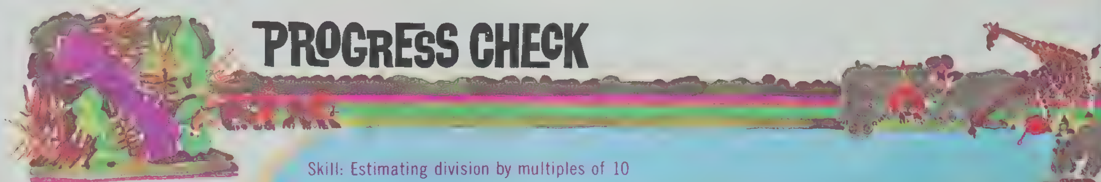
Success here will make the next step in division a lot easier.

Check yourself on the rounding skill.
Round each to the nearest ten.

	a		b		c		d	
1.	32	30	34	30	35	40	36	40
2.	93	90	94	90	95	100	97	100
3.	11	10	25	30	53	50	79	80

Check yourself on multiplication.
No fair using paper and pencil.
Do it in your head.

	a		b		c		d	
4.	10 × 1	10	10 × 10	100	100 × 10	1000	10 × 2	20
5.	10 × 5	50	100 × 5	500	200 × 5	1000	10 × 50	500
6.	20 × 6	120	40 × 6	240	300 × 6	1800	50 × 6	300
7.	40 × 8	320	400 × 8	3200	70 × 4	280	70 × 40	2800
8.	20 × 20	400	30 × 30	900	100 × 20	2000	200 × 30	6000



PROGRESS CHECK

Skill: Estimating division by multiples of 10

Check yourself on this skill. *Estimate* your answers.

	a	b	c	d
Group 1	$\overset{1}{10\overline{)10}}$	$\overset{10}{10\overline{)100}}$	$\overset{100}{10\overline{)1000}}$	$\overset{1000}{10\overline{)10000}}$
Group 2	$\overset{5}{20\overline{)100}}$	$\overset{3}{30\overline{)100}}$	$\overset{2}{40\overline{)100}}$	$\overset{2}{50\overline{)100}}$
Group 3	$\overset{1}{60\overline{)100}}$	$\overset{2}{60\overline{)120}}$	$\overset{2}{60\overline{)140}}$	$\overset{3}{60\overline{)160}}$

104

STOP See activity 1, page 122a.

See activity 2, page 122a.

ut your skills to work.

here were 7750 machine parts made. They were to e put into boxes. There were to be 50 in each box. ow many boxes would be needed to pack the parts?

50)7750 How many 50s in 7750?

100 × 50 = 5000 Not enough.
200 × 50 = 10,000 Too many.
More than 100 but less than 200 boxes will be needed.

155
5
50
100
50)7750 How many 50s in 7750?
5000 100 × 50
2750 How many 50s in 2750?
2500 50 × 50
250 How many 50s in 250?
250 5 × 50
0

. The company packed machine parts according to order. Here is the daily production run. The packing orders are also listed.

Machine parts made each day	Monday 8250	Tuesday 8500	Wednesday 7800	Thursday 8750	Friday 8550
Number to be packed in each box	50 per carton	20 per carton	40 per carton	70 per carton	10 per carton
Number of boxes	a 165	b 425	c 195	d 125	e 855

. About how many thousands of parts did the company make that week? About 43,000

105

lesson Page 105

goal Practice in dividing by a 2-digit multiple of 10

page 105 Since the divisor is a multiple of 10, rounding it is not necessary. The page provides sufficient direction for independent learners to go on by themselves.

You will want to work through the example with those for whom division is a real chore. Give pupils your moral support as you guide their thinking.

lesson Pages 106, 107, 108

goal Practice in rounding divisors and estimating quotients

page 106 Focus on rounding the divisor. Where did the 80 come from in the example? Complete **1a** through **d** as a group.

Next, examine the table together. Note that the youngsters are not required to compute the quotients. The style of thinking that is advanced on this page is the approach used for all division computation. Take your time. Make sure everyone is aboard. If necessary, complete the whole page together.

1. What do you say to yourself when you see this problem? $78\overline{)8307}$ ("Ugh" is not the correct answer.) You really should round 78 and then ask yourself, About how many 80s in 8307? Complete the following.

a $67\overline{)825}$ About how many 70s in 825? (Where did the 70 come from?) How many? 67 rounded to nearest ten;

b $58\overline{)2407}$ About how many ■s in 2407? How many? 40 60

c $42\overline{)376}$ About how many ■s in 376? How many? 9 40

d $74\overline{)6890}$ About how many ■s in 6890? How many? 90 70

2. Complete this table.

	Problem	Rounded division		Estimated Answer
a	$8\overline{)121}$	$10\overline{)121}$	10 × 10 = 100 **(Close!)** 20 × 10 = 200 **(Too many!)**	10
b	$12\overline{)333}$	$10\overline{)333}$	30 × 10 = 300 40 × 10 = 400	30
c	$52\overline{)510}$	$50\overline{)510}$	? 10 × 50 = 500 20 × 50 = 1000	? 10
d	$52\overline{)5100}$	? $50\overline{)5100}$	? 100 × 50 = 5000 200 × 50 = 10,000	? 100
e	$29\overline{)152}$	? $30\overline{)152}$	? 5 × 30 = 150 6 × 30 = 180	? 5
f	$29\overline{)1520}$	? $30\overline{)1520}$	? 50 × 30 = 1500 60 × 30 = 1800	? 50
g	$29\overline{)15200}$	? $30\overline{)15200}$	? 500 × 30 = 15,000 600 × 30 = 18,000	? 500

106

ting is important in doing long-division
ms. In fact, sometimes an estimate is
need. Pretend you are a player on a
g team in the major leagues. You are
to play in the World Series. If you *win*
ries, your team will win about $715 000.
oney is to be equally shared by about
ers, coaches, trainers, and the manager.

te each man's share. **THINK** There
out 40 people that will divide about
00. About how many 40s in 700 000?
0 000 = $400 000 Too small!
20 000 = $800 000 Very close!

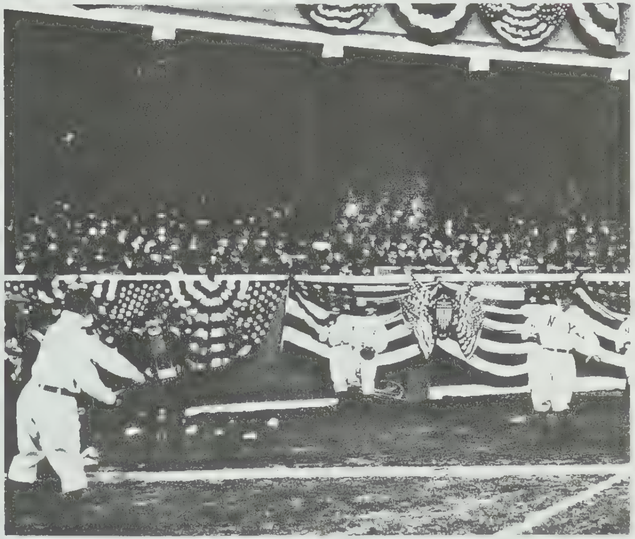
) is a good estimate.

ose the Series, your team shares only
$450 000. Estimate each man's share.
ould round the $450 000, but let's try
it as it is.)

10 000 = $400 000 Too small!
20 000 = $800 000 Too large!

share closer to $10 000 or to $20 000?
s a good estimate for your share?

1. Would it make sense to compute the exact amount? This would depend on your purpose. A spectator wouldn't need to, but the man who actually gives the money would need to.
2. Can you compute the exact amount? Think about these questions before you answer. Not until the Series is completed
   - a Will 34, 35, 36, or more men share in the World Series bonus? Don't know
   - b Are the winner's and loser's bonus dollars known exactly before the Series? No



107

goal Deciding whether an estimate or an exact answer is needed

page 107 Have your pupils write down an estimate of what they think a winning player and a losing player could earn in a World Series. See who comes closest to a recent year's figure of $18 215 for the winners and $13 687 for the losers.

Discuss the World Series problem. Where did the 40 come from in the estimation? Guide the youngsters to understanding that an estimate is good enough when exact numbers are not known or when the exact answer is not necessary.

Some baseball fans may be interested in researching the figure for the most recent year—perhaps even the highest figure ever paid the winners and the losers.

goal Applications requiring an estimated answer

page 108 The emphasis is on using rounding and estimating skills—pupils **should not** compute answers. Have them get their answers independently even though they may share the reading task. Some of the problems sound a little silly. It's hard to get a laugh when you're doing division, so have fun with this page. Make sure that you ask why an estimate is a sufficient answer for these problems.

There is no one way to estimate. The purpose of estimating is to get an answer that is reasonable. Get an estimated answer for these problems.

1. Izzy was dizzy from trying to figure out hov hamburgers to order for the scout picnic. H ordered 275 hamburgers for 39 people. Est number of hamburgers he ordered for each Was Izzy dizzy when he ordered? 7; yes!

2. Ann promised to drive you to camp. Camp 322 km from home. Ann says she can trave 75 km/h . Estimate the hours needed for th About 4 hours

3. Sue and Lou have 585 tickets to sell for the tournament. They want to sell them all in 1 Estimate how many tickets they need to se About 60

4. Harry and 30 of his classmates spent Satur tickets for performances of their school pla pupil sold about the same number of ticket ticket sales totalled 1001. About how many each pupil sell? About 30

5. The money Harry and the others collected $1418. About how much did each of the 31 collect for ticket sales? About $50

6. Mr. Nurvis, the teacher, is planning a parad class is to march. 16 children can be in one There are 736 children. About how many ro children can there be? About 40

Estimates can help you find a reasonable answer.
Now it's time to find some exact answers.

Pollution is a health problem almost everywhere. One way to fight pollution is to recycle materials after they have been used. The people at Crystal Creek save and sell glass. They get paid \$21 for each ton of glass. Last week they got a check for \$525. How many tons of glass did they sell?

Think How many 21s in 525?

Round 21 to 20.
How many 20s in 525?

$10 \times 20 = 200$ Not enough.
$20 \times 20 = 400$
$30 \times 20 = 600$ Too many.

They sold between 20 and 30 tons.

Now you divide.

$$\begin{array}{r} 25 \\ 5 \\ \underline{20} \\ 21\overline{)525} \\ \underline{420} \\ 105 \\ \underline{105} \\ 0 \end{array} \quad \begin{array}{l} \\ \\ \\ \\ 20 \times 21 \\ \\ 5 \times 21 \\ \\ \end{array}$$

The parts of a division problem have special names.

You already know what a remainder is.

$$\begin{array}{r} 25 \\ 21\overline{)525} \end{array}$$

In the problem above, the divisor is ⟶ 21
This is called the dividend.
The answer is the quotient.

Hardly anyone remembers the word *dividend*.
But do try to remember the word *divisor* and the word *quotient*.

Find these quotients.

1. $18\overline{)306}$ 17
2. $27\overline{)435}$ 16 R3
3. $35\overline{)525}$ 15
4. $42\overline{)588}$ 14
5. $24\overline{)676}$ 28 R4
6. $13\overline{)546}$ 42

goal Application of rounding and estimation skills to division

page 109 The focus on finding what number is **not enough** and what number is **too many** is the secret to division skill. After completing the computation in the example, check whether the quotient computed is somewhere between these two numbers. Emphasize using this estimate to check the **reasonableness** of the exact answer. Does that mean that the quotient is accurate? (No, only that it is reasonable.)

This program's materials stress concept and computational skill. Emphasis is seldom placed on technical terminology – only when it is necessary to communicate. This is one of those times. Stress using the terms **divisor** and **quotient.** Pupils will see these two words on most standardized tests.

Problems 1 through 6 should be completed independently. Watch for frustration.

goal Extension of estimation skill and application of this skill to division

page 110 Discuss the examples. Some youngsters may have difficulty deciding which estimate is closer to the actual quotient. Review the relationship of multiplication to division with these youngsters.

factor × factor = product $\quad \text{factor}\overline{)\text{product}}^{\text{factor}}$

Compare the product of the estimate with the dividend (product) of the division problem. Which estimated product is closer to the dividend?

The completed chart should be used when computing the three problems chosen. This is independent work.

The better you get at making an estimate, the fewer steps you will have to do in division. Start looking at your estimates more carefully. Make a good enough estimate to save yourself some work.

Complete this.

THE PROBLEM	DIVISOR SHOULD BE ROUNDED TO	FIRST ESTIMATES	ESTIMATED QUOTIENT
$37\overline{)814}$ (22)	40	$20 \times 40 = 800$ $30 \times 40 = 1200$	> 20 < 30
$17\overline{)901}$ (53)	20	$40 \times 20 = 800$ $50 \times 20 = 1000$	> 40 < 50
$72\overline{)1872}$ (26)	? 70	? $20 \times 70 = 1400$ $30 \times 70 = 2100$	?
$27\overline{)783}$ (29)	? 30	? $20 \times 30 = 600$ $30 \times 30 = 900$	?
$29\overline{)1348}$ (46 R 14)	? 30	? $40 \times 30 = 1200$ $50 \times 30 = 1500$	?
$36\overline{)673}$ (18 R 25)	? 40	? $10 \times 40 = 400$ $20 \times 40 = 800$	>10 <20 (closer to 20)
$34\overline{)992}$ (29 R 6)	? 30	? $30 \times 30 = 900$ $40 \times 30 = 1200$	>30 <40 (closer to 30)
$86\overline{)5834}$ (67 R 72)	? 90	$60 \times 90 = 5400$ $70 \times 90 = 6300$	>60 <70 (closer to 70)

After you estimate the least possible quotient and the greatest possible quotient, you can think some more. Should the quotient be closer to the least number or to the greatest number?

Pick three problems only. Find the exact quotient.

110

The students at Bad Air School decided to collect and sell aluminum cans for recycling. They collected cans, but they forgot to ask how much they would be paid for each ton. It took a long time, but they finally had 14 tons of aluminum. They sold it for $2254. How much money per ton did they get?

Think

How many 14s in 2254? Round 14 to 10.
How many 10s in 2254?

$100 \times 10 = 1000$ Not enough.
$200 \times 10 = 2000$ That's better.
$300 \times 10 = 3000$ Too many.

They got between $100 and $200 per ton.

They got $161 per ton for the aluminum. The estimate says this is a reasonable answer.

Find out exactly

```
      161
        1
       60
      100
 14)2254
     1400   100 × 14
      854
      840    60 × 14
       14
       14     1 × 14
        0
```

Would you have estimated 50? It would have been O.K.

1. Copy and complete these division problems.

a
```
        ?    170
              70
 45)7650     100
    4500  (? × 45)   100
    3150
    3150  (? × 45)   70
       0
```

b
```
       ?   29
            9
 23)667    20
    460  (? × 23)   20
    207
    207  (? × 23)   9
      0
```

c
```
        ?    380
              80
 12)4560     300
    3600  (? × 12)   300
     960
     960  (? × 12)   80
       0
```

2. Answer these.

a How many 19s in 950? 50

b How many 18s in 5400? 300

goal Application of rounding and estimation skills to dividing by a 2-digit divisor

page 111 Here's another complete example you can use to guide the pupil's thinking. Some pupils may have difficulty understanding why the estimate is between $100 and $200 rather than between $200 and $300.

$200 \times 10 = 2000$ That's better.

In fact, it's very close to the dividend 2254. Consider, however, that 14 was rounded down to 10. These two signals **combined** indicate that the quotient probably is between 100 and 200. Be patient. Estimation skills will be practiced and refined.

The remainder of the page is independent work. Do not require pupils to write the **thinking** steps at the side unless this makes them feel more secure.

100×14 These numbers are
60×14 automatically recorded in
1×14 the top-stacking method.

goal Extension of rounding and estimation skills and application of the skills to division

page 112 Rounding and estimation skills are the key to success in division. Thus far only the divisor has been rounded. We've reached the last step – rounding both the divisor **and** the dividend. This step is reached in the "Estimate" column of the chart. Watch for weak students who lose the original problem at this point. The next column should bring them back on target. Discuss why the quotient is between 10 and 20. How do they know? As soon as a pupil feels confident enough to go on alone, have him complete the chart independently.

Problem 7 is strictly for independent exploration. Discuss after it is completed. Did anyone notice something special about these problems?

Complete the table

1. $18\overline{)285}$	$\overset{10}{20\overline{)300}}$	10 × 18 = 180	10, 20
2. $17\overline{)536}$	$\overset{20}{20\overline{)500}}$	?	?
3. $28\overline{)2967}$	?	?	? 100, 200
4. $73\overline{)12351}$	?	? 100 × 73 = 7300	100, 200
5. $33\overline{)4311}$	? $30\overline{)4000}$	100 × 33 = 3300	100, 200
6. $27\overline{)663}$	? $\overset{20}{30\overline{)700}}$	20 × 27 = 540	20, 30

Something to think about

7.
- **a** 6 tens + 3 tens = ?, 60 + 30 = ? 9 tens; 90
- **b** 6 tens − 3 tens = ?, 60 − 30 = ? 3 tens; 30
- **c** 6 tens × 3 tens = ?, 60 × 30 = ? 18 hundreds; 1800
- **d** 6 tens ÷ 3 tens = ?, 60 ÷ 30 = ? 2 ones; 2
- **e** 9 hundreds + 9 tens = ?, 900 + 90 = ? 9 hundreds and 9 tens; 990
- **f** 9 hundreds − 9 tens = ?, 900 − 90 = ? 8 hundreds and 1 ten; 810
- **g** 9 hundreds × 9 tens = ?, 900 × 90 = ? 81 thousands; 81,000
- **h** 9 hundreds ÷ 9 tens = ?, 900 ÷ 90 = ? 1 ten; 10
- **i** 8 hundreds + 4 hundreds = ?, 800 + 400 = ? 12 hundreds; 1200
- **j** 8 hundreds − 4 hundreds = ?, 800 − 400 = ? 4 hundreds; 400
- **k** 8 hundreds × 4 hundreds = ?, 800 × 400 = ? 32 ten thousands; 320,000
- **l** 8 hundreds ÷ 4 hundreds = ?, 800 ÷ 400 = ? 2 ones; 2

112

he man who bought the scrap metal would not pay for part of a tonne.
here are 1000 kg in 1 t .
ou can understand why he wouldn't pay for 40 kg .

Would he pay for 500 kg? Why? Is that fair?

No; it's only part of a tonne; those are the rules; everyone knows about them ahead of time.

Would he pay for 990 kg? Why?

Maybe; 990 kg are reasonably close to a tonne.

1.

ow much would you pay for 1 pack of gum if e price were 2 for 15¢? Why? 8¢; you can't pay $\frac{1}{2}$¢

2.

If you were dividing 3 pieces of cake between 2 people, what could you do with the one piece remaining? You could divide it

3.

ou were supposed to divide your 15 books mong 4 visitors. What do you do with the nes remaining? Keep 3 yourself, draw straws

4.

Suppose you had 5 bags of peaches. Each bag had 4 peaches in it. Four people were to have equal shares. What do you do with the extra bag? Open the bag and give each person one peach.

5.

hat would you do if you had to share 35¢ ith your brother?
ip the quarter first and see who gets the extra penny

As you use division in the real world, you'll find many ways to handle a remainder. When you practise division, write the remainder beside the answer.

PROGRESS CHECK

Divide Skill Emphasis on dividing by a 2-digit number

1. 5)34 — 6 R4	2. 10)101 — 10 R1	3. 90)8100 — 90	4. 10)1000 — 100
5. 70)950 — 13 R40	6. 2)0 — 0	7. 33)7524 — 228	8. 27)1572 — 58 R6

*Accept any good answers for these problems

113

lesson Pages 113, 114

goal Examining division remainders in real-world situations; **Progress Check**—division skills

page 113 The remainder situations should provide a good basis for sharing ideas. Youngsters need to realize that sometimes a decision must be made on what to do with a remainder. Ask for opinions as to what is fair.

The Progress Check is independent work. Watch especially for responses to problem 6. Group those pupils who miss this specific problem. Hold out an empty hand and say that you're going to divide the candy you have in your hand between two children. Ask how many pieces each of the two youngsters will get.

Work individually with pupils who missed problems other than 6. Ask each one to **think aloud** as he works, so that you can determine at which step he is making his error. Lead by asking what to do next rather than by telling him the steps. See page 121 for additional practice.

STOP See activity 3, page 122a.

A problem for problem solvers! Mark a 3 × 3 square on the floor. This square is a 3-passenger automatic elevator. Ten people want to ride the elevator. How many trips must the elevator make to carry all the passengers?

goal Practice in dividing by a 2-digit divisor

page 114 No better way to learn how to catch one's own errors than to find the errors of others—besides it's more fun to catch someone else's mistakes. The youngsters are told right on the page that only one problem is correct. The others may have as few as one or as many as five mistakes. They had better be alert checkers! Pupils should work independently.

When the page is completed, discuss possible reasons for some of the mistakes. What kinds of help does a person making that mistake need?

One problem below is correct. Finding someone else's mistake is a challenge. Division mistakes are the hardest of all to find. The error may be in multiplication, subtraction, or addition. Find the errors. (There may be more than one in a problem.) Then write the name of the operation done incorrectly.

For example:

```
     185 R17
       5
      80
     100
43)7872
    4300
    3672   7872 − 4300 = 3572, not 3672
    3440   Subtraction error
     232
     215
      17
```

1.

```
                Should be 25
     205 R0
       1
       4
     200   Should be 20
35)875
     700   Should be 7000
           Multiplication error
     175
     140
      35
      35
       0
```

2.

```
     101 R35   Should b
       2
       9
      90
57)5682
    4130   Should be 51
           Multiplication
     552
     413   Should be 51
     149   Should be 139
     114
      35
```

3.

```
      83 R5   No
       3      errors
      20
      60
34)2827
    2040
     787
     680
     107
     102
       5
```

4.

```
     123 R17   Should be
               140 R50
       3
      20
     100
56)7890
    5600
    1290   Should be 2290
           Subtraction error
    1120
     170
     153   Should be 168
           Multiplication error
      17
```

5.

```
     199 R20   Should be
               198 R14
       9   Should be 8
      90
     100
24)4766
    2400
    2366   Multiplication error
    2166   Should be 2160
     206   Should be 200 Subtraction error
     186   Should be 216
           Multiplication error
      20
```

6.

```
     112 R 6   Should be
               112 R 6
       2
      10
     100
78)8752
    7800
     952
     780
     162   Should be 172
           Subtraction er
     156
       6
```

7. Which error could have been spotted with a good estimate? Error in problem 1

114

See activity 4, page 122b.

The students who collected aluminum cans want to buy $5000 worth of equipment for the school. They need more money—$2760 more. Now how many more tons of aluminum to collect if the price stays at $160 per ton?

Think How many 160s in 2760?

Round 160 to 200.
How many 200s in 2760?

$10 \times 200 = 2000$ Not enough.
$20 \times 200 = 4000$ Too many.

They will need more than 10 but less than 20 tons. Find out exactly.

$$\begin{array}{r l}
17 \text{ R}40 & \\
7 & \\
10 & \\
160\overline{)2760} & \\
1600 & 10 \times 160 \\
1160 & \\
1120 & 7 \times 160 \\
40 &
\end{array}$$

The question was, How many *tons*? What are you going to do with the remainder? Collect more or sell as 17 tons.

If they sell 17 tons, will they get as much money as they need? What is the answer to this problem?
No; they need to sell 18 tons

Here are some computation problems. You won't have to worry about what to do with the remainder on these.

1. $250\overline{)7500}$ 30
2. $340\overline{)7161}$ 21 R21
3. $170\overline{)1192}$ 7 R2
4. $110\overline{)5507}$ 50 R7
5. $260\overline{)2349}$ 9 R9

The next ones are for very brave people.

6. $620\overline{)84,940}$ 137
7. $130\overline{)11,220}$ 86 R40
8. $250\overline{)31,250}$ 125
9. $305\overline{)15,250}$ 50
10. $430\overline{)23,456}$ 54 R236

lesson Page 115

goal Introduction to dividing by a 3-digit divisor

memo Dividing by a 3-digit divisor is introduced here, but mastery is not expected at this time. The pupil is being given a taste of what is to come in chapters where the development will be expanded. Consider skipping this page with pupils who have been having difficulty and are just beginning to experience success. This type of division represents another big computational step.

page 115 The page gives sufficient guidance to be used as independent work with very capable learners. Use your discretion in handling the page and in making assignments.

lesson Pages 116, 117

goal Review of checking division computation by multiplication

memo Page 116 and the top half of page 117 should be worked together.

page 116 This is primarily a discussion page for examining the relationship of multiplication to division. This is not a new idea so there shouldn't be any problems. Go right on to page 117.

When you use division for something important, you have to be right. An estimate tells you if your answer is reasonable. How do you tell if your answer is right? Try multiplication.

1. $4 \times 7 = 28$ True or false?
   $7 \times 4 = 28$ True or false?
   $28 \div 7 = 4$ True or false?
   $28 \div 4 = 7$ True or false?

2. $6 \times 9 = 54$ True multiplication sentence
   $9 \times 6 = 54$ True multiplication sentence
   $54 \div 6 = 9$ True division sentence
   Write another true division sentence using 6, 9, and 54. $54 \div 9 = 6$

3. $14 \times 11 = 154$
   Write another true multiplication sentence using 11, 14, and 154. $11 \times 14 = 154$
   $154 \div 11 = 14$
   Write another true division sentence using 11, 14, and 154. $154 \div 14 = 11$

4. $7018 \div 22 = 319$. This is a true sentence.
   Write another true division sentence using 22, 319, and 7018. $7018 \div 319 = 22$

5. Divide.

$$\begin{array}{r} \underline{12} \\ 2 \\ 10 \\ 13\overline{)156} \\ \underline{130} \\ 26 \\ \underline{26} \\ 0 \end{array}$$

Check Does $156 \div 13 = 12$?

$$\begin{array}{r} 12 \\ \underline{\times\ 13} \\ 36 \\ \underline{120} \\ 156 \end{array}$$

13 × 12 does equal 156!
Does $156 \div 12 = 13$? Yes

Answer this one QUICKLY! $12\overline{)156}$? 13

Continue with checking.

116

Can any true sentences be written using the 3 numbers given in each problem?
a) 12, 3, 4
b) 12, 6, 2
c) 2, 2, 4 (Don't forget addition and subtraction here.)
d) 10, 20, 200
e) 60, 180, 3
f) 11, 12, 132
g) 50, 70, 3500
h) 90, 40, 3500 (Only an inequality can be written for this one.)

6. $9\overline{)65}$ with quotient 7; -63; remainder 2 65 ÷ 9 = 7 R2 Check it!

9 × 7 = 63 and 63 + 2 = 65
or (9 × 7) + 2 = 65 is another way of writing it.

7. $8\overline{)49}$? 6 R1 49 ÷ 8 = 6 R1 Check it. Use multiplication. (8 × 6) + 1 = 49

DIVIDE AND CHECK

8. 301 ÷ 17 or $17\overline{)301}$ 17 R12

9. 625 ÷ 15 or $15\overline{)625}$ 41 R10

10. 532 ÷ 25 or $25\overline{)532}$ 21 R7

PROGRESS CHECK

Skills: Dividing 4-digit by 2-digit number; checking division with multiplication

1. Divide and check. 1565 ÷ 95 or $95\overline{)1565}$ 16 R45

2. Divide and check. 2345 ÷ 35 or $35\overline{)2345}$ 67

3. Divide and check. 3486 ÷ 42 or $42\overline{)3486}$ 83

*4. Divide and check. 1111 ÷ 100 or $100\overline{)1111}$ 11 R11

117

STOP See activity 5, page 122b.

See activity 6, page 122b.

goal Practice in checking division computation by multiplication; **Progress Check** – dividing by a 2-digit divisor

page 117 If multiplication is to truly check an answer, the multiplication must actually be computed. Simply writing related statements without verifying the numbers checks only the understanding of how multiplication is related to division. Stress that a problem that fails to check is a signal to go back and redo the work – not simply to accept the existence of an error. The error can be in the division or it can be in the multiplication-check step too.

Provide any additional help needed before assigning the Progress Check. Please remember – dividing and checking doubles the number of problems. A few problems go a long way!

After completion of the Progress Check, work individually with those who made mistakes on problems 1 through 3. Have the youngster explain how he computed. This will help you determine the type of help he needs. Watch for such trouble spots as:

- Division facts not mastered
- Poor understanding of place value
- Errors in multiplication, subtraction, or addition
- Remainder forgotten

Any one of these trouble spots can cause the whole division problem to be incorrect, and yet the pupil may understand the basic division procedure. Refer to page 121 for additional problems. This may be a good place to enlist the help of peer tutors.

lesson Pages 118, 119, 120

goal Introduction to a shorter division algorithm

memo Pages 118, 119, and 120 are **recommended for all pupils.** Many pupils are now ready, willing, and able to move to a shorter division algorithm. Others are just beginning to experience success. Upon completion of these pages, let each learner choose the algorithm he feels most comfortable with. The focus remains on **accuracy**.

page 118 Nice and easy. Don't rush. Discuss as you go along. Hold off comparing the **old** algorithm and the **new** until page 120. Praise students who already see the similarities—but don't suggest them. Let this discovery be theirs. Go right on to page 119.

The form that you have been using to write a division problem is a good one. You just about can't get lost while you are dividing, subtracting, multiplying, and adding. You can always use that form if you want to.

There is another form that saves some writing. If you understand it and like it, you can use it.

750 people must get from the train station to the stadium. One bus holds 50 people. How many busloads will there be?

There is no change in the thinking steps.
THINK How many 50s in 750?
The estimate is more important than ever.

$10 \times 50 = 500$
$20 \times 50 = 1000$
More than 10 buses but less than 20 buses will be needed.

You start to write the problem in the same way.

Write only the 1 in the tens place.

```
   15
50)750
   50↓   1 ten × 50 = 50 tens   Write 50.
   250   Now you are ready for the ones.
   250   5 × 50 = 250   Write the 5 in the quotient.
     0
```

It does look a bit like magic.
But don't be afraid of something new. There is no rush.
Take time out to do the problem the old way.
Did you get the same quotient?

118

Study another example.

19)874 Estimate first. Round 19 to 20.
How many 20s in 874?
$40 \times 20 = 800$ That's close.
$50 \times 20 = 1000$ Too many.
There are more than 40 but less than 50 20s in 874.

Now compute.

```
    4
19)874
   76      4 tens × 19 = 76 tens    O.K. so far?
   11
```

You are ready for the ones in the next step.

```
    46
19)874
   76↓
   114   (6 · 19 = 114) Write the 6 in the quotient.
   114
     0
```

When you use this method, finish the multiplication before you write the digit in the quotient. You will want to make sure you are right.

The only place where you can have trouble is sloppy writing. For example:

```
    4
19)8 74      ← WHERE IS THE TENS PLACE?
   76        ← WHAT IS TO BE SUBTRACTED FROM WHAT?
```

If you have problems like this, you'd better stick with the other form.

goal Continued development of a shorter division algorithm

page 119 Continue your nice and easy discussion and development of the shorter algorithm. Take time to answer questions. None of your pupils are sloppy writers, are they? (Don't we all wish that the answer would be no.) Lined paper turned sideways provides place-value columns. This does help — sometimes.

goal Completing the development of a shorter division algorithm

page 120 Now is the time to compare the **old** method with the **new**. Remember—the individual pupil is free to choose the form that he or she wants to use to compute. Some youngsters need a little prodding to try anything new, but no one should become frustrated.

Look at the two ways of writing a division problem. Look to see how they are alike. How are they different?

The problem is $27\overline{)1836}$.

The first step for both is an estimate.

Think Round 27 to 30.
How many 30s in 1836?
$60 \times 30 = 1800$
$70 \times 30 = 2100$
It's less than 70 but more than 60.

First step

$$\begin{array}{r} 60 \\ 27\overline{)1836} \\ \underline{1620} \end{array} \quad 60 \times 27$$

First step

$$\begin{array}{r} 6 \\ 27\overline{)1836} \\ \underline{162} \end{array} \quad 6 \text{ tens} \times 27 = 162 \text{ tens}$$

How are the first steps alike? How are they different? Same multiplication problem; in the first you write 6 in the second you write 6 tens by placing 6 in the te

Next step

$$\begin{array}{r} 68 \\ \underline{8} \\ 60 \\ 27\overline{)1836} \\ \underline{1620} \\ 216 \\ \underline{216} \\ 0 \end{array} \quad \begin{array}{l} \\ \\ \\ \\ 60 \times 27 \\ \\ 8 \times 27 \\ \\ \end{array}$$

Next step

$$\begin{array}{r} 68 \\ 27\overline{)1836} \\ \underline{162} \\ 216 \\ \underline{216} \\ 0 \end{array} \quad \begin{array}{l} \\ \\ 6 \text{ tens} \times 27 = 162 \text{ tens} \\ \\ 8 \times 27 = 216 \\ \\ \end{array}$$

How are the last steps alike? How are they different? Same multiplication problem; in the first you write 8 in the second you write 8 ones by placing 8 in the on

Try just two problems, using the shorter method. If you don't like it, you can use the other method for the rest of these problems.

1. How many 63s in 882? 14
2. How many 39s in 702? 18
3. How many 47s in 799?
4. How many 78s in 858? 11
5. How many 23s in 437? 19
6. How many 36s in 396?
7. How many 57s in 855? 15
8. How many 68s in 884? 13
9. How many 82s in 984?

120

EXTRA PRACTICE

1. Round each number to the nearest ten. Be careful.

a 587 590	**b** 681 680	**c** 43 40	**d** 9007 9010
e 11,111 11,110	**f** 209 210	**g** 330 330	**h** 1006 1010

2. Round each number to the nearest hundred.

a 1567 1600	**b** 1199 1200	**c** 956 1000	**d** 1096 1100

3. Estimate answers.

a 8)16 2	**b** 8)170 20	**c** 8)161 20	**d** 80)161 2
e 70)79 1	**f** 70)790 10	**g** 70)7900 100	**h** 7)790 100
i 82)4107 50	**j** 82)417 5	**k** 53)395 8	**l** 350)3905 10

4. Estimate. Divide. Use either form. Check. Estimated answer in parentheses

a 5)17 (3) 3 R2	**b** 5)170 (30) 34	**c** 50)170 (3) 3 R20	**d** 50)1700 (30) 34
e 40)181 (4) 4 R21	**f** 40)1810 (40) 45 R10	**g** 39)181 (4) 4 R25	**h** 39)1810 (40) 46 R16
i 27)550 (20) 20 R10	**j** 27)555 (20) 20 R15	**k** 270)5550 (20) 20 R150	**l** 27)5550 (200) 205 R15
m 69)5400 (70) 78 R18	**n** 69)5420 (70) 78 R38	**o** 690)5400 (7) 7 R570	**p** 690)5420 (7) 7 R590

121

lesson Pages 121, 122

goal Practice in rounding, estimating quotients, and dividing

page 121 Some pupils may have already made use of parts of this page for additional practice. Save the page for those who need more help after the Checkout. Pupils who are operating successfully may skip the page entirely.

goal **Checkout**—rounding, estimating quotients, and dividing by a 2-digit divisor

page 122 The Checkout contains every type of skill necessary to compute division efficiently. These skills are identified in the answer key to help you select the types of additional help and practice necessary.

Sometimes practice with only a few more problems is necessary to help **set** a skill in a youngster's mind. This extra work will help if he knows basically how to operate. Practice without understanding reinforces faulty learning. You will have to decide which pupils need more practice and which pupils need your attention. You know the pupils who are ready to go on. Page 121 provides additional practice.

Make sure that you praise the youngsters. This has been a long, hard chapter. They did do a great job, didn't they?

CHECKOUT

Your goal in this chapter was to gain skill in division. Did you? Prove it. Do the following problems.

Skill: Rounding to nearest ten

1. Round each number to the nearest ten.

   a 49 **b** 125 **c** 764 **d** 5777 5780

Skill: Round to nearest hundred

2. Round each number to the nearest hundred.

   a 549 500 **b** 279 300 **c** 1487 1500 **d** 5550 5600

Skill: Rounding and estimating quotients

3. There are 856 students at Martin Luther King Elementary School. About how many hundreds of students are there? (Round to the nearest hundred.) Abou

4. A total of 964 people paid for tickets to the game. About how many hundreds of people bought tickets? (Round to the nearest hundred.) About 1000 or 10 hu

Skill: Estimating quotients

5. Estimate answers. Show what you did to get an estimate.

   a 5×97 $5 \times 100 = 500$ **b** 18×27 $20 \times 30 = 600$

   c $6\overline{)691}$ $\overset{100}{6\overline{)700}}$ **d** $19\overline{)99}$ $\overset{5}{20\overline{)100}}$

   e $67\overline{)131}$ $\overset{2}{70\overline{)130}}$ **f** $57\overline{)2988}$ $\overset{50}{60\overline{)3000}}$

Skills: Estimating quotients; dividing by 2-digit number; checking division with m

6. Estimate. Divide. Check.

   a $\overset{(7)\ 7}{30\overline{)210}}$ **b** $\overset{(7)\ 7}{31\overline{)217}}$ **c** $\overset{(100)\ 116\ R75}{78\overline{)9123}}$

   d $\overset{(80)\ 75\ R22}{83\overline{)6247}}$ **e** $\overset{(80)\ 72\ R42}{44\overline{)3210}}$ **f** $\overset{(400)\ 380}{12\overline{)4560}}$

122

See activity 7, page 122b.

See activity 8, page 122b.

RESOURCES

another form of evaluation

for progress check– page 104

Check yourself on this skill.
Estimate your answers.

	(a)	(b)	(c)	(d)
Group 1.	$30\overline{)30}$ 1	$30\overline{)300}$ 10	$30\overline{)3000}$ 100	$30\overline{)30{,}000}$ 1 000
Group 2.	$20\overline{)200}$ 10	$30\overline{)200}$ 6	$40\overline{)200}$ 5	$50\overline{)200}$ 4
Group 3.	$60\overline{)200}$ 3	$60\overline{)220}$ 3	$60\overline{)240}$ 4	$60\overline{)260}$ 4

for progress check– page 113

Divide.

1. $7\overline{)44}$ 6 R2
2. $20\overline{)124}$ 6 R4
3. $50\overline{)3500}$ 70
4. $40\overline{)4000}$ 100
5. $60\overline{)730}$ 12 R10
6. $46\overline{)828}$ 18
7. $24\overline{)3480}$ 145
8. $36\overline{)5278}$ 146 R22

for progress check– page 117

1. Divide and check. 7738 ÷ 53 or $53\overline{)7738}$ 146
2. Divide and check. 5493 ÷ 27 or $27\overline{)5493}$ 203 R12
3. Divide and check. 6218 ÷ 46 or $45\overline{)6075}$ 135

*4. Divide and check. 2325 ÷ 200 or $200\overline{)2225}$ 11 R25

for checkout– page 122

1. Round each number to the nearest ten.
   a) 53 50 b) 435 440
   c) 837 840 d) 4608 4610
2. Round each number to the nearest hundred.
   a) 638 600 b) 585 600
   c) 2754 2800 d) 4320 4300
3. The movie theater has seats for 528 people. About how many hundreds of people can sit in the theater? (Round to the nearest hundred.) About 500
4. There are 365 pages in the book. About how many hundreds of pages are there? (Round to the nearest hundred.) About 400
5. Estimate answers. Show what you did to get an estimate.
   a) 7 × 63 (7 × 60 420) b) 32 × 58 (30 · 60 = 1800)
   c) $5\overline{)484}$ ($5\overline{)500}$ 100) d) $33\overline{)95}$ ($30\overline{)100}$ 3) e) $76\overline{)843}$ ($80\overline{)800}$ 10) f) $24\overline{)6345}$ ($20\overline{)6000}$ 300)
6. Estimate. Divide. Check.
   a) $60\overline{)360}$ (6) 6 b) $42\overline{)856}$ (20) 20 R16
   c) $21\overline{)4852}$ (200) 231 R1 d) $57\overline{)6391}$ (100) 112 R7
   e) $86\overline{)8924}$ (100) 103 R66 f) $18\overline{)5645}$ (300) 313 R11

activities

1. things wood cubes

Label the faces of each cube to provide three types of cubes:

- 2, 3, 4, 5, 6, 7
- 40, 50, 60, 70, 80, 90
- 200, 300, 400, 500, 600, 700, 800

Two cubes are rolled at a time. The product is the pupil's score. Pupils predetermine the score needed to win. Group pupils with similar practice needs. To generate appropriate practice, vary the cubes given to a group.

2. Have the pupils collect information about the freeway system in your state.

- The approximate width of a freeway from fence to fence
- The approximate width of a lane of traffic
- The number of miles of freeway in the entire state (A state road map will prove helpful.)

The state highway department can furnish some of this information. Don't overlook the possibility of actually measuring if you have an overpass for pedestrians only nearby.

Use the information found to answer the following questions:

- About how many square miles of land are taken up by freeways in your state?
- About how much corn (or local crop) would grow on that much land in one year?

Extension questions:

- How many cubic yards of concrete did it take to build the state freeway system?
- How much would that much concrete cost?

3. Individual activity (Provide the pupil with the following practice problems and cross-number puzzle.)

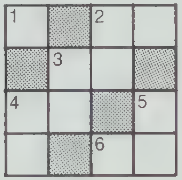
Divide. Write only the remainder in the cross-number puzzle.

ACROSS

1. $7\overline{)41}$ (5 R6)
2. $50\overline{)536}$ (10 R36)
3. $82\overline{)394}$ (4 R66)
4. $40\overline{)392}$ (9 R32)
6. $30\overline{)264}$ (8 R24)

DOWN

2. $55\overline{)421}$ (7 R36)
3. $72\overline{)422}$ (5 R62)
4. $83\overline{)2029}$ (24 R37)
5. $90\overline{)684}$ (7 R54)



4. things spirit masters

Stump Your Neighbors. Have each pupil make up a long-division word problem. Make sure the divisor has 2 digits and the dividend 4 digits.

After each pupil has prepared his division problem, you will need to edit the problems so that they can be understood. The problems must be numbered 1, 2, 3, 4, . . . Have a committee of three pupils write all the problems on spirit masters that you provide. Divide the work.

Have everyone work the problems at some time. A correction committee can check the papers. The problem that stumps the most pupils wins.

Too many problems? Then have half the group do half of the problems; the rest can do the other half.

5. things spirit master

Prepare a spirit master as shown.

Match each problem with its answer.

Problems	Answers	Code Letter
1. 1091 ÷ 51	31	T
2. 3968 ÷ 62	21 R20	G
3. 1638 ÷ 26	82 R55	A
4. 5385 ÷ 65	63	E
5. 1085 ÷ 35	64	R

Write the code letter found after the answer in the appropriate box.

Problem	1	2	3	4	5
Answer					
Code Letter					

6. things clock with second hand

Individual activity (Provide the pupil with the following directions.)

1. Be a clock watcher. Count the number of breaths that you take in one minute.
2. Use the information that you found in step 1 to complete the following chart.

Length of time	Estimated number of breaths I take
1 minute 1 hour 1 day 1 week 1 month 1 year	

7. things spirit master

Prepare a spirit master as shown. Time for imaginations. Today you are a stock clerk in a grocery store. Items must be marked with prices before they are placed on the shelf. Your job—to figure the price of each item.

Price per carton	Number in a carton	Price per item	O.K.'d by storekeeper
\$12.50 \$ 7.00	12 10		

Select a peer tutor to act as storekeeper and verify the prices. Make a decision about how remainders affect prices.

8. things clock with second hand; metrestick or yardstick

Individual activity (Provide the pupil with the following directions.)

1. Walk a straight path for 10 seconds. No fair running.
2. Measure the distance you walked with a metrestick or yardstick.
3. Use the information you found in steps 1 and 2 to complete the following chart.

Length of time	Estimated distance I can walk
10 seconds 1 minute 1 hour 3 hours	

4. About how long would it take you to walk a mile? A mile is 5280 feet.
5. Could you walk a kilometre in half an hour?

additional learning aids

operation—chapter objectives 1, 2, 3

SRA products

Mathematics Learning System, Activity Masters, level B, SRA (1974)
  Spirit masters: W-7, 8, 20, 26, 27, 28, 30, 31; P-6, 7

Computapes, SRA (1972)
  Module 4, Lessons: MD 35, 36
Computational Skills Development Kit, SRA (1965)
  Division cards: 9, 10
Cross-Number Puzzles (Whole Numbers), SRA (1966)
  Division card: 10
diagnosis: an instructional aid—Mathematics Level B, SRA (1972)
  Probe: M-4
Skill through Patterns, level 5, SRA (1974)
  Spirit masters: 36, 37, 46, 47, 48, 49, 50, 51, 53, 54, 55, 68, 71, 72

other learning aids (described on page 144e)—
Numble, Rally with Remainders

6 MATH SENTENCES

before this chapter the learner has—

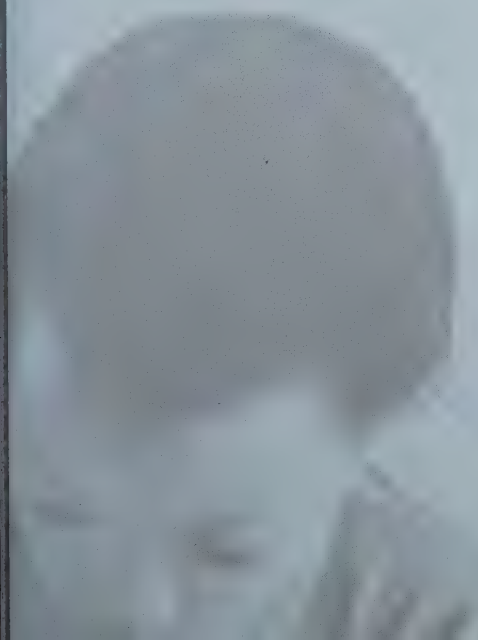
1. Identified simple math sentences as true, false, or open
2. Found solutions to open math sentences with one operation and one placeholder
3. Written a math sentence to show that a solution to a one-step word problem is true

in chapter 6 the learner is—

1. Reviewing true, false, and open math sentences
2. Mastering finding a solution to an open math sentence with one operation and one placeholder
3. Mastering writing a math sentence to show that a solution to a one-step word problem is true
4. Writing word problems to match math sentences
5. Solving two-step word problems
6. Eliminating extraneous information from word problems
7. Deciding whether a word problem gives enough information to solve the problem
8. Extrapolating information from a chart to use in solving word problems

in later chapters the learner will—

1. Master solving a one-step word problem with extraneous information
2. Locate information given in a chart or table



Notes & Things

The math sentence in this chapter is used to summarize how the word problem has been solved rather than to determine how to solve the word problem. This distinction is important. The reason for taking this approach needs some explanation.

If an adult uses an open math sentence to help solve a problem, he probably uses it to sort out and organize information in a rather complex situation. An adult rarely writes an open sentence for a situation if the answer is obvious. (He might complete a computational check for accuracy, but it would be unusual to complete an algebraic manipulation of a simple problem.) Remember, please, that theoretically the adult has mastered computation. He can read, he can reason, and he may have been exposed to at least one year of high school algebra. Yet we expect children to do much more than adults do with a problem situation.

Number stories that can be put in texts for children of this age are generally quite simple because the child may not have mastered all computational skills, his reading may be limited, he really hasn't had enough experience (practice) to do a lot of reasoning, and he is just getting to know what a number sentence is. Many times a child can read (or listen to) a number story and somehow know the answer even though he may not have any idea how he got the answer. For goodness sake, let's not discourage this intuitive approach to problem solving. We may discourage it if we insist that the child write an open sentence to summarize what the story says. Finding the answer is the ultimate goal, after all!

To give children the necessary experience with math sentences, let the true sentence be a way to show that an answer is right. Please let them put the puzzle together with all the pieces of the puzzle at hand. A child will continue to reinforce his knowledge of math sentences. He will be forced to critically examine his own reasoning. If he can't find a true sentence to confirm his answer, he will know that his answer probably is not correct. It is *his* job to find the correct answer rather than your job to point out that his answer is wrong.

G. Polya's book, *How to Solve It,* has been the foundation for much of the thought in this series. Polya's model for problem solving is, however, designed for the student who has had a wide variety of experiences. The intent of this series is to provide those experiences on which analogies and generalizations can be based. Developing a good problem solver is a long process. Knowing how to write a math sentence is only a small but important part of the process. Developing skill in devising and interpreting math sentences is also a long process. Mastery of this concept will require many more experiences than this single chapter can provide. But this chapter is a beginning.

things

file cards and recipe box

For the extra activities you will want to have these things available:

felt pens or crayons of 3 colors

goal Think about and explore ideas through a picture clue

page 123 The symbol painted on the roadway is primarily for automobiles but does it tell the person walking across that road anything?

Math sentences are made up entirely of symbols. Each different symbol has a meaning. Trying to figure out that meaning can be a challenge.

Get everyone started thinking about the role of symbols by talking about those that both a pedestrian and a driver of a vehicle must know. It is unfortunate that highway signs are not the same for every country in the world – they aren't the same even for every major city in a country. Both the color and the symbol printed on the sign itself may vary. If possible get a copy of the traffic symbols used in your area from the local highway department. It will be good learning experience for everyone. Have each person start a personal sketchbook of local signs that govern walkers, bike riders, and drivers. If the youngsters see a sign with a bold, black slash through a symbol and learn that the black mark means "no," then the slash mark through a symbol used in math books certainly must also have the same meaning for them.

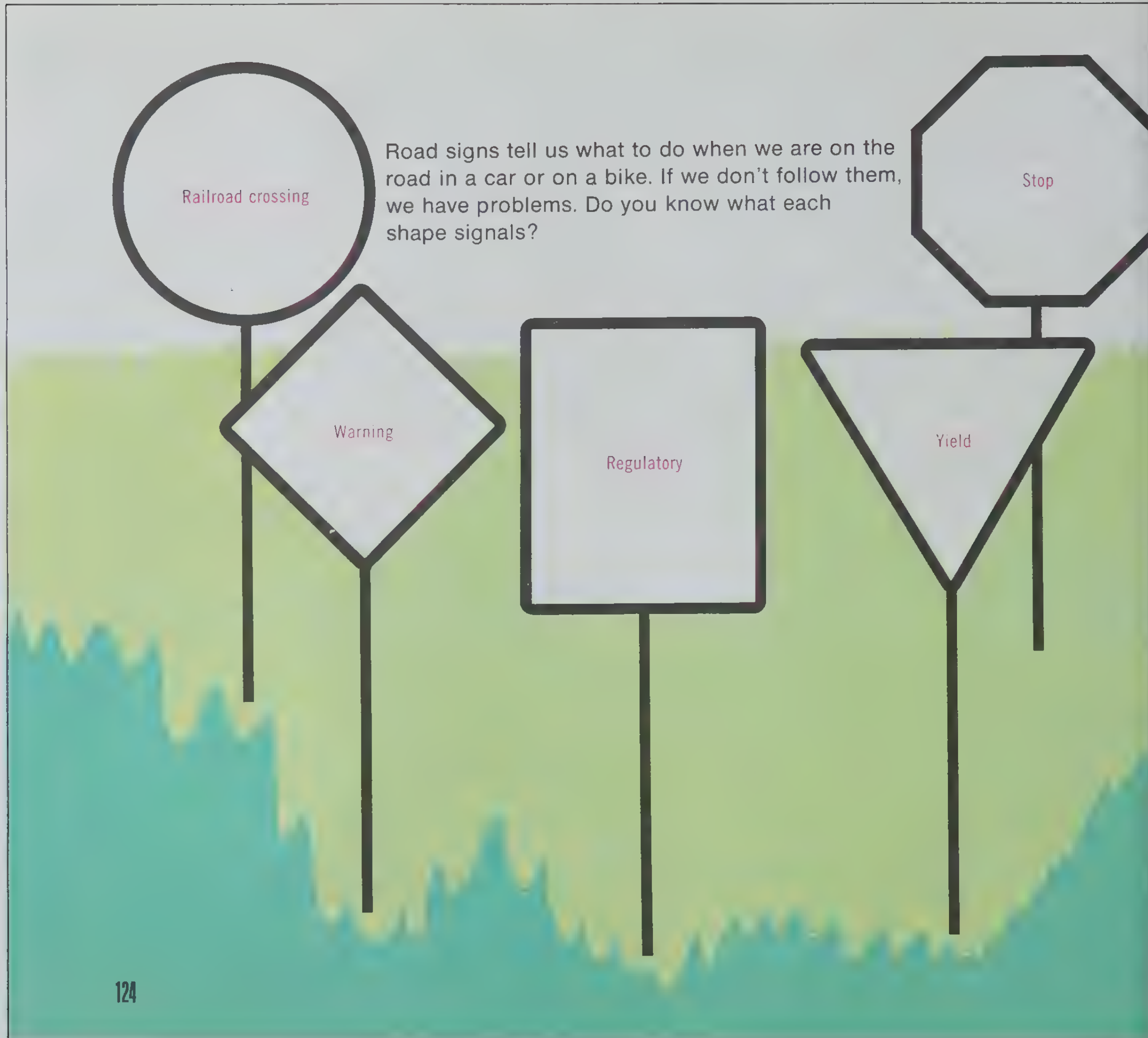
lesson Pages 124, 125, 126

goal Examining road signs to introduce the theme of the chapter

memo Pages 124 and 125 work together.

page 124 Challenge your pupils to identify the different road signs shown. What does each sign say? Where would one be likely to find each sign? Why are these road signs used?

Go on to page 125.



Pretend the picture below shows a new road. You are in charge of placing traffic signs. Where should they go? What should the sign signal? Placement of signs will vary. Discuss why signs are placed as they are.

GOAL

s to find the signals that will help you solve a math problem

The caution light has turned to green

GO

125

goal Continuing the introduction of the chapter theme

page 125 Those road signs must now be placed along a road. Pupils should complete that job independently. The important aspect of this exercise is that each person be able to justify his decisions. Each sign has a purpose. Which placement of signs would make sense? Which placement of signs would serve to move traffic efficiently, yet protect people and property? Certainly more than one plan can be considered right.

Now, on to examining signs and symbols in math.

goal Review of the operation and relation symbols of mathematics

page 126 Some pupils may have had no previous experience with math sentences. The top half of the page is designed to get everyone thinking about operation and relation symbols. Notice that the pupils are to make **true** sentences. **True** may be a new math word to them. If so, write examples such as the following on the chalkboard:

$2 + 3 = 5$ is a true sentence.
$2 = 3 - 5$ is a sentence, but it is false.
$2 - 3 = 5$ is also a false sentence.

A relation symbol signals a sentence. But the sentence is not necessarily true.

Problem 2 confirms the importance of a relation symbol in making a sentence. Point out that no operation symbols have been used with the numbers. But each set can be made into a sentence that will be true or false, depending upon the relation symbol that is used.

Problems 1 and 2 are independent work.

Math has symbols too. If we don't follow the directions they give, we have a problem. Without the symbols, we don't have a problem at all—at least not a math problem.

1. Pretend you are writing a math book. You are in charge of placing the operation and relation symbols to make true sentences. Where should they go in each set of three digits? What should the symbols signal? Accept still other appropriate answers.

a $3 \overset{+}{\ } 5 \overset{>}{\ } 7$	**b** $17 \overset{=}{\ } 8 \overset{+}{\ } 9$	**c** $4 \overset{-}{\ } 2 \overset{<}{\ } 3$
d $7 \overset{+}{\ } 10 \overset{=}{\ } 17$	**e** $2 \overset{\times}{\ } 4 \overset{=}{\ } 8$	**f** $2 \overset{+}{\ } 2 \overset{=}{\ } 4$
g $9 \overset{=}{\ } 3 \overset{\times}{\ } 3$	**h** $21 \overset{\div}{\ } 7 \overset{=}{\ } 3$	**i** $7 \overset{\times}{\ } 0 \overset{=}{\ } 0$
j $4 \overset{=}{\ } 12 \overset{\div}{\ } 3$	**k** $8 \overset{-}{\ } 0 \overset{=}{\ } 8$	**l** $1 \overset{+}{\ } 5 \overset{=}{\ } 6$

2. You have written math sentences with three numbers and one operation symbol and one relation symbol. Write more true sentences using only the relation symbols >, <, and =.

a $0 \overset{<}{?} 1$	**b** $10 \overset{>}{?} 9$	**c** $\frac{2}{2} \overset{=}{?} 1$
d $\frac{1}{3} \overset{=}{?} \frac{2}{6}$	**e** $1\frac{1}{2} \overset{>}{?} 1$	**f** $8 \overset{>}{?} 7$
g $\frac{2}{8} \overset{>}{?} \frac{1}{8}$	**h** $99 \overset{>}{?} 97$	**i** $\frac{3}{2} \overset{=}{?} 1\frac{1}{2}$
j $\frac{6}{8} \overset{=}{?} \frac{3}{4}$	**k** $\frac{9}{9} \overset{=}{?} \frac{7}{7}$	**l** $\frac{0}{4} \overset{=}{?} \frac{0}{5}$

a How long is segment A? 6 cm

b How long is segment B? 3 cm

c How long are A and B together? 9 cm

d Write a math sentence. 6 + 3 = 9

a How long is segment C? 9 cm

b How long is segment D? 3 cm

c How much longer is segment C than segment D? 6 cm

d Write a math sentence. 9 − 3 = 6

a What is the length of this book to the nearest centimetre? 26 cm

b What is the width of this book to the nearest centimetre? 21 cm

c How much longer is it than wide? 5 cm

d Write a math sentence. 26 − 21 = 5

You must be 16 years old to drive a car.

a How old are you? 10 or 11

b How many years must you wait to drive a car? 6 or 5

c Write a math sentence. 16 − 10 = 6 or 16 − 11 = 5

Sometimes simple little problems can fool you. Look out for this one.

a How many pages in this chapter? 22

b What page are you doing? Page 127 (Fifth page of chapter)

c How many pages more to do in this chapter? 17

d Write a math sentence. 22 − 5 = 17

127

lesson Page 127

goal Writing math sentences to check a computation

memo A different approach to math sentences is taken in the materials of this program. The math sentence is not required as the first step in finding the solution to a problem. (See **Notes and things**, page 122d.)

page 127 The problems are quite simple. The pupil is encouraged to use his reasoning power in order to get an answer to the problem and then to write a math sentence in order to check his answer. This process will give the pupil the necessary practice in writing sentences. It will also give him the freedom to manipulate objects, draw pictures, or use any technique that helps him find an answer that makes sense.

The approach introduced on this page will be used throughout the chapter.

lesson Pages 128, 129, 130

goal Analysis of true, false, and open math sentences

page 128 There's much to talk about on this page. For problem 2, plug in some values for ***n***; then ask whether the sentences are **true** or **false**. Make sure you plug in some values to make the sentence true and some to make it false so that the pupils can see that this sentence is **open**.

For problems 3 and 4, have pupils write example sentences to back up their answers.

A math sentence may be

TRUE
or
FALSE

1.

Is each of these sentences **TRUE?** Yes

a $10 + 5 = 15$

b $14 \times 2 > 25$

c $18 \div 3 = 6$

d $10 < 16$

Is each of these sentences **FALSE?** Yes

e $10 + 5 = 16$

f $3 \times 9 > 30$

g $14 \div 7 < 1$

h $15 < 10$

2.

What about these sentences? True or false? You can't tell.

a $15 + 10 = n$ **b** $14 \times n > 32$ **c** $n < 5$

Do you know whether they are true? false? Your answer depends on what n stands for. Right? These are called open sentences. No; no Yes

3.

Do all math sentences have to have an operation symbol such as +, −, ×, or ÷? No

4.

Do all math sentences have to have a relation symbol such as =, ≠, >, or <? Yes

5.

What do all open sentences have that true sentences and false sentences do not have? An unknown number

6.

Mark gets a weekly allowance of 25 cents. Mark says that if he saves all of his allowance each week, he will have $4.00 in 16 weeks.
Is his statement true? false?

A math sentence can help you decide.

$16 \times 25 = 400$ Is that a true sentence? Yes

Is Mark's statement true or false?

128

Challenge your pupils to follow these directions. Take each true sentence in problem 1. Change only the relation symbol. What happened to the sentence? (False sentence) Change the relation symbol again. What happened to the sentence? (False again)

Example: $10 + 5 = 15$ $10 + 5 < 15$
$10 + 5 > 15$

How many relation symbols make a sentence true? (1) How many relation symbols make a true sentence false? (2)

Problems 1–3: Accept other appropriate math sentences.

1 Jan practiced 30 minutes before school and 20 minutes after school. She said she practiced for an hour.

a Was her statement true or false?

b Write a math sentence to back up your answer. $30 + 20 < 60$ or $30 + 20 = 50$

2 Sam delivers papers every day of the week. He walks 20 blocks each morning. He said that he walked more than 100 blocks every week just delivering papers.

a Was his statement true or false?

b Write a math sentence to back up your answer. $20 \times 7 > 100$ or $20 \times 7 = 140$

True or False

3 Ma Dell made 28 cupcakes. She gave them to her 7 children. She said each one got 5 cupcakes.

a Is her statement true or false?

b Write a math sentence to back up your answer. $28 \div 7 < 5$ or $7 \times 5 = 35$

4 Simon walked several blocks to the store. Then he walked 4 more blocks. He said, "I have walked 13 blocks in all." How many blocks did he walk to the store? Does an open sentence tell the number of blocks to the store? No

$$n + 4 = 13$$

a Can you make that open sentence true? Yes

b What numbers will make it true? 9

c What numbers will make it false? All numbers except 9

5 Find a number that will make each open sentence true.

a $24 \div n = 3$ 8

b $8 + n = 20$ 12

c $2 \times n = 2$ 1

d $14 - n = 14$ 0

e $6 + n < 8$ 0, 1

f $5 - n > 4$ 0

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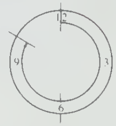
goal Writing a math sentence to check a word-problem solution

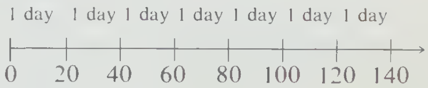
page 129 Here is an excellent opportunity to let everyone see that different people solve problems in different ways. If one pupil volunteers his way of **thinking** about a problem, perhaps another pupil will have another way. Most of these problems lend themselves to simple pictures as well. A picture may be the perfect aid for those youngsters who are uptight about word problems.

Problem 1

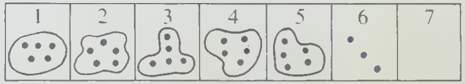


Draw a clock without hands. 30 minutes, then 20 more minutes. Is that 1 hour?

Problem 2

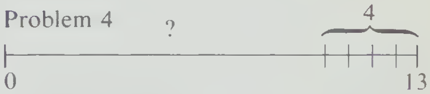


Problem 3



Draw 28 dots. Put 5 in each ring. Do all 7 children have 5?

Problem 4



In problem 4, $n + 4 = 13$ does indirectly tell you how many blocks to the store. But since it is an open sentence, you must solve the sentence first. This is a rather tricky question.

goal Finding solutions for open sentences; identifying true, false, and open sentences

page 130 Take time to explore how a **replacement** for an open sentence may or may not be the **solution**. A sentence may have no solution, one solution, or more than one solution.

Be sure to talk about problem 4 with everyone. An EQUATION must have an equal sign as the relation symbol, along with something that indicates a quantity on each side of the equal sign. That makes it an equation as well as a sentence. Have the youngsters make up equations and other math sentences to back up their answers.

A replacement for a letter, a box, or a question mark can make a sentence true or false. A replacement that makes the sentence true is a solution to the sentence.

1. Find solutions for each open sentence.

a $27 \div n = 3$ 9

b $6 \times 8 = r$ 48

c $y - 12 = 7$ 19

d $24 + 40 = s$ 64

e $24 \div 4 = q$ 6

f $\frac{1}{2} + b = 1\frac{1}{2}$ 1

g $\frac{1}{4} + b = \frac{3}{4}$ $\frac{2}{4}$ or $\frac{1}{2}$

h $37 - n = 15$ 22

i $n + \frac{1}{4} = \frac{3}{4}$ $\frac{2}{4}$ or $\frac{1}{2}$

2. Which of these sentences are true? Which are false? Which are open?

a $\frac{4}{5} + \frac{1}{10} > 1$ False

b $32 \div a = 8$ Open

c $7 \times 6 \neq 40$ True

d $9 \times 5 < 40$ False

e $\frac{1}{3} > \frac{4}{12}$ False

f $96 \div 32 = 3$ True

g $18 \div b = 6$ Open

h $100 \times 17 = n$ Open

i $154 - n = 67$ Open

j $36 \div 6 = 5$ False

k $4 \times b = 28$ Open

l $c \times 7 = 49$ Open

3. Find solutions for the open sentences in problem 2.

b 4 g 3 h 1700 i 87 k 7 l

4. Are all equations math sentences? Yes
Are all math sentences equations? N
Can an equation be an open sentence? Yes
What relation symbol must every equation have? =

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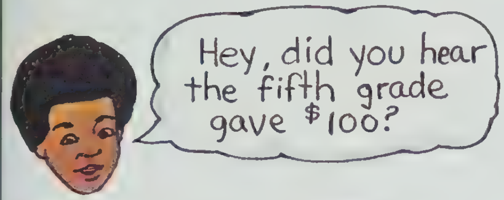
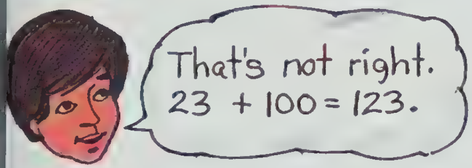
Have each pupil write an open sentence for which he can find a solution. Solutions should be written on a different piece of paper. Have the pupils exchange only sentences and solve each other's sentences. Then have them compare solutions. If the solutions do not agree, exchange with a third person.

ason read in the school newspaper that he fifth grade had made a contribution to the ctivity fund. This raised the total to $140.

veryone knew there was only $23 in the fund efore the fifth grade's contribution.





Simon guessed again. "Maybe it was $120." Jason hated to argue, but he had to say, "No. $23 + 120 = 143$." Simon guessed again. "How about $117?" Was this guess more accurate? Was it right? Yes; yes—$23 + 117 = 140$

Let's use math sentences to recap the discussion.

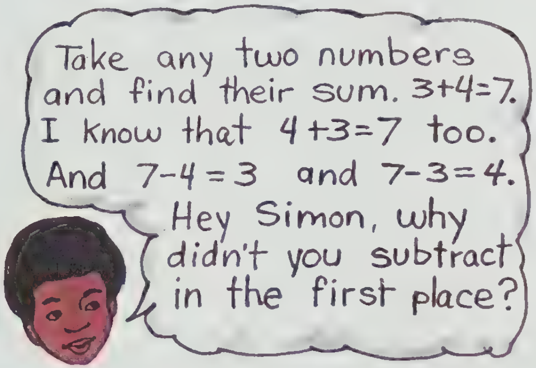
$23 + n = 140$

Can n be 100? Nope! $23 + 100 = 123$

Can n be 120? Wrong again! $23 + 120 = 143$

Can n be 117? Right! $23 + 117 = 140$

Simon was probably good at estimating. Jason was good at computing. He remembered something special about addition and subtraction.



$23 + n = 140$ can be thought of as $n = 140 - 23$.

Is that magic, or does it work all the time?
It works all the time.

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lesson Pages 131, 132

goal Using subtraction to solve open addition sentences

page 131 The dialogue that starts on this page is an example of a pupil using his reasoning power to estimate an answer. Here computation proves the estimates to be a little off, but the problem is correctly interpreted. Ask the pupils what ***n*** represents in the equations.

Have your pupils give examples of equations to prove the relationship of subtraction to addition, as shown at the bottom of the page. Begin with an open addition sentence. Then find the solution by subtraction.

goal Practice in finding solutions for open sentences; deciding whether an estimate or an accurate answer is needed

page 132 Problems 2 and 3 require the pupils to solve open sentences with inequalities. Solve a few problems such as $15 + n > 27$ and $8 + n < 9$ as a group before beginning the assignment. Then problems 1 through 4 may be done independently.

Simon said he wasn't guessing to find an answer; he was estimating. What's the difference between a good guess and an estimate? Sometimes an estimate isn't good enough. An accurate answer must be computed.
An estimate is usually gotten by some procedure; a guess is not.

Jason saw an addition equation with one number to be added missing, so he subtracted to find the answer.

$$23 + n = 140$$
$$n = 140 - 23$$

1. Use Jason's method to solve the following.

   a $621 + n = 896$ — $n = 896 - 621$ — Solution: 275
   b $256 + n = 450$ — $n = 450 - 256$ — Solution: 194
   c $127 + n = 216$ — $n = 216 - 127$ — Solution: 89

2. Find solutions for these sentences.

   a $163 + n > 200$ — $n > 200 - 163$ — Solution: Numbers greater than 37
   b $256 + n < 257$ — $n < 257 - 256$ — Numbers less than 1
   c $525 + n > 532$ — $n > 532 - 525$ — Numbers greater than 7

3. Find a solution for each sentence.

   a $128 + n > 212$ — Numbers greater than 84
   b $128 + n < 212$ — Numbers less than 84
   c $n = 212 - 128$ — 84
   d $89 + n > 265$ — Numbers greater than 176
   e $89 + n < 265$ — Numbers less than 176
   f $n = 265 - 89$ — 176
   g $248 + n > 285$ — Numbers greater than 37
   h $248 + n < 285$ — Numbers less than 37
   i $n = 285 - 248$ — 37

4. Which is needed—an accurate answer or an estimated answer?
   This would depend on your purpose for needing these answers.

   a How much money you should take to the store Estimated
   b How much money you can spend at the store Accurate
   c How long it takes to get to school Estimated
   d How long it takes to get home Estimated
   e How many people in your city Estimated
   f How many people are registered to vote Accura
   g How many people do vote Accurate

You will use this information on the next page.

Every math sentence must have what mathematicians call a relation symbol. There are lots of these symbols. The relation symbols you use the most are $=$, $>$, and $<$. In a sentence they tell how one number relates to another.

Look back at the situations in problem 4 on the page before.

a and b Let t stand for the money you take.
Let s stand for the money you spend.

Can t equal s? Yes Can t be greater than s? Yes Can t be less than s? What would happen? No (unless you have a charge account or write a check)

c and d Let t stand for the time it takes to go to school.
Let h stand for the time it takes to go home.

Can t equal h? Yes Can t be greater than h? Yes Can t be less than h? Yes Why? Traffic patterns may change, you may take a different route, etc.

e, f, and g Let p stand for people.
Let r stand for those registered to vote.
Let v stand for those that do vote.

Can p equal r? (if you don't count people under 18 as people) Can p be greater than r? Yes Can r be greater than v? Yes

Look out! Can p equal v? No Can v be greater than r? No Can p be less than r? No

This is another one of those times when we have to think carefully. Can the math sentences on this page be called true or false? Why? Don't know; they contain unknown numbers.

Go back over the situations. Assign a number to each letter. (Be reasonable when you put the numbers into the story situation.) Find out if the sentence can be true or false when you are working with numbers.

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lesson Pages 133, 134

goal Relating open math sentences to real situations

page 133 This page is strictly for group discussion. The questions play on somewhat abstract word and number relationships. You may have some pupils who have not had enough experience to understand these situations. Substitute a value for the letter immediately if you feel your pupils cannot handle the abstraction. Or perhaps you will want to use the clue words from which the letters were taken.

The answer to **4a** and **b** "Can $t < s$?" is **no** mathematically, but **yes** with charge accounts. You might be ready for such an answer from our liberated younger generation.

goal Finding solutions for open math sentences; **Progress Check** – finding solutions for open sentences and identifying true, false, and open sentences

page 134 Discuss the top of the page. These computations should be completed by the pupils working independently. You may wish to hold the Progress Check for another day.

Look for subtraction errors with pupils who have troubles with problems 1 through 8 of the Progress Check. Check whether the pupil knows **how** to find n also.

Examine the pupils' errors for problems 9 through 16. Incorrect **true** or **false** answers signal computational troubles. Incorrect **open** answers signal lack of understanding of an open sentence. And there is one more trouble spot. Look for errors resulting from reading inequality signs in reverse.

Not every math sentence needs the operation symbol $+$, $-$, $\times$, or $\div$. Lots of math sentences have them, though.

1. Find a solution for each sentence.

a $500 + 250 = n$ 750 **b** $200 + 175 = n$ 375 **c** $125 + 175 = n$ 300 **d** $395 + 195 = n$ 590

No trouble, right? All you had to do was add. Keep on.

e $400 + n = 650$ 250 **f** $600 + n = 867$ 267 **g** $135 + n = 241$ 106 **h** $275 + n = 474$ 199

Not quite so easy, right? Did you really *add* to find the value of n?
No, you really subtracted.

If both Simon and Jason would get to work, it would help.

$300 + n = 475$ Simon says the solution is less than 475.
Jason says you should subtract to find how much it is.
$n = 475 - 300$ Now that's not so bad. What's the solution? 175

PROGRESS CHECK

Now it's your turn. Find the solution. Skill: Finding solutions to open sentences

1. $536 + n = 860$ 324	**2.** $728 + n = 887$ 159	**3.** $467 + n = 996$ 529	**4.** $143 + n = 971$ 828
5. $752 + n = 980$ 228	**6.** $685 + n = 900$ 215	**7.** $245 + n = 570$ 325	**8.** $489 + n = 600$ 111

Skill: Identifying true, false, or open sentences
Which of the following are true? Which are false? Which are open?

9. $3 \times 9 > 28$ False	**10.** $365 - 214 = 151$ True	**11.** $9 \times 8 = 54$ False	**12.** $9 + 6 > 15$ False
13. $72 \div n = 9$ Open	**14.** $3 + 9 > 15 - 3$ False	**15.** $35 < 9 \times 4$ True	**16.** $25 + 25 = n$ Open

134

See activity 1, page 144a.

Challenge capable pupils with these ideas:
$\neq$ is read "is not equal to"
$\ngtr$ is read "is not greater than"
$\nless$ is read "is not less than"
Which of these sentences are true? Which are false? $7 + 3 \ngtr 10$ $7 \nless 3$ $3 \nless 7$ $3 + 0 \neq 3$ $5 + 7 \neq 8$ $5 \ngtr 3$

If there are 33 rows and there are 6 trees in each row, how many trees in all?

Jason wrote an open sentence to find the answer.

$$33 \times 6 = t$$

What is the answer to the problem?

$$33 \times 6 = 198$$

Is this answer close to Simon's estimate? Yes

Solve these. You may want to start by writing an open sentence. Maybe you won't. But you are asked to write a true math sentence to show the right answer.
Accept other appropriate sentences.

1. There were 6 trucks. Each truck hauled 100 bundles of towels. How many bundles in the 6 trucks in all? $6 \times 100 = b$
   $6 \times 100 = 600$

2. Ma Dell made 60. She put the same number into each of 5 boxes. How many in each box? $60 \div 5 = b$
   $60 \div 5 = 12$

3. There were 24 in one room. There were 36 in another room. How many in all? $24 + 36 = a$
   $24 + 36 = 60$

4. Sam had 36. He gave 15 away. How many did he have left? $36 - 15 = g$
   $36 - 15 = 21$

135

lesson Pages 135, 136

goal Practice in solving word problems and writing math sentences to verify the answer

page 135 Discuss the example with everyone. Three main skills are used:

- Estimating
- Computing the answer
- Writing a math sentence

Estimating is a good way to start working a problem if the answer is not obvious. And Simon does a good job with this skill. Problems 1 through 4 should be worked independently. Estimation is not necessary for these problems.

goal Looking for word clues in word problems

page 136 Most pupils don't realize how they are influenced by word clues. These word clues sometimes are so strong that we act without thinking.

Have the youngsters jot down answers to problem 1. Then examine the word clues in the remaining problems. Do the clues always signal the correct operation? Did anyone get fooled? Please note that problem **2e** requires two steps. The answers to these problems should be done independently.

The word problems that result from problem 3 will be the beginning of a problem-solving box. Let your pupils be authors. A simple recipe box and file cards can be the source of good practice material. Put the problem on the front of the card and the answer on the back.

Sometimes when you solve word problems, you have to be a

You have to find word clues to know what arithmetic to do.

1. What operation *might* these words signal?

- **a** How many in all? Addition or multiplication
- **b** How much faster? Subtractio
- **c** How many were left? Subtraction
- **d** He added more. Addition or subtractio
- **e** What is the total? Addition or multiplication
- **f** How many in each? Divisio

2. See if your clues work. Find the answers.

- **a** 25 boxes. 5 in each box. How many in all? 125
- **b** 42 came early. 50 more got there. How many in all? 9
- **c** He ran the distance in 3 minutes. She ran the distance in $2\frac{1}{2}$ minutes. How much faster was she?
- **d** There were 20. 5 went home. How many were left? 1
- **e** Each bag would hold 5. There were 6 bags. There were 35 to be packed. How many were left? 5 Two steps: $5 \times 6 = 30$ 35 −
- **f** There were 50. He added some more. Now there were 60. How many did he add? 10
- **g** There are 25 in one book, 30 in the second book, and 10 in the third book. What is the total number in the three books? 65

3. You make up a simple story that has this question at the end: How many in each?

Lots of times word clues do help you decide how to compute. But always read the whole problem, or you might get fooled.

Word problems can be deceiving. Here are some problems in riddle form to prove the point.

Question: If one man has two sacks of grain and another has three sacks, which man has the heavier load?

Answer: The man with two sacks of grain, because the other man just has three sacks.

Question: Some months have thirty days, some thirty-one, but how many have twenty-eight days?

Answer: 12

Write a story for $32 \div 4 = n$

Joe's thought:

If 32 pennies are divided equally among 4 boys, how much money will each have?

Jackie's idea:

How many girls will get packages of gum if each girl gets 4 packages of gum and there are 32 packages in all?

Two stories. One math sentence. Same answer? Yes

Write a story for $6 \times 5 = n$

Hank writes:

There are 6 teams and 5 players on each team. How many players?

Pete says:

Last summer I worked for 6 days and was paid $5 a day. How much money should I have received?

Two stories. One math sentence. Same answer? Yes

1. You write a story to fit $32 \div 4 = n$.

2. Write a story for any two of these. Then solve every sentence.

a	$66 \div 11 = n$	6	**b**	$15 \times 4 = n$	60	**c**	$35 + a = 92$	57
d	$133 \div n = 19$	7	**e**	$190 \times n = 950$	5	**f**	$n - 42 = 77$	119

You can make up a story for just about any math sentence. But the important skill is to complete a true math sentence. A true math sentence is one good way to find out if your solution to the problem is correct.

lesson Pages 137, 138

goal Writing word problems to match given math sentences

page 137 Just as two people might choose two different but correct math sentences to summarize their thinking for a problem, two different stories might result in only one sentence. The pupils should be encouraged to maintain their own style of thinking but be challenged to prove that their thinking is reasonable.

Discuss the first half of the page with everyone. The material presented on this page should not be difficult, so pupils should have little trouble completing the bottom half independently.

goal Writing math sentences for word problems

page 138 Checking for mistakes and correcting the errors should be independent work. Save the last sentence on the page to talk about.

Here's a copy of Jerome's homework. He was supposed to write open sentences to fit the stories and then solve the sentences. He made some mistakes. Some answers are not even reasonable. Find the

mistakes

he made.

1 Of the 24 students in the class, 16 were never absent. How many were sometimes absent? O.K.

$24 - 16 = a$
$8 = a$

2 Mark bought some model airplane kits. Each kit had 37 pieces. There were 185 pieces all together. How many kits did Mark buy? $185 \div 37 = k$ $5 = k$

$185 \times 37 = K$
$6845 = K$

3 17 of the airplanes that arrived today at Swegis Airport were jets. 43 of the airplanes that arrived today were not jets. How many airplanes arrived today? $a = 43 + 17$ $a = 60$

$a = 43 - 17$
$a = 26$

4 If 1 ounce of prevention is worth 16 ounces of cure, how much is 32 ounces of prevention worth? $c = 16 \times 32$ $c = 512$

$c \times 16 = 32$
$c = 32 \div 16$
$c = 2$

5 Each of the 27 booths at the carnival had 17 prizes. How many prizes in all? O.K.

$27 \times 17 = p$
$459 = p$

6 144 avocados were packed in a box with 12 on each layer of the box. How many layers? $144 \div 12 = l$ $12 = l$

$144 \times 12 = l$
$1728 = l$

How could Jerome straighten out his thinking on problems 2, 3, 4, and 6? Use estimati

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Solve

Write a true math sentence that recaps the story.
Accept other appropriate sentences.

1. The 29 students of Miss Brown's fifth-grade class were planning a field trip. 17 in the class did not want to go to the local museum. Did the remaining students have enough votes for a majority? 29 − 17 = 12; no, not enough

2. Jason owns a large terrarium in which there are 16 brown, warty toads. He says he has twice as many soft, slimy snails as he has brown, warty toads. How many soft, slimy snails does he have? 2 × 16 = 32

3. Rita gets a weekly allowance of 25¢. If she saves all of her allowance each week, how long will it take her to save $3.00? 300 ÷ 25 = 12

4. Plastic phonograph records are made in a press. The press can make 1 record a minute. How many records could 3 presses make in 11 minutes? 3 · 11 = 33

5. A football field is 120 yards long and about 53 yards wide. It has field lines every 10 yards. This is part of a story. Write four versions of the story so that you have an addition problem in the first story, a subtraction problem in the second story, a multiplication problem in the third story, and a division problem in the fourth story.

Example: The quarterback started at the goal line. He ran 40 yards. On the next play he gained 20 yards. On the third play he lost the ball. How many yards did he gain on the two plays? 40 + 20 = 60

139

lesson Pages 139, 140, 141

goal Writing true math sentences to match a word problem

page 139 Pupils may complete this page as independent study. They will probably need 15 to 20 minutes. Computation is required.

The majority of pupils **will** find an answer first. Look out for the one who knows how to tackle the problem, can write a correct math sentence, but has the wrong answer because of some computation difficulties.

You may want to group pupils who have reading difficulties and have a peer tutor work with them.

goal Solving two-step word problems

page 140 Begin by having pupils describe how they would –

- Scratch an itchy foot if their shoe were on.
- Repair a flat tire on their bike.
- Eat a banana.
- Eat a peanut that is in its shell.
- Go from one room to another if the door were closed.

Get them to recognize that these tasks have something in common – they all require you to take more than one step to get the task done.

In the problems on this page more than one math sentence will be needed to get the job done. Encourage everyone by pointing out that since there is more than one way to look at a problem, their math sentences may not be the same – **but** their computed answers should be the same. Take time to talk about problem 4. Do help with reading if necessary.

Sometimes you get into a situation where you have to find the solution to two sentences to get the problem solved. The solution to the first sentence is part of the second sentence. You might think of these as two-step problems. Accept other appropriate sentences.

1 Jane went to the store for her mother. She bought bread for 32 cents and a can of soup for 17 cents. How much change should she get if she gave the clerk a dollar?

First you figure out what the total bill was. Then you can find how much change. $32 + 17 = 49$ $\$1.00 - \$.49 = \underline{\$.51}$

2 Ping-Pong balls were packed for shipment with 36 balls in a tray. There were 4 trays in each layer and 8 layers in the carton. How many balls are there in each carton?

First you figure out how many balls in each layer. Then you can find how many in the 8 layers.
$4 \times 36 = \underline{144}$ $8 \times 144 = \underline{1152}$

3 Robbie wants to buy 2 records that cost \$3.67 each. He has only \$7.14. Can he buy 2? If he can't, how much more does he need?

First you find out how much 2 records cost. Then you can find out how much money he needs.
$2 \times \$3.67 = \underline{\$7.34}$ $\$7.34 - \$7.14 = \underline{\$.20}$

4 Bill earned money babysitting. He charged 50 cents an hour. He took care of Junior Jones for 3 hours Tuesday. He watched Sissy Smith for 2 hours Wednesday. How much money did he earn these two days?

There certainly is more than one way you can organize the information to find the answer to this one. Write true sentences to show what your two steps were.
$3 + 2 = 5$ $5 \times \$.50 = \$2.50$

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Challenge your sharpies to go back through the problems and try to write one true math sentence for each problem that includes both steps. For openers, you may need to give them an example.

1. $100 - (32 + 17) = 51$
2. $(4 \times 36) \times 8 = 1152$
3. $(2 \times 3.67) - 7.14 = 20$
4. $(3 + 2) \times .50 = 2.50$ **or** $(3 \times .50) + (2 \times .50) = 2.50$

Would you believe that sometimes you can have too much information? You don't need all the facts that are available.

Some of the following problems have information that you don't need. Write a true math sentence to show the information that you used to get the solution.

1 The fuel tank in Bob's car holds 40 ℓ. If Bob drives under 88 kilometres per hour (km/h), he gets 9 kilometres per litre (km/ℓ). If he drives over 88 km/h, he gets only 7 km/ℓ.

a How many kilometres could he drive with a full tank if he kept under 88 km/h all the time?

9 x 40 = 360 (km)

b How far could he travel with a full tank if he drove over 88 km/h the entire time? 7 x 40 = 280 (km)

c If he drives under 88 km/h the entire time, how many more kilometres can he travel on a full tank than if he drives over 88 km/h the entire time?

360 - 280 = 80 (km)

2 The fuel tank in Susan's motorcycle holds 9 ℓ, but she gets almost 160 km/ℓ. If Susan rode 155 km Monday and 347 km Tuesday, what distance did she ride altogether?

155 + 347 = 502 (km)

3 The company pays Mr. Slick 8¢ per kilometre to cover expenses for his car. Oil costs 60¢ per litre. His car used 1 ℓ last week. Fuel is 14¢/ℓ. He bought 450 ℓ last week. He drove 1900 km last week. How much did the company pay him for car expenses last week?

1900 x $0.08 = $152.00

141

goal Extracting from word problems only the information necessary to solve the problems

page 141 Extraneous information is included in these word problems. Before turning to the text, read the following short story to the group. Tell the youngsters you want to know how many crumbs each ant carried. They will need paper and pencil.

As the ant procession headed toward the huge chocolate cake, the fourth ant in line tripped and fell, causing all thirty-six ants to fall. Relieved that they weren't carrying cake at the time, the ants began to line up again. But ant number 4 forgot his place in line. Not knowing what to do, he took the place of number 5, who in turn took the space assigned to number 6. Number 6 had no choice but to go to number 7's place, and so on down the line. By the time the ants had finally figured out who went where, the cake was stale. Number 36 was completely lost. If 3600 crumbs were to be carried away and all the ants carried the same load, how many crumbs of stale cake did each ant carry? (100)

The point of the story is that there may be a great many words and numbers, but the job is to select the proper information to answer a specific question. The youngsters will get more practice with the problems on the page. You decide how best to handle the page—as independent study or as a group that shares the reading task and the thinking tasks too.

lesson Pages 142, 143

goal Determining whether the statement of a problem gives sufficient information to solve it

memo Pages 142 and 143 are extension pages, included to challenge your more capable learners. You will want to be selective in assigning these pages for individual study or for work with them as a group.

page 142 These problems involve three tasks:

- Deciding whether sufficient information is given
- Deciding what additional information is needed and then making up the needed information
- Writing a true math sentence

Watch for pupils who are on the right track but are confused by the number of zeros involved.

For Experts Only

Write a true math sentence for the stories that have enough information. Make up information for those stories that need more. Then write a true math sentence for them, too.

1 There are 330 000 000 trees growing in Leafland. 320 100 000 of these are oaks. 6 600 000 of the trees are elms. The rest of the trees are willows. How many willows are there in Leafland?

320,100,000 + 6,600,000 = 326,700,000
330,000,000 − 326,700,000 = 3,300,000

2 One column of newsprint is 33 cm long. There are 3 lines to a centimetre. About how many words would fit in the column?

Need to know how many words to a line.
Example: 8 words to a line, so 8 x 3 = 24 24 x 33 = 792

3 Herbie the humpback whale ate 5000 herring for lunch. Each herring had just eaten 7000 little shellfish. Each little shellfish had just eaten 130 000 teeny, tiny plants. How many teeny, tiny plants did Herbie eat?

7000 × 130,000 = 910,000,000
5000 × 910,000,000 = 4,550,000,000,000

4 There are 340 children in Marsha's school. 130 buy lunch at the school. Some eat lunch at home. How many bring their lunch to school?

Need to know how many eat at home.
Example: 200 go home for lunch,
so 130 + 200 = 330
340 − 330 = 10

5 Tim put 3 problems on th[e] board. Mary put on 7. Sa[m] put on twice as many as Tim and Mary togethe[r]. How many problems are [on] the board?

3 + 7 = 10
2 × 10 = 20
10 + 20 = 30

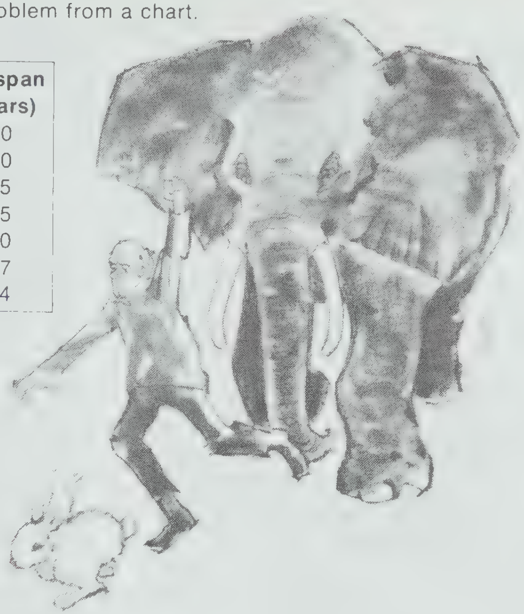
142

ometimes we have to get information to solve a problem from a chart.

ere is an example.

he chart lists
even kinds of
nimals, including
an. It tells two
ings about each
nd: how fast it
an run and how
ng it can live.

Animal	Speed (km/h)	Lifespan (years)
Lion	80	20
Zebra	64	20
Rabbit (tame)	56	5
Cat (tame)	48	15
Human being	44	70
Elephant	40	47
Pig (tame)	18	14



nswer the following questions.
ou may want to use math sentences to help you.

How much faster is a lion than a zebra?
$80 - 64 = \underline{16}$ (km/h)

According to this chart, would you be able to catch a pet rabbit running away from you? No $(44 < 56)$

An angry elephant is chasing you. Who can run faster, you or the elephant? You
How much faster? $44 - 40 = \underline{4}$ (km/h)

What is the difference between the speeds of the fastest and the slowest animals listed? $80 - 18 = \underline{62}$ (km/h)

What is the difference between the longest lifespan and the shortest lifespan for animals listed on the chart?
$47 - 5 = \underline{42}$ (years)
$70 - 5 = \underline{65}$ (years)
Both answers may be given.
Discuss whether human beings are animals.

6. Anita has two pets — a rabbit and a cat. Which pet is likely to live longer? How many years longer? The Cat $15 - 5 = \underline{10}$ (years)

7. The zebra at Zenith Zoo has lived $\frac{7}{20}$ of its expected lifespan. How old is the zebra? $\frac{7}{20} \times 20 = \underline{7}$ (years old)

goal Extrapolating information from a chart to use in solving word problems

page 143 Independent learners have all the information necessary to handle this job.

If you feel some of your pupils need more experience reading charts and extrapolating information, you may want to work as a group.

lesson Page 144

goal **Checkout** – identifying and completing true sentences; solving word problems

page 144 You'll want to discuss the various directions and then have the youngsters complete the page independently.

When checking errors to determine the additional help needed, look for –

- Problem 1 – computational errors
- Problem 2 – careless guesses or a specific symbol misused consistently
- Problem 3 – reversed symbols for **greater than** and **less than**
- Problem 4 – lack of understanding of the problem (Ask the pupil to draw a picture for each story to check this.)

CHECKOUT

Skill: Identifying true math sentences

1. Which are true sentences? d

   a $14 \times 4 = 42$ **b** $63 > 8 \times 8$ **c** $63 < 7 \times 9$

   d $\frac{3}{5} > \frac{1}{3}$ **e** $\frac{3}{5} - \frac{1}{3} > 1$ **f** $74 - 27 = n$

Skill: Using symbols to make true sentences

2. Copy each set of three digits. Write an operation and a relation symbol to complete a true sentence. Accept appropriate answers.

   a 3 +4 <12 (× =) **b** 0 =8 −8 (+ =) **c** 5 ×4 >11 (+ <)

   d 4 =2 +2 (− = / = ×) **e** 5 ×3 =15 **f** $\frac{1}{4}$ + $\frac{3}{4}$ = 1 (< +)

3. Use >, <, or = to make true sentences.

   a 16 >15 **b** $\frac{6}{8} = \frac{3}{4}$ **c** $\frac{3}{8} < \frac{1}{2}$

Skill: Solving word problems

4. Solve the following problems. Then write a true sentence to show that your answer is correct.
   Accept other appropriate sentences.

   a 16 cars.
   8 in each row.
   How many rows?
   $16 \div 8 = \underline{2}$

   b 12 meals.
   3 meals each day.
   How many days?
   $12 \div 3 = \underline{4}$

   c 9 boats.
   5 people in each.
   How many people?
   $9 \times 5 = \underline{45}$

   d 10 packages.
   10 kg each.
   How many kg?
   $10 \times 10 = \underline{100}$

144

See activity 2, page 144a.

See activity 3, page 144a.

RESOURCES

another form of evaluation

for progress check–page 134

Find the solution.

1. $475 + n = 600$ $n = 125$
2. $396 + n = 682$ $n = 286$
3. $143 + n = 950$ $n = 807$
4. $220 + n = 593$ $n = 373$
5. $754 + n = 760$ $n = 6$
6. $400 + n = 826$ $n = 426$

Which of the following are true? Which are false? Which are open?

7. $99 - 60 = 30$ False
8. $47 + n = 68$ Open
9. $45 \div 9 = 5$ True
10. $4 + 7 < 4 \times 5$ False
11. $100 = 20 \times 4$ False
12. $5 \times t > 16$ Open

for checkout–page 144

1. Which are true math sentences? a, c, e

   a) $\frac{1}{4} + \frac{3}{4} = 1$ b) $n + 47 = 95$
   c) $25 > 6 \times 4$ d) $63 \div 9 = 6$
   e) $14 < 7 \times 3$ f) $t = 13 + 70$

2. Write an operation and a relation symbol to complete a true sentence for each set of three digits. Accept appropriate answers.

   a) $2 \times 3 = 6$ b) $8 - 5 = 3$
   c) $3 = 9 - 6$ d) $6 + 5 < 12$
   e) $7 - 7 = 0$ f) $\frac{2}{3} + \frac{1}{3} = 1$

3. Use the relation symbol >, <, or = to make true sentences.

   a) $9 < 12$ b) $\frac{1}{3} > \frac{1}{5}$ c) $\frac{4}{6} = \frac{2}{3}$

4. Solve the following problems. Then write a true sentence to show that your answer is correct. Accept other appropriate sentences.

   a) 36 children.
   9 teams. $36 \div 9 = 4$
   How many children on each team?

   b) Each pencil costs 5¢.
   She buys 6 pencils.
   How much money?   5¢ × 6 = 30¢

   c) 24 cookies.
   8 club members. $24 \div 8 = 3$
   How many cookies for each?

activities

1. Challenge the pupil to follow these directions:

1. Write a true addition sentence. $(17 + 6 = 23)$
2. Change this sentence to a missing addend sentence. $(n + 6 = 23$ **or** $17 + n = 23)$
3. Write a subtraction sentence using the same numbers as in the missing addend sentence. $(23 - 6 = n$ **or** $23 - 17 = n)$ Write n in the answer position.
4. Solve the sentence in step 3.
5. Substitute the answer from step 4 for n in the sentence for step 3. Check the sentence. Is the sentence true?

2. things small cards; felt pens or crayons of 3 colors

Use a pen of 1 color. Make 2 cards for each numeral from 0 through 20. Use a pen of a different color to make a set of cards for the operation signs +, −, ÷, and ×. With the third pen make a set of cards for the relation symbols >, <, and =.

Have pupils work in pairs. The operation-symbol and relation-symbol cards are spread out faceup. The numeral cards are shuffled and placed in a stack facedown. Pupils alternate taking the top 3 numeral cards and combining them with an operation symbol and a relation symbol to form a true sentence. The pupil can do this in as many ways as he is able for the 3 numbers drawn. The other pupil checks each sentence and can correct any incorrect sentence or add any sentences the first pupil does not form.

Points are earned as follows:

- Forming a correct sentence – 2 points
- Correcting a false sentence – 3 points

Pupils predetermine how many rounds to play or the number of points needed to win.

3. Individual activity (Provide the pupil with the following.)

Remember that: $\neq$ is read "is not equal to"
$\ngtr$ is read "is not greater than"
$\nless$ is read "is not less than"

Use only the numbers from 0 through 10. How many solutions can you find for each of these sentences?

a) $3 + n \neq 10$ b) $3 \nless n$
c) $3 \ngtr n + 4$ d) $5 + n \ngtr 9$

additional learning aids

problem solving and applications

–chapter objectives 1, 2, 3, 4, 5, 6, 7, 8

SRA products

Mathematics Learning System, Activity Masters, level B, SRA (1974)
Spirit masters: W-8, 15, 16, 24, P-1, 3, 6, 8, 9, 10, 11

diagnosis: an instructional aid– Mathematics Level B, SRA (1972)
Probes: M-19, 20, 21

Mathematics Involvement Program, SRA (1971)
Cards: 85, 285, 16

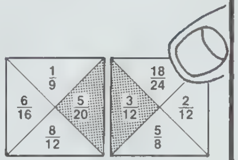
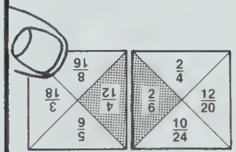
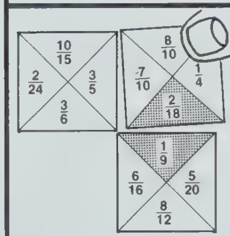
Skill through Patterns, level 5, SRA (1974)
Spirit masters: 5, 45, 46, 55

other learning aids (described on page 144e)–
Foo, Heads Up*, True or False

*Trademark of Creative Publications

Use these as models for fractional part number lines.

playing board for fraction squares

fraction squares	Here are the rules for the game:	1 Get a buddy. Sit together.	2 Here is what you play with: (1) a playing board, (2) 25 square pieces.	3 Each take twelve pieces. Put them faceup. The last piece goes faceup on the center of the playing board.
4 You start. Put one piece faceup on the playing board.	5 There are two rules: (1) The side of one piece must be touching the side of another piece.	(2) The touching sides must have equivalent fractions.		or
	6 Play a piece—get one point.	**center**	7 Take turns. Each turn you play one piece.	8 It is O.K. if you play a piece next to two or more other pieces.
	9 You do not have to form two or more pairs of equivalent fractions. One is enough. But more than one is better.	You score one point for each pair of equivalent fractions. You can score as many as four points on one play this way.	10 Suppose there is no piece you can play on your turn. Discard a piece and pass.	11 Suppose you make a mistake. Remove the piece and discard it when you are caught. Then pass.
12 Play until both of you have no more pieces. Then the game is over. The player with more points wins.				

playing pieces for fraction squares

$\frac{8}{10}$ $\frac{7}{10}$ $\frac{1}{4}$ $\frac{2}{18}$	$\frac{1}{9}$ $\frac{6}{16}$ $\frac{5}{20}$ $\frac{8}{12}$	$\frac{16}{24}$ $\frac{18}{30}$ $\frac{4}{14}$ $\frac{20}{24}$	$\frac{10}{15}$ $\frac{2}{24}$ $\frac{3}{5}$ $\frac{3}{6}$	$\frac{5}{6}$ $\frac{4}{12}$ $\frac{3}{18}$ $\frac{8}{16}$
$\frac{18}{24}$ $\frac{3}{12}$ $\frac{2}{12}$ $\frac{5}{8}$	$\frac{10}{20}$ $\frac{3}{15}$ $\frac{14}{16}$ $\frac{12}{18}$	$\frac{2}{5}$ $\frac{6}{18}$ $\frac{15}{25}$ $\frac{2}{3}$	$\frac{18}{20}$ $\frac{2}{10}$ $\frac{2}{16}$ $\frac{1}{2}$	$\frac{4}{8}$ $\frac{5}{25}$ $\frac{1}{7}$ $\frac{12}{16}$
$\frac{2}{4}$ $\frac{2}{6}$ $\frac{12}{20}$ $\frac{10}{24}$	$\frac{6}{9}$ $\frac{4}{24}$ $\frac{3}{9}$ $\frac{3}{4}$	$\frac{9}{12}$ $\frac{9}{15}$ $\frac{14}{20}$ $\frac{7}{9}$	$\frac{5}{12}$ $\frac{5}{9}$ $\frac{1}{5}$ $\frac{14}{21}$	$\frac{4}{6}$ $\frac{6}{24}$ $\frac{2}{20}$ $\frac{15}{20}$
$\frac{12}{30}$ $\frac{1}{6}$ $\frac{7}{21}$ $\frac{9}{10}$	$\frac{14}{18}$ $\frac{1}{8}$ $\frac{3}{8}$ $\frac{4}{10}$	$\frac{15}{18}$ $\frac{8}{24}$ $\frac{4}{20}$ $\frac{20}{25}$	$\frac{12}{15}$ $\frac{12}{14}$ $\frac{4}{16}$ $\frac{6}{12}$	$\frac{7}{14}$ $\frac{6}{10}$ $\frac{1}{3}$ $\frac{10}{12}$
$\frac{16}{20}$ $\frac{2}{7}$ $\frac{1}{12}$ $\frac{5}{10}$	$\frac{8}{20}$ $\frac{1}{10}$ $\frac{6}{30}$ $\frac{4}{5}$	$\frac{6}{8}$ $\frac{2}{14}$ $\frac{6}{7}$ $\frac{6}{15}$	$\frac{10}{16}$ $\frac{2}{8}$ $\frac{10}{18}$ $\frac{24}{30}$	$\frac{12}{24}$ $\frac{7}{8}$ $\frac{5}{15}$ $\frac{10}{25}$

7 FRACTIONS

before this chapter the learner has—

1. Renamed a fraction as an equivalent fraction
2. Found the simplest name for a fraction
3. Renamed appropriate fractions as mixed or whole numbers
4. Added and subtracted two fractions with like denominators

in chapter 7 the learner is—

1. Mastering renaming answers in simplest form
2. Adding and subtracting fractions with unlike denominators
3. Adding and subtracting mixed numbers
4. Multiplying a whole number and a fraction
5. Multiplying two fractions

in later chapters the learner will—

1. Master finding a common denominator for two frequently used fractions
2. Master writing an appropriate fraction as a mixed or whole number
3. Master finding the sum or difference of any two fractions
4. Master finding the sum or difference of any two mixed numbers
5. Master finding the product of any two fractions

Notes & Things

This chapter on fractions will give the learners enough review and practice in renaming fractions to enable the majority of them to attain mastery. The skill will be used in addition, subtraction, and multiplication.

Each learner will review the method for finding equivalent fractions, but the beginning emphasis will remain on finding the simplest name for the answer to an addition or subtraction problem. It is hoped the pupils have discovered by now the value of inspecting to find if there is a common factor in the numerator and denominator. This informal approach allows fraction names for whole numbers and mixed numbers to be included for renaming.

There is no pressure here on the learner to use the greatest common factor in the renaming process. In fact, there is no isolated practice with the greatest common factor. The learner, through much experience, will come to see that taking time to find the greatest common factor saves a lot of work.

The instructional material is carefully controlled and sequenced. The learner has lots of examples to get involved with as his thinking is guided. First there is addition and subtraction of fractions with common denominators, and then mixed numbers. Finally the learner reaches the point where renaming is used for quite a different reason from giving an answer its simplest name. Renaming becomes necessary to add or subtract fractions with unlike denominators. The denominators in all problems have been limited by having one denominator a multiple of the other. That's a big enough step for now. Mastery is not expected in this chapter, but each child should feel quite confident about the procedure itself.

If the youngsters are tired of fractions halfway through the chapter, please STOP on page 164 and go to another chapter. Page 164 starts the study of multiplication, and everyone's mind needs to be fresh and open to start this new operation. Chapter 11, "Finding, Organizing, and Reporting Information," or chapter 12, "Geometry–Symmetry," would be good relief.

Fortunately the algorithm for multiplication is probably the easiest of all algorithms to learn. Nearly all children will operate correctly with no difficulty. After all, they have wanted to add the numerators *and* the denominators, but the book said they couldn't. (That is a frequent error in the addition operation.) The temptation was strong for subtraction, BUT . . . At last, they get to multiply both numerators and then both denominators. No worry about a common denominator, for goodness sake!

And then the big folks and the book take all the simplicity out of multiplication–it's necessary to rename the product in simplest form. Take a look at the size of some of those numerators and denominators, $\frac{7}{8} \times \frac{2}{3} = \frac{14}{24}$. They are unreal! Renaming is a big job.

The youngster may think, Sometimes they want 'em bigger (to get common denominator for + or −), sometimes they want 'em smaller (simplest name), and sometimes they just want 'em different (other names for 1). I wish they'd make up their minds.

Multiplication with renaming is featured in the remaining part of the chapter. Please be patient with the class. It will probably be a while before the whole thing makes sense to them (if ever).

things

inch rulers
set of felt fractional parts (optional)

For the extra activities you will want to have these things available:

- spirit masters
- rubber jar rings
- masking tape
- colored plastic sticks
- yarn, string, or ribbon
- pictures of racing cars
- egg cartons
- dried beans of 2 kinds

goal Talk about and explore ideas through a picture clue

page 145 What a glorious kite! Now the question is – will if fly? Why does the surface of the kite have reinforcing parts that divide it into fractional parts? What are some good designs for kites? This question will let you know whether you have any high-flying experts in your groups. If no one comes up with any ideas, abandon the kite discussion and take a look at things in the classroom where braces serve to divide an object into fractional parts. Try questions such as these.

- One windowpane is what fractional part of the entire window?
- One sheet of slate is what fractional part of the whole chalkboard?
- One bookshelf is what fractional part of the storage space of the whole bookcase?
- Three drawers of the cabinet are what part of the whole cabinet?

Let your pupils think of their own questions from there. Someone may observe that a person is a fractional part of all the people in the room.

Do you suppose anyone can figure out what fractional part all the math books are of all the books in the room? Wow!

lesson Page 146

goal **Survey**—ability to rename fractions and to apply renaming skills

page 146 Problems 1 through 4 each focus on a specific renaming skill. These skills are reviewed, practiced, and applied throughout the chapter. Anticipate that your pupils will be operating at various levels of ability. The following chart will help you determine group assignments.

Problem	Review and Practice	Application
1	Page 147	
2	147	Pages 149–157
3	148	
4	Pages 158–159	Pages 160–164

Problem 5 sets the learning goals of the chapter—adding and subtracting fractions with unlike denominators and exploring the multiplication of fractions.

Renaming a fraction can mean a lot of things.
Look at the different kinds of renaming.

THINK about *what* you do as you rename these.

1. What is the simplest name for each of these?

   a $\frac{2}{4}$ $\frac{1}{2}$ **b** $\frac{14}{12}$ $1\frac{1}{6}$ **c** $\frac{6}{9}$ $\frac{2}{3}$ **d** $\frac{5}{5}$ 1 **e** $\frac{10}{15}$ $\frac{2}{3}$

2. Rename each whole number as a fraction.

   a $3 = \frac{?}{2}$ 6 **b** $1 = \frac{?}{8}$ 8 **c** $1 = \frac{?}{3}$ 3

   d $1 = \frac{?}{10}$ 10 **e** $2 = \frac{?}{4}$ 8 **f** $4 = \frac{?}{4}$ 16

3. Try to rename each of these mixed numbers as a fraction.

   a $1\frac{1}{10}$ $\frac{11}{10}$ **b** $2\frac{1}{5}$ $\frac{11}{5}$ **c** $2\frac{2}{7}$ $\frac{16}{7}$ **d** $10\frac{11}{12}$ $\frac{131}{12}$

   That's enough.

4. And would you believe there is still one more type of renaming? Rename each pair of fractions so that they have a common denominator.

   a $\frac{1}{3}$ and $\frac{1}{6}$ $\frac{2}{6}$ and $\frac{1}{6}$ **b** $\frac{3}{4}$ and $\frac{5}{8}$ $\frac{6}{8}$ and $\frac{5}{8}$ **c** $\frac{3}{6}$ and $\frac{1}{2}$ $\frac{3}{6}$ and $\frac{3}{6}$

   That's enough.

5. If you put it all together, you can do problems like these. Try them.

   a $1\frac{3}{4} + 1\frac{1}{8}$ $2\frac{7}{8}$ **b** $8\frac{5}{6} - 2\frac{1}{3}$ $6\frac{1}{2}$ **c** $2\frac{2}{5} + 1\frac{1}{10}$ $3\frac{1}{2}$

   d $3\frac{1}{2} - 1\frac{1}{6}$ $2\frac{1}{3}$ **e** $4\frac{1}{2} + 2\frac{1}{4}$ $6\frac{3}{4}$ **f** $2\frac{7}{8} - 1\frac{3}{4}$ $1\frac{1}{8}$

Your goal is to become really good at renaming fractions.

146

Did you make any mistakes on problem 1 on the page before?
Maybe this will help.

That's a common factor.

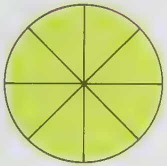
$\frac{2}{4}$ Is there a common factor? $\frac{2 \div 2}{4 \div 2} = \frac{1}{2}$

$\frac{8}{12}$ Is there a common factor?
2 is a common factor, but so is 4.
Use the greatest common factor and save time. $\frac{8 \div 4}{12 \div 4} = \frac{2}{3}$

Practice on these. Write the simplest name for each.

	a	b	c	d	e	f	g	h	i	j
1.	$\frac{3}{6}$ $\frac{1}{2}$	$\frac{4}{8}$ $\frac{1}{2}$	$\frac{6}{12}$ $\frac{1}{2}$	$\frac{5}{10}$ $\frac{1}{2}$	$\frac{7}{14}$ $\frac{1}{2}$	$\frac{3}{9}$ $\frac{1}{3}$	$\frac{2}{10}$ $\frac{1}{5}$	$\frac{6}{8}$ $\frac{3}{4}$	$\frac{10}{12}$ $\frac{5}{6}$	$\frac{8}{8}$ $\frac{1}{1}$ or 1

The last problem looks easy, BUT $\frac{8}{8} = 1$.
Maybe you had to draw a picture for yourself.
But drawing pictures takes a long time.
The denominator tells how many parts in one.
If the numerator is the same number as the denominator, then the fraction says you are considering the whole thing.



Renaming 2 is not a problem.
THINK One whole thing is divided into $\frac{8}{8}$.

Then 2 whole things are the same as $\frac{8}{8} + \frac{8}{8} = \frac{8+8}{8} = \frac{16}{8}$

Practice on these.
Rename each whole number as a fraction.

	a	b	c	d	e	f
2.	$1 = \frac{?}{6}$ 6	$2 = \frac{?}{2}$ 4	$3 = \frac{?}{3}$ 9	$5 = \frac{?}{8}$ 40	$4 = \frac{?}{3}$ 12	$2 = \frac{?}{3}$ 6
3.	$1 = \frac{?}{9}$ 9	$3 = \frac{?}{4}$ 12	$2 = \frac{?}{6}$ 12	$4 = \frac{?}{4}$ 16	$3 = \frac{?}{5}$ 15	$5 = \frac{?}{10}$ 50

147

lesson Pages 147, 148

goal Review of and practice in finding the simplest name for a fraction and in renaming a whole number as a fraction

page 147 The page provides review and practice for the two renaming skills checked in problems 1 and 2 on page 146. You decide which pupils can operate independently. Make sure the youngsters are directed to appropriate types of practice that were indicated by their errors on the previous page. No one should practice a skill already mastered. Pupils who made no mistakes in problems 1 and 2 of page 146 should skip this entire page. It would be great if you could enlist these pupils as peer tutors.

goal Renaming mixed numbers as fractions and fractions as mixed numbers

page 148 Only pupils who had trouble with problem 3 on page 146 will need to work this page. With these youngsters you will want to get out those circular felt regions for some hands-on experience. Lead them from the felt parts to the abstracted diagrams illustrated on the pupil page.

Some youngsters may see a relationship between these skills and multiplication and division. Great! But don't take time to teach these shortcuts now. This is the time to develop understanding.

Renaming a mixed number as a fraction is *not* a very big step.
$2\frac{1}{10}$ says that you have 2 whole things and $\frac{1}{10}$ more.
2 is the same as $\frac{10}{10}+\frac{10}{10}$, or $\frac{20}{10}$. You have $\frac{1}{10}$ more.
So $\frac{20}{10}+\frac{1}{10}=\frac{21}{10}$. AND $\frac{21}{10}$ is the fraction name for $2\frac{1}{10}$.

Time for practice. Rename each mixed number as a fraction.

	a	b	c	d	e	f	g
1.	$1\frac{5}{6}$ $\frac{11}{6}$	$1\frac{3}{4}$ $\frac{7}{4}$	$2\frac{1}{2}$ $\frac{5}{2}$	$3\frac{1}{6}$ $\frac{19}{6}$	$2\frac{3}{10}$ $\frac{23}{10}$	$3\frac{1}{3}$ $\frac{10}{3}$	$2\frac{3}{5}$ $\frac{13}{5}$
2.	$3\frac{3}{8}$ $\frac{27}{8}$	$2\frac{2}{3}$ $\frac{8}{3}$	$4\frac{1}{4}$ $\frac{17}{4}$	$2\frac{2}{5}$ $\frac{12}{5}$	$2\frac{5}{6}$ $\frac{17}{6}$	$1\frac{3}{4}$ $\frac{7}{4}$	$3\frac{1}{6}$ $\frac{19}{6}$

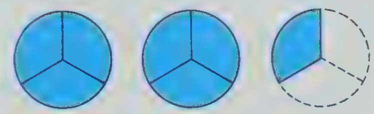
Renaming a fraction as a mixed number isn't hard either. Start slowly.

Which of these fractions can be renamed as mixed numbers?

	a	b	c	d	e	f	g
3.	$\underline{\frac{5}{3}}$	$\frac{3}{5}$	$\underline{\frac{7}{5}}$	$\frac{5}{7}$	$\underline{\frac{7}{3}}$	$\frac{3}{7}$	$\frac{0}{4}$
4.	$\frac{2}{3}$	$\underline{\frac{3}{2}}$	$\frac{6}{9}$	$\underline{\frac{9}{6}}$	$\underline{\frac{12}{5}}$	$\frac{5}{12}$	$\underline{\frac{9}{8}}$

Use $\frac{7}{3}$ as an example. **Think** How many $\frac{3}{3}$s in $\frac{7}{3}$?

There are 2 and $\frac{1}{3}$ more.
$\frac{7}{3}$ can be renamed $2\frac{1}{3}$.



Practice. Rename each fraction as a mixed number.

	a	b	c	d	e	f	g
5.	$\frac{3}{2}$ $1\frac{1}{2}$	$\frac{7}{5}$ $1\frac{2}{5}$	$\frac{9}{4}$ $2\frac{1}{4}$	$\frac{13}{4}$ $3\frac{1}{4}$	$\frac{8}{5}$ $1\frac{3}{5}$	$\frac{5}{3}$ $1\frac{2}{3}$	$\frac{11}{6}$ $1\frac{5}{6}$
6.	$\frac{7}{3}$ $2\frac{1}{3}$	$\frac{9}{2}$ $4\frac{1}{2}$	$\frac{16}{5}$ $3\frac{1}{5}$	$\frac{19}{8}$ $2\frac{3}{8}$	$\frac{11}{6}$ $1\frac{5}{6}$	$\frac{15}{4}$ $3\frac{3}{4}$	$\frac{21}{10}$ $2\frac{1}{10}$

1 Add. Rename the sums as mixed numbers.

Example: $\frac{2}{3} + \frac{2}{3} = \frac{4}{3}$ THINK $\frac{3}{3} = 1$. How many $\frac{3}{3}$s in $\frac{4}{3}$? 1

There's 1 and $\frac{1}{3}$ more. $\frac{4}{3}$ can be renamed $1\frac{1}{3}$.

a $\frac{5}{6}+\frac{2}{6}$ $1\frac{1}{6}$ b $\frac{3}{9}+\frac{7}{9}$ $1\frac{1}{9}$ c $\frac{4}{5}+\frac{3}{5}$ $1\frac{2}{5}$ d $\frac{5}{7}+\frac{6}{7}$ $1\frac{4}{7}$ e $\frac{7}{8}+\frac{4}{8}$ $1\frac{3}{8}$

2 The next set of problems requires renaming—lots of it.

Example: $\frac{3}{8} + \frac{7}{8}$ First you add. Your answer is $\frac{10}{8}$. Now you rename $\frac{10}{8}$ as $1\frac{2}{8}$.

And you can rename again, because $\frac{2}{8} = \frac{1}{4}$.

Your final answer is $1\frac{1}{4}$.

Try these. Your challenge is to find the simplest name for each sum.

a $\frac{3}{4}+\frac{3}{4}$ $1\frac{1}{2}$ b $\frac{5}{6}+\frac{5}{6}$ $1\frac{2}{3}$ c $\frac{7}{8}+\frac{5}{8}$ $1\frac{1}{2}$ d $\frac{2}{3}+\frac{2}{3}$ $1\frac{1}{3}$ e $\frac{1}{5}+\frac{4}{5}$ 1

f $\frac{5}{6}+\frac{3}{6}$ $1\frac{1}{3}$ g $\frac{4}{5}+\frac{2}{5}$ $1\frac{1}{5}$ h $\frac{5}{8}+\frac{5}{8}$ $1\frac{1}{4}$ i $\frac{9}{10}+\frac{3}{10}$ $1\frac{1}{5}$ j $\frac{5}{12}+\frac{11}{12}$ $1\frac{1}{3}$

Bet you are wondering: **When do I use all this stuff?**
Time out to look at some situations.

3 He had to practice $\frac{3}{4}$ of an hour on his music.
Homework would take another $\frac{3}{4}$ hour.
How long before he would be free? $1\frac{1}{2}$ hours

4 She needed $1\frac{7}{8}$ yards of fabric for the skirt.
And $\frac{3}{8}$ more yards of the same fabric for trim.
How much fabric should she buy? $2\frac{1}{4}$ yards

There is something new in that last problem. What is it? A mixed number to be computed.
Make sure you know how to add with mixed numbers.

149

lesson Pages 149, 150, 151, 152

goal Application of renaming skills to renaming sums

memo Pages 149 and 150 work together.

page 149 Actually, no new skill is introduced on the page. Renaming fractions as mixed numbers and finding the simplest name for a fraction have been reviewed and practiced. Adding fractions with like denominators is a familiar task by now. Here the youngster puts it all together.

Pupils should understand that these are all names for the same number.

$\frac{10}{8}$ $1\frac{2}{8}$ $1\frac{1}{4}$

One name is as correct as the other. Usually the problem determines which name is to be used. Since every fraction can be renamed with many different names, the simplest name is preferred when computing answers.

Problem 4 introduces adding a mixed number and a fraction. The problem is diagramed and discussed at the top of the next page. Please encourage the pupils to go right on to that diagram.

goal Extension of addition skills to mixed numbers

page 150 The first example uses the horizontal algorithm for adding a mixed number and a fraction. Here the mixed number is renamed as a fraction before the actual addition is performed. This renaming step can be eliminated when the vertical algorithm is used. The vertical algorithm introduced in the second example is much more efficient to use. Please point out the ease of first adding the fractions and then adding the whole numbers when using this form. (Think of the renaming involved when the horizontal algorithm is used with two mixed numbers. That's enough to turn anyone off!)

The problems themselves involve these skills:

- Adding fractions with like denominators
- Adding whole numbers
- Renaming the sum

There are no new skills – only the algorithm is different.

Let the picture help you get an answer for $1\frac{7}{8} + \frac{3}{8}$.

Let's look at that last problem. The amount for the skirt can be thought of as $\frac{15}{8}$. $\frac{3}{8}$ more is needed. $\frac{15}{8} + \frac{3}{8} = \frac{18}{8}$ AND that answer has to be renamed. $\frac{18}{8} = 2\frac{2}{8}$ – Ugh! You have to rename AGAIN. What is the answer? $2\frac{1}{4}$

That is a real-world problem for you. Sometimes it seems that you never find the answer. Be patient. You'll make it.

Try another problem.

They wanted to make fudge and a chocolate cake. The fudge recipe called for $2\frac{1}{4}$ cups of sugar. The cake needed $1\frac{3}{4}$ cups of sugar. How much sugar do they need to make both?

Skip the picture this time. Save even more time by writing the problem this way:

$$\begin{array}{r} 2\frac{1}{4} \\ +\ 1\frac{3}{4} \\ \hline 3\frac{4}{4} \end{array}$$

First add the fractions.
Then add the whole numbers.
And finally rename the answer.
How many cups of sugar are needed? 4

Your turn. Add these. Name each in simplest form.

1. $1\frac{1}{2} + 1\frac{1}{2}$	**2.** $2\frac{2}{3} + 1\frac{1}{3}$	**3.** $2\frac{5}{6} + 4\frac{1}{6}$	**4.** $1\frac{3}{5} + 2\frac{2}{5}$	**5.** $1\frac{3}{8} + 1\frac{5}{8}$
3	4	7	4	3

Do you think that set of problems was rigged? Why?
Yes; all answers are whole numbers.

150

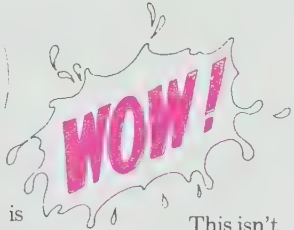
You practised only one type of renaming. Look at all types.

$$\begin{array}{r} 2\frac{3}{8} \\ +\,1\frac{1}{8} \\ \hline 3\frac{4}{8} \end{array}, \text{ or } 3\frac{1}{2}$$

There is nothing hard here.

$$\begin{array}{r} 1\frac{3}{4} \\ +\,1\frac{1}{4} \\ \hline 2\frac{4}{4} \end{array}, \text{ or } 3$$

The answer $2\frac{4}{4}$ is another name for 2 + 1. That's not hard.



This isn't as bad as it looks, but take your time with this one.

$$\begin{array}{r} 1\frac{5}{6} \\ +\,1\frac{2}{6} \\ \hline 2\frac{7}{6} \end{array}, \text{ or } 3\frac{1}{6}$$

Study this example:

$$\begin{array}{r} 2\frac{2}{3} \\ +\,1\frac{2}{3} \\ \hline 3\frac{4}{3} \end{array}$$

What is the simplest name?

$\frac{4}{3} = 1\frac{1}{3}$

$3 + 1\frac{1}{3} = 4\frac{1}{3}$ That's your answer in simplest form.

Take time to practise only this type of renaming.
Name each in simplest form.

LOOK OUT

	a	b	c	d	e	f	g
1.	$3\frac{5}{4}$ $4\frac{1}{4}$	$2\frac{5}{3}$ $3\frac{2}{3}$	$6\frac{8}{5}$ $7\frac{3}{5}$	$9\frac{4}{3}$ $10\frac{1}{3}$	$5\frac{13}{10}$ $6\frac{3}{10}$	$4\frac{11}{6}$ $5\frac{5}{6}$	$1\frac{17}{9}$ $2\frac{8}{9}$
2.	$2\frac{6}{4}$ ($3\frac{2}{4}$) $3\frac{1}{2}$	$4\frac{12}{10}$ ($5\frac{2}{10}$) $5\frac{1}{5}$	$6\frac{9}{6}$ ($7\frac{3}{6}$) $7\frac{1}{2}$	$3\frac{12}{8}$ ($4\frac{4}{8}$) $4\frac{1}{2}$	$3\frac{14}{10}$ ($4\frac{4}{10}$) $4\frac{2}{5}$	$1\frac{8}{6}$ ($2\frac{2}{6}$) $2\frac{1}{3}$	$2\frac{10}{4}$ ($4\frac{2}{4}$) $4\frac{1}{2}$

151

goal Practice in renaming sums in simplest form

page 151 The focus is on renaming sums. Three types of renaming can result in adding mixed numbers. All three are presented on the page. You'll have to decide how best to handle these skills with your particular pupils. For many, the examples given will be sufficient; others may need more help. Watch out for the last problem—two renaming skills are involved.

goal Practice in adding mixed numbers and renaming the sums

page 152 Now is the time for the youngsters to show off their ability to add mixed numbers and fractions **and** their ability to rename the sums. Watch for two types of errors:

- Error in addition, but the incorrect sum renamed correctly
- Correct addition, but an error in renaming the sum

The pupil should practice only the skill that is giving him trouble.

Add. Write the simplest name for each sum.

	a	b	c	d	e
1.	$1\frac{3}{5} + 2\frac{2}{5}$	$4\frac{2}{3} + 1\frac{1}{3}$	$3\frac{5}{6} + 1\frac{1}{6}$	$7\frac{3}{8} + 2\frac{5}{8}$	$3\frac{4}{7} + 4\frac{3}{7}$
	$(3\frac{5}{5})$ 4	$(5\frac{3}{3})$ 6	$(4\frac{6}{6})$ 5	$(9\frac{8}{8})$ 10	$(7\frac{7}{7})$ 8

The next ones may look different, but compute the same way. (Watch for correct computation, but an error in renaming.)

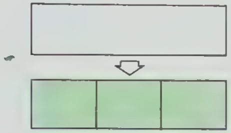
	a	b	c	d	e
2.	$5\frac{4}{5} + \frac{3}{5}$	$4\frac{5}{9} + \frac{7}{9}$	$2\frac{7}{8} + \frac{5}{8}$	$3\frac{3}{5} + \frac{4}{5}$	$1\frac{5}{7} + \frac{3}{7}$
	$(5\frac{7}{5})\,6\frac{2}{5}$	$(4\frac{12}{9} = 5\frac{3}{9})\,5\frac{1}{3}$	$(2\frac{12}{8} = 3\frac{4}{8})\,3\frac{1}{2}$	$(3\frac{7}{5})\,4\frac{2}{5}$	$(1\frac{8}{7})\,2\frac{1}{7}$

Back to word problems again.

3. Sara bought $2\frac{1}{3}$ dozen plain doughnuts and $1\frac{1}{3}$ dozen jelly doughnuts for the party. How many dozen doughnuts did she buy?
   $3\frac{2}{3}$ (probably 4 dozen)

4. Jamie took $10\frac{1}{2}$ minutes to do question 1. He took only 8 minutes for question 2.
   - **a** How much time did he take? $18\frac{1}{2}$ minutes
   - **b** He had 20 minutes to do the whole page. How much time was left? $1\frac{1}{2}$ minutes

That's one way to get you to think about subtraction. Believe it or not, subtraction comes next!

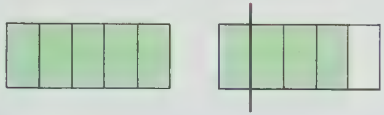


$1 - \frac{2}{3} = \underline{?}$ Can this be done? Rename 1 as $\frac{3}{3}$.

Why rename? Now can you subtract? $\frac{3}{3} - \frac{2}{3} = \underline{?}$
You can't subtract unless you rename; yes; $\frac{1}{3}$

1. Subtract. (You will have to rename the whole number.)

a $1 - \frac{3}{5}$ $\frac{2}{5}$ **b** $1 - \frac{7}{9}$ $\frac{2}{9}$ **c** $2 - \frac{1}{2}$ $1\frac{1}{2}$ **d** $3 - \frac{3}{4}$ $2\frac{1}{4}$ **e** $1 - \frac{7}{12}$ $\frac{5}{12}$ **f** $4 - \frac{7}{9}$ $3\frac{2}{9}$



$1\frac{4}{5} - \frac{3}{5}$ can also be pictured.
How many fifths shaded? 9
Take away $\frac{3}{5}$. How many fifths remain? $1\frac{4}{5} - \frac{3}{5} = ?$ $1\frac{1}{5}$

This problem could have been written →
Use this vertical form whenever you want to.

$$\begin{array}{r} 1\frac{4}{5} \\ -\ \frac{3}{5} \\ \hline ? \end{array}$$ $1\frac{1}{5}$

2. Subtract.

a $1\frac{2}{3} - \frac{1}{3}$ $1\frac{1}{3}$ **b** $1\frac{4}{5} - \frac{3}{5}$ $1\frac{1}{5}$ **c** $2\frac{1}{3} - \frac{1}{3}$ 2 **d** $2\frac{3}{10} - \frac{2}{10}$ $2\frac{1}{10}$ **e** $1\frac{5}{9} - \frac{4}{9}$ $1\frac{1}{9}$ **f** $1\frac{5}{6} - \frac{4}{6}$ $1\frac{1}{6}$

You might get out of practice if you don't do some renaming. Don't let that happen. Subtract some more. Write the simplest name for each answer.

g $1\frac{3}{4} - \frac{1}{4}$ $1\frac{1}{2}$ **h** $1\frac{9}{10} - \frac{7}{10}$ $1\frac{1}{5}$ **i** $2\frac{5}{6} - \frac{1}{6}$ $2\frac{2}{3}$ **j** $3\frac{5}{8} - \frac{3}{8}$ $3\frac{1}{4}$ **k** $3\frac{11}{12} - \frac{1}{12}$ $3\frac{5}{6}$ **l** $4\frac{9}{10} - \frac{1}{10}$ $4\frac{4}{5}$

lesson Pages 153, 154, 155

goal Introduction to subtracting a fraction from a whole number and from a mixed number

page 153 You'll want to talk about this page together. The horizontal form used at the top is great when the whole number is 1 or when the entire whole number is renamed as a fraction.

$1 - \frac{3}{5} = \frac{5}{5} - \frac{3}{5} =$

$3 - \frac{3}{4} = \frac{12}{4} - \frac{3}{4} =$

When the vertical form is used, however, only 1 whole need be renamed.

$$\begin{array}{r} 3 \\ -\ \frac{3}{4} \\ \hline \end{array} \qquad \begin{array}{r} 2\frac{4}{4} \\ -\ \frac{3}{4} \\ \hline \end{array}$$

No need to discuss this unless one of the youngsters brings it up. Having to pick from two algorithms can be confusing for some. Watch for youngsters who do not know which fraction name to use when renaming the whole number for subtraction.

The problems in **2a** through **f** will help you identify pupils who have trouble with the subtraction algorithm. Renaming troubles will show up in problems **2g** through **l**.

goal Practice in subtracting mixed numbers and renaming the difference when possible

page 154 Addition and subtraction parallel each other in work with whole numbers. This parallelism extends into the world of fractions. Nothing new—only the operation has been changed.

The vertical form is best when you subtract mixed numbers. Study the example first.

$$\begin{array}{r} 4\frac{4}{5} \\ -\,1\frac{1}{5} \\ \hline 3\frac{3}{5} \end{array}$$

First subtract the fractions.
Then subtract the whole numbers.

You know you might have to rename in subtraction, too.

$$\begin{array}{r} 3\frac{3}{4} \\ -\,2\frac{1}{4} \\ \hline 1\frac{2}{4} \text{ or } 1\frac{1}{2} \end{array} \qquad \text{Try this one:} \quad \begin{array}{r} 4\frac{2}{5} \\ -\,2\frac{2}{5} \\ \hline 2\ ? \end{array}$$

Is your answer a mixed number?
No. It's a whole number.

Show how good you are at subtraction. Do these. Give the simplest name for your answers.

	a	b	c	d	e	f	g
1.	$5\frac{7}{8} - 1\frac{1}{8}$	$1\frac{7}{9} - 1\frac{1}{9}$	$2\frac{5}{6} - 1\frac{1}{6}$	$4\frac{3}{5} - 1\frac{3}{5}$	$2\frac{5}{6} - 2\frac{5}{6}$	$6\frac{5}{9} - 3\frac{2}{9}$	$3\frac{5}{8} - 1\frac{1}{8}$
	$(4\frac{6}{8})$ $4\frac{3}{4}$	$(\frac{6}{9})$ $\frac{2}{3}$	$(1\frac{4}{6})$ $1\frac{2}{3}$	3	0	$(3\frac{3}{9})$ $3\frac{1}{3}$	$(2\frac{4}{8})$ $2\frac{1}{2}$
2.	$10\frac{9}{10} - 8\frac{3}{10}$	$6\frac{5}{8} - 4\frac{3}{8}$	$7\frac{2}{5} - 4\frac{1}{5}$	$8\frac{4}{9} - 2\frac{2}{9}$	$10\frac{4}{5} - 2\frac{1}{5}$	$6\frac{3}{10} - 5\frac{1}{10}$	$8\frac{1}{8} - 6\frac{1}{8}$
	$(2\frac{6}{10})$ $2\frac{3}{5}$	$(2\frac{2}{8})$ $2\frac{1}{4}$	$3\frac{1}{5}$	$6\frac{2}{9}$	$8\frac{3}{5}$	$(1\frac{2}{10})$ $1\frac{1}{5}$	2

3. The dog chewed up $\frac{2}{5}$ of the rope. How much of the rope is left? $\frac{3}{5}$

4. Bill had $\frac{1}{4}$ of the pizza. Jan had $\frac{1}{4}$. Dan had $\frac{1}{4}$. Is there any left for you? How much? Yes; $\frac{1}{4}$

SUBTRACT

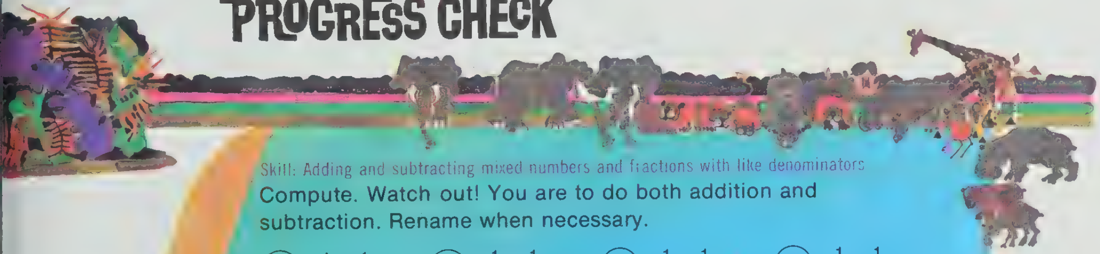
1. Rename when necessary.

a $\begin{array}{r} 2\frac{2}{3} \\ -\ 1\frac{1}{3} \\ \hline 1\frac{1}{3} \end{array}$ b $\begin{array}{r} 2\frac{1}{2} \\ -\ 1\frac{1}{2} \\ \hline 1 \end{array}$ c $\begin{array}{r} 4\frac{3}{4} \\ -\ 2\frac{1}{4} \\ \hline 2\frac{1}{2} \end{array}$ d $\begin{array}{r} 3\frac{5}{6} \\ -\ 1\frac{1}{6} \\ \hline 2\frac{2}{3} \end{array}$ e $\begin{array}{r} 4\frac{11}{12} \\ -\ 2\frac{5}{12} \\ \hline 2\frac{1}{2} \end{array}$

2. Recopy these in vertical form if you want to.
Don't forget to rename.

a $3\frac{4}{5} - 1\frac{2}{5}$ $2\frac{2}{5}$ b $5\frac{7}{10} - 3\frac{3}{10}$ $2\frac{2}{5}$ c $1\frac{7}{12} - 1\frac{5}{12}$ $\frac{1}{6}$ d $2\frac{7}{8} - 1\frac{3}{8}$ $1\frac{1}{2}$

PROGRESS CHECK



Skill: Adding and subtracting mixed numbers and fractions with like denominators

Compute. Watch out! You are to do both addition and subtraction. Rename when necessary.

1. $1\frac{1}{2} + \frac{1}{2}$ 2 2. $2\frac{1}{2} + \frac{1}{2}$ 3 3. $3\frac{1}{2} + \frac{1}{2}$ 4 4. $1\frac{1}{2} - \frac{1}{2}$ 1

5. $2\frac{1}{2} - \frac{1}{2}$ 2 6. $3\frac{1}{2} - \frac{1}{2}$ 3 7. $2\frac{1}{4} + \frac{3}{4}$ 3 8. $2\frac{3}{4} - \frac{1}{4}$ $2\frac{1}{2}$

9. $1\frac{3}{4} - \frac{1}{4}$ $1\frac{1}{2}$ 10. $\frac{5}{6} + \frac{5}{6}$ $1\frac{2}{3}$ 11. $\frac{5}{6} - \frac{5}{6}$ 0 12. $\frac{5}{6} - \frac{0}{6}$ $\frac{5}{6}$

See activity 1, page 175a.

See activity 2, page 175a.

goal Practice in subtracting mixed numbers; **Progress Check** – adding and subtracting mixed numbers and fractions with like denominators and then renaming the answer when possible

page 155 The top of the page provides practice for pupils who need it. With some pupils you may want to skip the practice and go directly to the Progress Check.

Some pupils will need to rewrite the problems in the Progress Check into vertical form; others will use the horizontal form. Please require no more than is necessary. Those capable of computing mentally should be encouraged to do so. If there is an error in their answer, ask that their work be shown.

Look for four common types of errors:
- Disregard of the operation sign; correct answer for the wrong operation
- Computation errors
- Renaming errors
- No attempt to rename an answer that can be renamed

You may want to have your pupils look for related problems in the Progress Check.

1 and 4	7, 8, and 9
2 and 5	10, 11, and 12
3 and 6	

lesson Pages 156, 157

goal Introduction to renaming in the subtraction of mixed numbers

page 156 Until now, renaming has been done most often **after** finding the answer. You'll want to talk about the need sometimes to rename before being able to subtract. Look out for those youngsters who mentally reverse the two fractions and go right on—to an incorrect answer.

The youngster has this to do. $\begin{array}{r} 4\frac{1}{5} \\ -2\frac{2}{5} \\ \hline \end{array}$ But he thinks this. $\begin{array}{r} 4\frac{2}{5} \\ -2\frac{1}{5} \\ \hline 2\frac{1}{5} \end{array}$

This reversing can be handled quickly. Ask that the pupil pretend he has only $\frac{1}{5}$ of a pizza. Insist that he give you $\frac{2}{5}$ of the pizza. Keep on with similar examples until frustration builds and the child sees that there is no way he can give you the part you request. Use manipulatives if necessary.

Point out that renaming the answer may still be necessary in some problems.

Are you ready?

Try this one.

$$\begin{array}{r} 4\frac{1}{5} \\ -2\frac{2}{5} \\ \hline \end{array}$$

Can you subtract the fractions? No

There is a curve ball in the subtraction game.

When this problem comes up in whole numbers, you rename—a ten as ten ones, for example. Can $4\frac{1}{5}$ be renamed? Rename one of the whole numbers as $\frac{5}{5}$. How many fifths then? Yes; $3\frac{6}{5}$

$4\frac{1}{5}$ *Rename* → $3\frac{1}{5} + \frac{5}{5}$ *Rename* → $3\frac{6}{5}$

$-2\frac{2}{5}$ *Rewrite* → $-2\frac{2}{5}$ $-2\frac{2}{5}$

$1\frac{4}{5}$ **O.K. now?** Yes

Try these. You'll have to rename.

1. LOOK $\begin{array}{r} 5\frac{1}{3} \\ -2\frac{2}{3} \\ \hline \end{array}$ **NO!** → $\begin{array}{r} 4\frac{4}{3} \\ -2\frac{2}{3} \\ \hline \end{array}$ **O.K. now.** $2\frac{2}{3}$

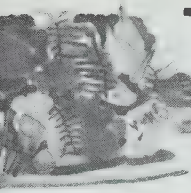
2. $\begin{array}{r} 3\frac{2}{7} \\ -1\frac{5}{7} \\ \hline \end{array}$ → $\begin{array}{r} 2\frac{9}{7} \\ -1\frac{5}{7} \\ \hline 1\frac{4}{7} \end{array}$

3. $\begin{array}{r} 6\frac{1}{6} \\ -3\frac{5}{6} \\ \hline 2\frac{1}{3} \end{array}$ $5\frac{7}{6}$ **4.** $\begin{array}{r} 4\frac{1}{5} \\ -2\frac{2}{5} \\ \hline 1\frac{4}{5} \end{array}$ $3\frac{6}{5}$ **5.** $\begin{array}{r} 7\frac{2}{5} \\ -2\frac{3}{5} \\ \hline 4\frac{4}{5} \end{array}$ $6\frac{7}{5}$ **6.** $\begin{array}{r} 3\frac{1}{4} \\ -1\frac{3}{4} \\ \hline 1\frac{1}{2} \end{array}$ $2\frac{5}{4}$ **7.** $\begin{array}{r} 5 \\ -2 \\ \hline 2 \end{array}$

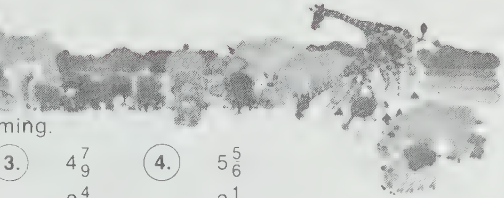
If you're not sure, ask about these.

8. $4 - 1\frac{3}{4}$ $2\frac{1}{4}$ $(3\frac{4}{4})$ **9.** $6 - 4\frac{3}{5}$ $1\frac{2}{5}$ $(5\frac{5}{5})$ **10.** $5 - 3\frac{1}{2}$ $1\frac{1}{2}$ $(4\frac{2}{2})$ **11.** $3 - 2\frac{1}{2}$ $(2\frac{2}{2})$

12. $3 - 1\frac{2}{3}$ $1\frac{1}{3}$ $(2\frac{3}{3})$ **13.** $4 - 2\frac{5}{6}$ $1\frac{1}{6}$ $(3\frac{6}{6})$ **14.** $8 - 7\frac{7}{8}$ $\frac{1}{8}$ $(7\frac{8}{8})$ **15.** $10 - 1\frac{1}{10}$ $(9\frac{10}{10})$



PROGRESS CHECK



Subtract. Watch out for renaming.

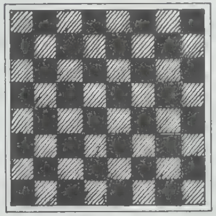
1. $7\frac{2}{5} - 2\frac{1}{5}$ $5\frac{1}{5}$
2. $3\frac{1}{4} - 1\frac{3}{4}$ $1\frac{1}{2}$
3. $4\frac{7}{9} - 2\frac{4}{9}$ $2\frac{1}{3}$
4. $5\frac{5}{6} - 3\frac{1}{6}$ $2\frac{2}{3}$
5. $2\frac{1}{5} - 1\frac{2}{5}$ $\frac{4}{5}$
6. $2\frac{3}{8} - 1\frac{5}{8}$ $\frac{3}{4}$
7. $9\frac{1}{4} - 3\frac{3}{4}$ $5\frac{1}{2}$

LOOK OUT!

8. $5 - 1\frac{1}{2}$ $3\frac{1}{2}$
9. $4\frac{2}{3} - 1$ $3\frac{2}{3}$

*10. $2\frac{3}{4} - 1\frac{1}{2}$ $1\frac{1}{4}$

*11. $3\frac{5}{8} - 1\frac{1}{4}$ $2\frac{3}{8}$



What fraction of the board is black?

$\frac{1}{2}$

goal **Progress Check**—subtracting mixed numbers and renaming differences when possible

page 157 Everyone should complete problems 1 through 9, but problems 10 and 11 include a special challenge. They require renaming the fractions with a common denominator before computing. Use these problems to help you identify pupils who can move through the next section of the chapter more quickly.

Watch for three common types of error:

$3\frac{1}{4} - 1\frac{3}{4} = 2\frac{2}{4}$ Mentally reversed fractions; then subtracted

$5 - 1\frac{1}{2} = 4\frac{1}{2}$ Brought down fraction; then subtracted the whole number

$2\frac{1}{5} = 2\frac{6}{5}$, $-1\frac{2}{5} = 1\frac{2}{5}$, $1\frac{3}{5}$ Renaming error

Try to identify whether the root of the trouble is in subtracting or in renaming. You may want the youngster to reason aloud so that you can hear any faulty thinking.

Supersleuths should be able to find a better way than counting to solve the problem.

STOP See activity 3, page 175b.

See activity 4, page 175b.

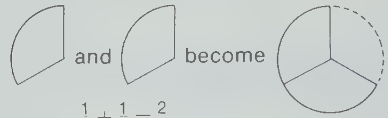
lesson Pages 158, 159

goal Introduction to adding fractions with unlike denominators

memo Pages 158 through 164 focus on adding and subtracting fractions with unlike denominators. In each case, one denominator is a factor of the other.

page 158 A big new idea is introduced on this page. You'll want to get everyone off to the right start. Manipulating felt parts is no more helpful than working with diagrams or paper and pencil—**unless** the felt parts are actually exchanged for same-size pieces and laid over a circular region to find the sum. This exchange to same-size parts parallels the renaming step on paper. Work with manipulating the felt parts only with youngsters who are having trouble and until they can act confidently. Then go right on to page 159.

When we add or subtract fractions less than one, the pie model works too.



$\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$

$\frac{1}{2}+\frac{1}{2}=\frac{2}{2}$ or 1

All you have to do is put the pieces together. Right?

$\frac{1}{2}+\frac{1}{3}=$ **?** How do we name it?

$\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$ Look at the denominators. They are the same.

$\frac{1}{2}+\frac{1}{2}=\frac{2}{2}$ Look at the denominators. They are the same.

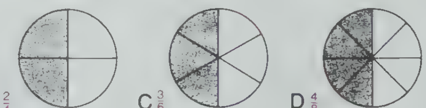
$\frac{1}{2}+\frac{1}{3}=$? The denominators are the troublemakers.

All we have to do is get the denominators of both fractions to show the same number of parts.

A

Look at the unshaded part. What fraction names it? $\frac{1}{2}$

What fraction names these unshaded parts?



B $\frac{2}{4}$ C $\frac{3}{6}$ D $\frac{4}{8}$

You know that $\frac{1}{2}$ can be named $\frac{2}{4}$ or $\frac{3}{6}$ or $\frac{4}{8}$ or $\frac{5}{10}$ or $\frac{6}{12}$ or $\frac{7}{14}$ or $\frac{8}{16}$ or $\frac{9}{18}$ or $\frac{10}{20}$ or . . .

You know that $\frac{1}{3}$ can be named $\frac{2}{6}$ or $\frac{3}{9}$ or $\frac{4}{12}$ or $\frac{5}{15}$ or . . . Hold everything! There is no need to go on.

$\frac{1}{2}+\frac{1}{3}$ can be renamed as $\frac{3}{6}+\frac{2}{6}$ You know how to name that answer. $\frac{5}{6}$

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$\frac{3}{8} + \frac{1}{4}$ ← Look. The denominators do not show the same number of parts.

Can $\frac{3}{8}$ be renamed as $\frac{?}{4}$? NOPE!
But can $\frac{1}{4}$ be renamed as $\frac{?}{8}$? Yes
So rename. Get common denominators.

Rename $\frac{3}{8} + \frac{1}{4}$, $\frac{2}{8}$

Rewrite $\frac{3}{8} + \frac{2}{8} = \underline{?}$ $\frac{5}{8}$

$\frac{1}{2} + \frac{1}{4}$ ← Get common denominators.

Can $\frac{1}{2}$ be renamed as $\frac{?}{4}$? Yes

Rename $\frac{1}{2} + \frac{1}{4}$, $\frac{2}{4}$

Rewrite $\frac{2}{4} + \frac{1}{4} = \underline{?}$ $\frac{3}{4}$

You will be finding a lot of common denominators.
Practice. Find a common denominator for each pair of fractions.

1. $\frac{1}{2}, \frac{1}{8}$ THINK Can $\frac{1}{2}$ be renamed as $\frac{?}{8}$? Do it. Yes; $\frac{4}{8}$
2. $\frac{1}{6}, \frac{1}{3}$ THINK Can $\frac{1}{6}$ be renamed as $\frac{?}{3}$? NO!
   Try again. Can $\frac{1}{3}$ be renamed as $\frac{?}{6}$? Do it. Yes; $\frac{2}{6}$
3. $\frac{2}{3}, \frac{1}{9}$ THINK Can $\frac{2}{3}$ be renamed as $\frac{?}{9}$? Do it. Yes; $\frac{6}{9}$
4. $\frac{3}{4}, \frac{1}{8}$ THINK Can $\frac{3}{4}$ be renamed as $\frac{?}{8}$? Do it. Yes; $\frac{6}{8}$
5. $\frac{1}{12}, \frac{2}{3}$ THINK Can $\frac{1}{12}$ be renamed as $\frac{?}{3}$? NO!
   Try again. Can $\frac{2}{3}$ be renamed as $\frac{?}{12}$? Do it. Yes; $\frac{8}{12}$
6. $\frac{1}{2}, \frac{1}{6}$ THINK Can $\frac{1}{2}$ be renamed as $\frac{?}{6}$? Do it. Yes; $\frac{3}{6}$

goal Practice in finding a common denominator

page 159 The two examples at the top of the page show when a common denominator is needed. The focus now, however, is on developing skill in finding a common denominator, rather than on performing the actual addition.

Your guidance will be important. There are important questions for you to ask. Which fraction should be renamed? Try folding strips of paper. Can fourths be folded into halves? How about halves—can they be folded into fourths? Repeat for thirds and sixths.

Renaming the pairs of fractions should be completed independently. This skill is prerequisite to all further work in the addition and subtraction of fractions.

lesson Pages 160, 161, 162

goal Practice in adding fractions with unlike denominators

page 160 The page provides sufficient information to enable learners who are operating with confidence to go on independently. You will want to guide the others.

If finding the common denominator is a big problem for some youngsters, work on this skill in isolation. The renaming and addition steps should cause no trouble once they are over the common-denominator hurdle. Simply examine pairs of fractions and have the youngsters name the common denominator. Then move on to the renaming step. Finally, have the addition completed.

Fractions must have *common denominators* before you can add.

	$\frac{1}{2} + \frac{3}{4} = \underline{?}$	Find the common denominator.
Rename, using the common denominator.	$\frac{1}{2} \downarrow \frac{2}{4}$	
Rewrite the problem.	$\frac{2}{4} + \frac{3}{4} = \frac{5}{4}$	Easy, isn't it?

1.

Here is the problem.	$\frac{2}{3} + \frac{1}{6} = \underline{?}$	Find the common denominator.
Rename, using the common denominator.	$\frac{2}{3} \downarrow \frac{?}{6}$ 4	
Rewrite the problem.	$\frac{?}{6}$ 4 $+ \frac{1}{6} = \frac{?}{6}$ 5	Now you're ready to add!

2.

Try another problem.	$\frac{1}{8} + \frac{1}{4} = \underline{?}$	Find the common denominator.
Rename, using the common denominator.	$\frac{1}{4} \downarrow \frac{?}{8}$ 2	
Rewrite the problem.	$\frac{1}{8} + \frac{?}{8}$ 2 $= \frac{?}{8}$ 3	Now you can add.

3.

Try one more.	$\frac{2}{3} + \frac{1}{9} = \underline{?}$	Find the common denominator.
Rename, using the common denominator.	$\frac{2}{3} \downarrow \frac{?}{?}$ $\frac{6}{9}$	
Rewrite the problem.	$\frac{?}{?} + \frac{?}{?} = \frac{?}{?}$ $\frac{6}{9} + \frac{1}{9} = \frac{7}{9}$	

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Now you're ready to practice addition on your own.
Remember—

Step 1 Write the problem; find the common denominator.

Step 2 Rename, using the common denominator.

Step 3 Rewrite the problem; then add.

Step 4 Write the simplest name for your answer.

1. $\frac{1}{2}+\frac{1}{4}$ $\frac{3}{4}$ 2. $\frac{1}{3}+\frac{7}{6}$ $(\frac{9}{6})$ $1\frac{1}{2}$ 3. $\frac{3}{8}+\frac{1}{2}$ $\frac{7}{8}$

4. $\frac{4}{9}+\frac{1}{3}$ $\frac{7}{9}$ 5. $\frac{3}{4}+\frac{5}{8}$ $(\frac{11}{8})$ $1\frac{3}{8}$ 6. $\frac{5}{10}+\frac{2}{5}$ $\frac{9}{10}$

7. $\frac{1}{3}+\frac{1}{6}$ $(\frac{3}{6})$ $\frac{1}{2}$ 8. $\frac{1}{2}+\frac{7}{10}$ $(\frac{12}{10})$ $1\frac{1}{5}$ 9. $\frac{5}{6}+\frac{1}{2}$ $(\frac{8}{6})$ $1\frac{1}{3}$

Subtraction is done the same way. Try these.
Don't forget the four easy steps.

10. $\frac{1}{2}-\frac{1}{6}$ $(\frac{2}{6})$ $\frac{1}{3}$ 11. $\frac{7}{12}-\frac{1}{4}$ $(\frac{4}{12})$ $\frac{1}{3}$ 12. $\frac{4}{3}-\frac{11}{12}$ $\frac{5}{12}$

13. $\frac{3}{5}-\frac{3}{10}$ $\frac{3}{10}$ 14. $\frac{7}{6}-\frac{2}{3}$ $(\frac{3}{6})$ $\frac{1}{2}$ 15. $\frac{5}{12}-\frac{1}{6}$ $(\frac{3}{12})$ $\frac{1}{4}$

16. Compute. Watch for the operation signs.

a $\frac{1}{2}+\frac{3}{10}$ $(\frac{8}{10})$ $\frac{4}{5}$ b $\frac{3}{8}+\frac{1}{4}$ $\frac{5}{8}$ c $\frac{2}{3}-\frac{4}{9}$ $\frac{2}{9}$ d $\frac{5}{6}+\frac{2}{3}$ $(\frac{9}{6})$ $1\frac{1}{2}$

e $\frac{11}{12}-\frac{3}{4}$ $(\frac{2}{12})$ $\frac{1}{6}$ f $\frac{4}{3}-\frac{5}{9}$ $\frac{7}{9}$ g $\frac{1}{12}+\frac{5}{6}$ $\frac{11}{12}$ h $\frac{4}{7}-\frac{3}{14}$ $\frac{5}{14}$

goal Practice in adding and subtracting fractions with unlike denominators

page 161 Before making an assignment, take a quick survey. Ask the pupil to number a paper from 1 through 9 and jot down the common denominator for each of the first nine problems. All correct? Good! This youngster is ready to go on independently.

Group the youngsters who need your guidance. Focus on one step at a time.

goal **Progress Check**—adding fractions with unlike denominators and then renaming the sums when possible

page 162 The first two problems provide review of and practice with the ideas developed on the preceding pages. You may want to discuss these problems or to include them in the Progress Check.

Types of errors to watch for include—
- Renaming with a common denominator
- Addition computation
- Correct sum renamed incorrectly

PROGRESS CHECK

Skill: Renaming sums

Practice your renaming skills as you add these problems.

(1.) $\frac{2}{3} + \frac{2}{3}$ Common denominator? You're ready to add.

$\frac{2}{3} + \frac{2}{3} = \frac{2+2}{3} = \frac{4}{3}$ Now what? Rename $\frac{4}{3}$ as $1\frac{1}{3}$.

That was too easy. That's no fun.

(2.) $\frac{2}{3} + \frac{5}{6}$ Common denominator? Find one. *Rename* $\frac{2}{3} = \frac{?}{6}$

Write the problem with a common denominator. $\frac{4}{6} + \frac{5}{6} = \frac{?}{6}$

NOW WHAT?

Rename $\frac{9}{6}$ as $1\frac{1}{2}$.

Skills: Adding fractions with unlike denominators; renaming sums

Your turn. Find the sums, renaming the fractions when you need to. Rename your answers.

(3.) $\frac{1}{5} + \frac{3}{10} = ?$ $(\frac{5}{10})$ $\frac{1}{2}$ (4.) $\frac{1}{3} + \frac{1}{9} = ?$ $\frac{4}{9}$ (5.) $\frac{1}{4} + \frac{3}{8} = ?$ $\frac{5}{8}$ (6.) $\frac{2}{3} + \frac{1}{6} = ?$ $\frac{5}{6}$

(7.) $\frac{1}{3} + \frac{5}{12} = ?$ $(\frac{9}{12})$ $\frac{3}{4}$ (8.) $\frac{1}{6} + \frac{5}{12} = ?$ $\frac{7}{12}$ (9.) $\frac{1}{4} + \frac{1}{12} = ?$ $(\frac{4}{12})$ $\frac{1}{3}$ (10.) $\frac{5}{9} + \frac{2}{3} = ?$ $(\frac{11}{9})$ $1\frac{2}{9}$

(11.) $\frac{4}{5} + \frac{1}{10} = ?$ $\frac{9}{10}$ (12.) $\frac{1}{2} + \frac{3}{8} = ?$ $\frac{7}{8}$ *(13.) $\frac{7}{10} + \frac{3}{100} = ?$ $\frac{73}{100}$ *(14.) $\frac{8}{10} + \frac{19}{100} = ?$ $\frac{99}{100}$

162

See activity 5, page 175b.

See activity 6, page 175b.

just for fun

Lucky Fractions

This lucky card contains all the answers to the addition and subtraction problems on this page. After you do a problem, look for your answer on the lucky card and put a marker on it. The first one has been done for you. Do only as many problems as you need to win the game.

The pattern shows LUCKY within the first 16 problems. Other patterns are possible.

L	U	C	K	Y	
$\frac{35}{10}$	$\frac{5}{8}$	$\frac{84}{100}$	$\frac{3}{20}$	$\frac{9}{12}$	16.
$\frac{5}{18}$	$\frac{5}{15}$	$\frac{7}{8}$	$\frac{3}{10}$	$\frac{3}{8}$	4.
$\frac{6}{5}$	$\frac{8}{12}$	$\frac{7}{10}$	$\frac{15}{6}$	$\frac{11}{12}$	11.
$\frac{1}{9}$	$\frac{7}{9}$	$\frac{11}{12}$	$\frac{17}{9}$	$\frac{5}{6}$	12.
$\frac{3}{100}$	$\frac{2}{4}$	$\frac{10}{6}$	$\frac{11}{14}$	$\frac{3}{10}$	15.

1. $\frac{2}{5}+\frac{3}{10}=\frac{7}{10}$

2. $\frac{5}{3}+\frac{2}{9}=$? $\frac{17}{9}$
3. $\frac{3}{4}-\frac{1}{4}=$? $\frac{2}{4}$
4. $\frac{7}{8}-\frac{1}{2}=$? $\frac{3}{8}$
5. $\frac{1}{5}+\frac{2}{5}+\frac{3}{5}=$? $\frac{6}{5}$
6. $\frac{7}{10}+\frac{14}{100}=$? $\frac{84}{100}$
7. $\frac{6}{5}-\frac{13}{15}=$? $\frac{5}{15}$
8. $\frac{4}{9}-\frac{1}{3}=$? $\frac{1}{9}$
9. $\frac{7}{10}-\frac{2}{5}=$? $\frac{3}{10}$
10. $\frac{43}{100}-\frac{4}{10}=$? $\frac{3}{100}$
11. $\frac{5}{6}+\frac{1}{12}=$? $\frac{11}{12}$
12. $\frac{1}{3}+\frac{1}{6}+\frac{2}{6}=$? $\frac{5}{6}$
13. $\frac{5}{6}+\frac{5}{3}=$? $\frac{15}{6}$
14. $\frac{5}{2}-\frac{5}{6}=$? $\frac{10}{6}$
15. $\frac{7}{10}-\frac{2}{5}=$? $\frac{3}{10}$
16. $\frac{1}{3}+\frac{5}{12}=$? $\frac{9}{12}$
17. $\frac{3}{4}-\frac{1}{12}=$? $\frac{8}{12}$
18. $\frac{4}{7}+\frac{3}{14}=$? $\frac{11}{14}$
19. $\frac{3}{8}+\frac{1}{4}=$? $\frac{5}{8}$
20. $\frac{6}{2}+\frac{5}{10}=$? $\frac{35}{10}$
21. $\frac{1}{9}+\frac{2}{3}=$? $\frac{7}{9}$
22. $\frac{4}{3}-\frac{5}{12}=$? $\frac{11}{12}$
23. $\frac{11}{18}-\frac{1}{3}=$? $\frac{5}{18}$
24. $\frac{13}{20}-\frac{5}{10}=$? $\frac{3}{20}$
25. $\frac{1}{8}+\frac{6}{8}=$? $\frac{7}{8}$

Five problems in a row, column or diagonal win.

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lesson Pages 163, 164

goal Practice in adding and subtracting fractions with unlike denominators

page 163 The game element does ease the monotony of practice, especially when it isn't necessary to do all the problems. Emphasize the last sentence of the instructions. The youngsters who estimate their answers are rewarded. Caution them to watch the operation signs.

Challenge capable youngsters to make a new game board and vary the game as follows:
- Change the order of the fractions.
- Use only the simplest name for each fraction.

goal Extension of renaming skills to subtracting mixed numbers with unlike denominators

page 164 Note the heading "For Experts Only." You will want to use the page only with pupils who have mastered renaming with a common denominator and who need an added challenge. Guide their thinking with questions. What operation is used to make a comparison? What step is necessary before subtraction is possible? Careful of the last problem—the whole number must be renamed as a mixed number.

For Experts Only

A ├── $2\frac{1}{2}$ units ──┤

B ├── $1\frac{1}{4}$ units ──┤

How much longer is segment A than segment B?

$$\begin{array}{rcr} 2\frac{1}{2} & \xrightarrow{\textit{Rename.}} & 2\frac{2}{4} \\ & \text{You need a common denominator.} & \\ -\,1\frac{1}{4} & \xrightarrow{\textit{Rewrite.}} & -\,1\frac{1}{4} \\ \hline & & \blacksquare\frac{\blacksquare}{\blacksquare} \text{ units} \end{array}$$

$1\frac{1}{4}$

Rename before you subtract. You might have to rename the answer, too.

C ├── $2\frac{1}{4}$ units ──┤

D ├── $1\frac{1}{8}$ units ──┤

1. How much longer is segment C than segment D? $1\frac{1}{8}$ units

E ├── $2\frac{7}{8}$ units ──┤

F ├── $1\frac{3}{4}$ units ──┤

2. How much longer is segment E than segment F? $1\frac{1}{8}$ units

G ├── $1\frac{3}{4}$ units ──┤

H ├── $1\frac{3}{8}$ units ──┤

3. How much longer is segment G than segment H? $\frac{3}{8}$ units

I ├── 3 units ──┤

J ├── $1\frac{7}{8}$ units ──┤

4. How much longer is segment I than segment J? $1\frac{1}{8}$ units

CAN YOU THINK ABOUT MULTIPLICATION AS ADDITION?

1. $4 \times 6 = \underline{?}$ You know the answer is 24. If you didn't know the answer, you could find it by adding.

   $4 \times 6 = 24$ could be written

   $6 + 6 + 6 + 6 = 24$

2. You can always find the product of two whole numbers by adding. But if you used addition to complete $232 \times 17 = \underline{?}$ adding 232 seventeens would take a long time. So you multiply.

3. Pumpkin pie is Lester's favorite. His mother served each of the 4 people at dinner $\frac{1}{5}$ of a pie. How much of the pie did they eat?

   $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$

   This can be rewritten as $4 \times \frac{1}{5} = \frac{\overset{4}{?}}{5}$. How much pie? $\frac{4}{5}$

4. Every day Tiny Thirston drinks $\frac{3}{8}$ of a quart of ZIP ZAP punch.

   a How much does he drink in 2 days?

   $\frac{3}{8} + \frac{3}{8}$

   or $2 \times \frac{3}{8} = \frac{\overset{6}{?}}{8}$ quart. How much? $\frac{6}{8}$ quart or $\frac{3}{4}$ quart

   b How much in a week?

   $\frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8}$

   or $7 \times \frac{3}{8} = \frac{\overset{21}{?}}{8}$ quarts. How many quarts is that? $2\frac{5}{8}$ quarts

Can the idea of repeated addition be used in multiplying fractions?

lesson Pages 165, 166, 167, 168

goal Exploring the multiplication of a fraction by a whole number

memo The pupil's only previous experience with multiplying fractions has been in renaming. Then both the numerator and the denominator were multiplied by the same number. Pages 165 through 168 introduce multiplying a whole number and a fraction. The rules change. The repeated-addition approach should make things obvious, but watch for signs of possible confusion.

page 165 Operations with fractions parallel those with whole numbers. The pupil's concept of multiplication as repeated addition developed with whole numbers therefore is also applicable when working with fractions. This is a good talk-together page. Continue right on to page 166.

goal Development of an algorithm for multiplying a whole number and a fraction

page 166 The example provides a guide for thinking. Be sure to take time for the questions in the book. The algorithm developed should help youngsters avoid falling into the trap of multiplying both the numerator and denominator, as is done when renaming a fraction. Sufficient help is given on the page for independent readers. You'll want to work with the others.

The emphasis here is on the concept of multiplication. It is not necessary to insist on answers in simplest form. There is time for this later. When checking, you may wish to question whether the product could be renamed.

$\frac{4}{5}+\frac{4}{5}+\frac{4}{5}=\frac{12}{5}$ — This is the sum of the numerators.

$3\times\frac{4}{5}=\frac{12}{5}$ — Where does this number come from? $3\times4=12$
(3 times the numerator 4 equals 12)

$\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=\frac{8}{3}$ — You know where this comes from.

$4\times\frac{2}{3}=\frac{8}{3}$ — What about this? $4\times2=8$
(4 times the numerator 2 equals 8)

$3\times\frac{3}{4}=\frac{?}{4}$ — What is the correct numerator? 9

1. Use multiplication or addition to solve these problems.

   a $2\times\frac{1}{3}=\underline{?}$ $\frac{2}{3}$ b $3\times\frac{2}{3}=\underline{?}$ $(\frac{6}{3})$ 2 c $4\times\frac{1}{2}=\underline{?}$ $(\frac{4}{2})$ 2

   d $5\times\frac{1}{6}=\underline{?}$ $\frac{5}{6}$ e $3\times\frac{3}{8}=\underline{?}$ $(\frac{9}{8})$ $1\frac{1}{8}$ f $2\times\frac{2}{5}=\underline{?}$ $\frac{4}{5}$

2. Look at the first problem again. $2\times\frac{1}{3}=\frac{2}{3}$
   Is the numerator in the answer the same as the product of $2\times\frac{1}{3}=\frac{2}{3}$? Yes
   Could it be rewritten as $\frac{2\times1}{3}=\frac{2}{3}$? Yes

3. Could other problems be rewritten this way? Yes
   Complete $4\times\frac{2}{9}=\underline{?}$ $\frac{8}{9}$ Now complete $\frac{4\times2}{9}=\underline{?}$ $\frac{8}{9}$
   Are both the answers the same? They should be. Yes

4. Complete.

   a $3\times\frac{2}{5}=\frac{3\times2}{5}=\underline{?}$ $(\frac{6}{5})$ $1\frac{1}{5}$ b $2\times\frac{2}{7}=\frac{2\times2}{7}=\underline{?}$ $\frac{4}{7}$ c $6\times\frac{4}{9}=\frac{6\times4}{?_{9}}=\underline{?}$ $(\frac{24}{9})$ $2\frac{2}{3}$

   d $5\times\frac{2}{3}=\frac{\overset{5}{?}\times\overset{2}{?}}{3}=\underline{?}$ $(\frac{10}{3})$ $3\frac{1}{3}$ e $9\times\frac{5}{6}=\frac{\overset{9}{?}\times\overset{5}{?}}{\underset{6}{?}}=\underline{?}$ $(\frac{45}{6})$ $7\frac{1}{2}$ f $7\times\frac{3}{10}=\frac{\overset{7}{?}\times\overset{3}{?}}{\underset{10}{?}}=\underline{?}$ $(\frac{21}{10})$ $2\frac{1}{10}$

166

1. Find these products.

a $3 \times \frac{1}{2} = \underline{?}$ That's easy, isn't it? $(\frac{3}{2})$ $1\frac{1}{2}$

b $\frac{1}{2} \times 3 = \underline{?}$ That's a new one.

How do you do it?

Try this $\frac{1}{2} \times 3$ means $\frac{1}{2}$ of 3. Suppose you want to find $\frac{1}{2}$ of 3 apples. One way to do it is to take $\frac{1}{2}$ of each apple. Then add the $\frac{1}{2}$s.

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 3 \times \frac{1}{2}$$

Hey—we know that product is $\frac{3}{2}$.
$3 \times \frac{1}{2} = \frac{3}{2}$ and $\frac{1}{2} \times 3 = \frac{3}{2}$

You knew that you could change the order of two whole-number factors and get the same product. Now you know you can change the order of two fraction factors, too.

$$3 \times \frac{1}{2} = \frac{3 \times 1}{2} = \frac{3}{2} \quad \text{and} \quad \frac{1}{2} \times 3 = \frac{1 \times 3}{2} = \frac{3}{2}$$

2. Find these products.

a $2 \times \frac{1}{5} = \frac{2 \times 1}{5} = \underline{?}$ $\frac{2}{5}$ b $\frac{1}{5} \times 2 = \frac{1 \times 2}{5} = \underline{?}$ $\frac{2}{5}$ c $\frac{4}{5} \times 3 = \frac{4 \times 3}{5} = \underline{?}$ $(\frac{12}{5})$ $2\frac{2}{5}$

d $\frac{3}{8} \times 4 = \frac{3 \times ?^{4}}{8} = \underline{?}$ $(\frac{12}{8})$ $1\frac{1}{2}$ e $\frac{5}{6} \times 7 = \frac{^{5}? \times ?^{7}}{?_{6}} = \underline{?}$ $(\frac{35}{6})$ $5\frac{5}{6}$ f $\frac{2}{7} \times 5 = \underline{?}$ $(\frac{10}{7})$ $1\frac{3}{7}$

g $\frac{5}{9} \times 7 = \underline{?}$ $(\frac{35}{9})$ $3\frac{8}{9}$ h $\frac{2}{3} \times 4 = \underline{?}$ $(\frac{8}{3})$ $2\frac{2}{3}$ i $\frac{2}{3} \times 6 = \underline{?}$ $(\frac{12}{3})$ 4

goal Exploring the multiplication of a whole number by a fraction

page 167 Once again, repeated addition is used as the entry. Then the commutative property is used. Please notice that the terminology is not introduced. The emphasis is on ideas rather than on words, but introduce the terminology if you wish.

You may want to add some common sense to the development on the page. Is there another way to think of $\frac{1}{2}$ of 3 apples? Two people are sharing 3 apples—each receives $\frac{1}{2}$ of the apples. Must each apple be cut in $\frac{1}{2}$ to share equally?

To find a fractional part of a set, the objects need not always be cut into parts. It depends on what the set is. Think about $\frac{1}{4}$ of a dozen cookies. How many?

You decide whether your pupils are ready to write their products in simplest form.

goal **Progress Check**—multiplying whole numbers and fractions

page 168 The Progress Check identifies ability to multiply with a fraction factor in either position. The focus should be on the operation for now. You decide whether your pupils are ready to write products in simplest forms as well.

Look for two basic types of errors:

- Multiplication fact errors
- Confusion about the algorithm

Example: $\frac{1}{2} \cdot 6 = \frac{6}{12}$

This pupil is not following the algorithm.

$$\frac{1}{2} \cdot 6 = \frac{1 \cdot 6}{2} = \frac{6}{2}$$

Hands-on work with fraction felt pieces or regions drawn on the blackboard should help.

The Supersleuth section is meant for discussion, rather than paperwork. What about $\frac{1}{2}$ of $\frac{1}{2}$ hour? What part of an hour is that? How many minutes? What about $\frac{1}{2}$ of $\frac{1}{4}$ of a pie? Can you estimate how much that would be? And $\frac{1}{2}$ of $\frac{1}{2}$ of a fence? Is that a lot or a little? What about $\frac{1}{2}$ of \$3.00? How much is that? And $\frac{1}{2}$ of $\frac{2}{3}$? Guess we better learn a method!

These are situations when multiplication of fractions is used. Many youngsters may be able to **reason** the answers without computing. Marvelous! Let their replies signal those pupils who may be ready to move more quickly through the remainder of the chapter.

PROGRESS CHECK

Compute. Skill: Multiplying whole numbers and fractions

1. $\frac{1}{2} \times 6 = ?$ $(\frac{6}{2})$ 3
2. $\frac{3}{10} \times 8 = ?$ $(\frac{24}{10})$ $2\frac{2}{5}$
3. $\frac{5}{11} \times 2 = ?$ $\frac{10}{11}$
4. $3 \times \frac{7}{9} = ?$ $(\frac{21}{9})$ $2\frac{1}{3}$
5. $4 \times \frac{2}{7} = ?$
6. $\frac{2}{3} \times 10 = ?$ $(\frac{20}{3})$ $6\frac{2}{3}$
7. $5 \times \frac{3}{2} = ?$ $(\frac{15}{2})$ $7\frac{1}{2}$
8. $\frac{7}{8} \times 5 = ?$ $(\frac{35}{8})$ $4\frac{3}{8}$
9. $\frac{2}{11} \times 7 = ?$ $(\frac{14}{11})$ $1\frac{3}{11}$
10. $10 \times \frac{9}{10} =$

Have you ever heard of situations like these?

1. You plan to study half of the next half hour.
   How much time would that be? $\frac{1}{2}$ of what?
   $\frac{1}{4}$ hour; $\frac{1}{2}$ of $\frac{1}{2}$ hour
2. Look—there's a fourth of the pie left.
   You're going to eat half of it.
   How much of the pie will you eat? $\frac{1}{2}$ of what?
   $\frac{1}{8}$ pie; $\frac{1}{2}$ of $\frac{1}{4}$ pie
3. You and your sister are painting a fence. $\frac{1}{2}$ of it is your share. You have painted $\frac{1}{2}$ of your share.
   How much have you painted? $\frac{1}{2}$ of what?
   $\frac{1}{4}$ of the fence; $\frac{1}{2}$ of $\frac{1}{2}$ of the fence
4. You have \$3 to spend. You have spent $\frac{1}{2}$ of it.
   How much of the money have you spent? $\frac{1}{2}$ of what?
   \$1.50; $\frac{1}{2}$ of \$3.00
5. You have $\frac{2}{3}$ left. You give your best friend $\frac{1}{2}$ of that.
   How much did you give your friend? $\frac{1}{2}$ of what?
   $\frac{1}{3}$; $\frac{1}{2}$ of $\frac{2}{3}$

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STOP See activity 8, page 175b.

See activity 9, page 175c.

1. Greedy shared his candy bar with his two friends. First he marked it into thirds. He could have shared it then, but he didn't.

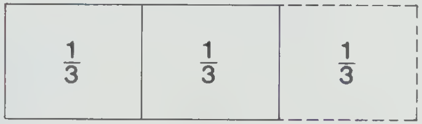
He marked each $\frac{1}{3}$ into 2 equal parts.
He gave each friend one of these parts.

Greedy gave $\frac{1}{2}$ of $\frac{1}{3}$ candy bar to each friend.
What fractional part did each friend get? $\frac{1}{6}$
The picture shows $\frac{1}{6}$. Does $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$? Yes

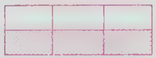
2. Another day Greedy wasn't so greedy. He decided to give $\frac{1}{2}$ of $\frac{2}{3}$ of his candy bar to each of his two friends.

He marked $\frac{1}{2}$ of each $\frac{1}{3}$.
Then he gave each friend $\frac{1}{2}$ of $\frac{2}{3}$ of his candy bar.

Does $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$? Yes
He gave each of his friends $\frac{2}{6}$ of his candy bar.
How much did he have left for himself? $\frac{2}{6}$ or $\frac{1}{3}$
Did he do it the hard way? Yes



3. You know that $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ and that $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$.
What is $\frac{1}{2} \times \frac{3}{3}$? $\frac{3}{6}$ or $\frac{1}{2}$
Draw a model to show your answer.
(First show $\frac{3}{3}$. Then show $\frac{1}{2}$ of the $\frac{3}{3}$.)



169

lesson Pages 169, 170, 171, 172

goal Introduction to the multiplication of two fractions

memo Pages 169 and 170 introduce a concept that is difficult for many pupils (and adults) to understand. Why is it that in multiplying two fractions the product diminishes rather than increases in size.

page 169 A good page to share ideas and thereby develop understanding. You might start things off by asking a key question. If Greedy gave each friend $\frac{1}{6}$ of his candy bar, how much did he keep for himself? Is Greedy a good name for this youngman?

Each youngster is free to use any shape region he wishes to make his model for problem 3. The only prerequisite is that he can explain his model.

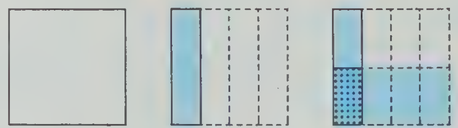
goal Development of the concept of the multiplication of two fractions

page 170 You may want the youngsters to make models of their own as you discuss the page. Square or rectangular regions are easier to work with—one factor can be shown vertically, the other horizontally.

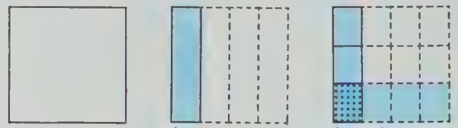
Here are some questions you may want to discuss:

- Is the denominator of the product larger or smaller than the denominator of either factor? (Larger)
- When you multiply whole numbers, the product usually increases. When you multiply with fractions, what increases? (The number of parts)
- As you multiply the number of parts, what happens to the size of the parts? (They become smaller.)

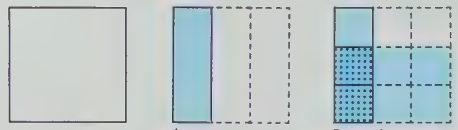
Encourage the youngsters to express in their own words how to find the product when they multiply two fractions. Look for an understanding of the concept and the ability to operate with it rather than for memorization of a rule.

1. 

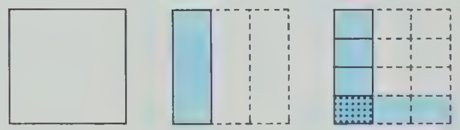
1 square $\quad \frac{1}{4}$ square $\quad \frac{1}{2}$ of $\frac{1}{4}$ square $\quad \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

2. 

1 square $\quad \frac{1}{4}$ square $\quad \frac{1}{3}$ of $\frac{1}{4}$ square $\quad \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$ $\quad$ What's $\frac{2}{3} \times \frac{1}{4}$? $\frac{2}{12}$ o

3. 

1 square $\quad \frac{1}{3}$ square $\quad \frac{2}{3}$ of $\frac{1}{3}$ square $\quad \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$ $\quad$ What's $\frac{1}{3} \times \frac{1}{3}$? $\frac{1}{9}$

4. 

1 square $\quad \frac{1}{3}$ square $\quad \frac{1}{4}$ of $\frac{1}{3}$ square $\quad \frac{1}{4} \times \frac{1}{3} = \underline{?}$ $\frac{1}{12}$ $\quad$ What's $\frac{3}{4} \times \frac{1}{3}$? $\frac{3}{12}$ o

5. Look at the *denominators* of the first problem. Does $2 \times 4 = 8$? Yes
Look at the *numerators* of the first problem. Does $1 \times 1 = 1$? Yes

6. Look at the denominators of each of the other problems. Then look at the numerators. Have you made a discovery?
Yes—to multiply 2 fractions, multiply the numerators, then the denominators.

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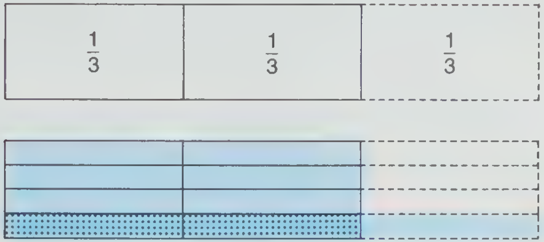
Study this problem and its model. Watch the numerators and denominators.



This model shows $\frac{2}{3}$.

The problem is $\frac{1}{4}$ of $\frac{2}{3}$.

Now the model has been divided into 4 equal parts. You're interested in one of the parts. How many parts in all now? 12 How many shaded? 2 $\frac{1}{4} \times \frac{2}{3} = \underline{?}$ $\frac{2}{12}$ or $\frac{1}{6}$

2. Draw a model to show $\frac{1}{4}$ of $\frac{1}{2}$ $(\frac{1}{4} \times \frac{1}{2})$. $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$ 

3. Try finding these products without drawing a model.

a $\frac{1}{2} \times \frac{1}{4}$ $\frac{1}{8}$ b $\frac{1}{2} \times \frac{1}{2}$ $\frac{1}{4}$ c $\frac{1}{2} \times \frac{3}{4}$ $\frac{3}{8}$

d $\frac{2}{4} \times \frac{1}{4}$ $\frac{2}{16}$ or $\frac{1}{8}$ e $\frac{2}{4} \times \frac{2}{4}$ $\frac{4}{16}$ or $\frac{1}{4}$ f $\frac{2}{4} \times \frac{3}{4}$ $\frac{6}{16}$ or $\frac{3}{8}$

4. If you could compute the products in the last problem, you can answer these questions.

a How much is $\frac{1}{3}$ of $\frac{1}{2}$ order of french fries? $\frac{1}{6}$

b $\frac{1}{2}$ of $\frac{3}{4}$ piece of cloth is how much? $\frac{3}{8}$

c How much is $\frac{1}{2}$ of $\frac{1}{2}$ hour? $\frac{1}{4}$

171

goal Practice in multiplying two fractions

page 171 Additional help for making models is given in the first problem. This should be sufficient direction to get everyone through making a model for problem 2. Remember—any model that a youngster can explain is satisfactory.

Encourage pupils who have difficulty with problem 3 to include this additional step:

$$\frac{1}{2} \times \frac{1}{4} = \frac{1 \times 1}{2 \times 4} = \frac{1}{8}$$

goal Practice in multiplying two fractions: **Progress Check** – multiplying two fractions

page 172 Pupils who need to write the computational algorithm should do so; those who are able to compute mentally should be encouraged to do so. Too much writing sometimes can stand in the way of success. The emphasis should be on finding the correct product, not on writing all the parts of an algorithm.

You may want to save the Progress Check for another day. Look for three common types of error:

- Multiplication fact error
- Miscopied problem
- Algorithm not understood

Focus the learner's attention on the type of error he has made before using page 173 for additional practice.

Page 174 provides a challenge for pupils who are operating with confidence.

Are you sure of your work when you multiply two fractions? If not, show your work like this:

$$\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$$

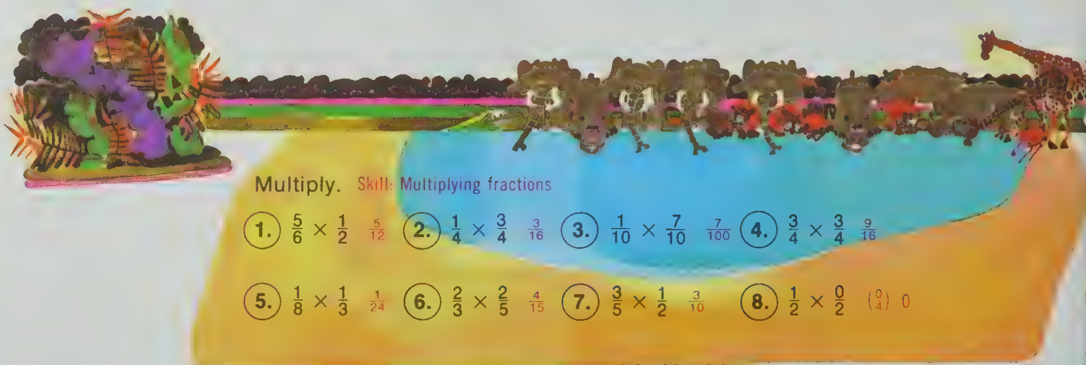
		a	b	c	d
Multiply.	1.	$\frac{1}{2} \times \frac{2}{5}$ $\left(\frac{2}{10}\right)$ $\frac{1}{5}$	$\frac{2}{5} \times \frac{1}{3}$ $\frac{2}{15}$	$\frac{1}{4} \times \frac{1}{4}$ $\frac{1}{16}$	$\frac{2}{9} \times \frac{2}{3}$ $\frac{4}{27}$
	2.	$\frac{2}{3} \times \frac{3}{4}$ $\left(\frac{6}{12}\right)$ $\frac{1}{2}$	$\frac{7}{8} \times \frac{3}{4}$ $\frac{21}{32}$	$\frac{2}{3} \times \frac{2}{3}$ 4 [illegible]	Look out! $\frac{4}{9} \times \frac{0}{2}$ $\left(\frac{0}{18}\right)$ 0

3. He bought $\frac{1}{2}$ loaf of bread. He gave $\frac{1}{4}$ of it to his brother. How much did he give away? $\frac{1}{8}$

4. She had $\frac{1}{4}$ of a dollar. She gave $\frac{1}{5}$ of it to her sister. What part of a dollar did she give to her sister? How many cents was that? $\frac{1}{20}$; 5¢

5. There was $\frac{2}{3}$ jar of jam. She used $\frac{1}{2}$ of it. How much did she use? $\left(\frac{2}{6}\right)$ $\frac{1}{3}$

6. He had $\frac{1}{2}$ of a dollar. He put $\frac{1}{10}$ of it in his bank. What part of a dollar did he save? $\frac{1}{20}$ How many cents was that? 5¢



Multiply. Skill: Multiplying fractions

1. $\frac{5}{6} \times \frac{1}{2}$ $\frac{5}{12}$ 2. $\frac{1}{4} \times \frac{3}{4}$ $\frac{3}{16}$ 3. $\frac{1}{10} \times \frac{7}{10}$ $\frac{7}{100}$ 4. $\frac{3}{4} \times \frac{3}{4}$ $\frac{9}{16}$

5. $\frac{1}{8} \times \frac{1}{3}$ $\frac{1}{24}$ 6. $\frac{2}{3} \times \frac{2}{5}$ $\frac{4}{15}$ 7. $\frac{3}{5} \times \frac{1}{2}$ $\frac{3}{10}$ 8. $\frac{1}{2} \times \frac{0}{2}$ $\left(\frac{0}{4}\right)$ 0

See activity 10, page 175c.

See activity 11, page 175c.

Compute. *Make sure every product has its simplest name.*
This set of problems has some interesting patterns.
Watch for them.

a $\frac{1}{5} \times \frac{1}{4}$ $\frac{1}{20}$ b $\frac{1}{5} \times \frac{2}{4}$ $\frac{1}{10}$ c $\frac{1}{5} \times \frac{3}{4}$ $\frac{3}{20}$ d $\frac{1}{5} \times \frac{4}{4}$ $\frac{1}{5}$

e $\frac{2}{5} \times \frac{1}{4}$ $\frac{1}{10}$ f $\frac{2}{5} \times \frac{2}{4}$ $\frac{1}{5}$ g $\frac{2}{5} \times \frac{3}{4}$ $\frac{3}{10}$ h $\frac{2}{5} \times \frac{4}{4}$ $\frac{2}{5}$

i Did you find any patterns? Which ones? All denominators are 20 before renaming

ch of these problems is different. Compute.
oducts should be named with their simplest names.

	a	b	c	d
2.	$\frac{1}{4} \times \frac{4}{5}$ $\frac{1}{5}$	$\frac{1}{6} \times \frac{2}{3}$ $\frac{1}{9}$	$\frac{2}{5} \times \frac{3}{4}$ $\frac{3}{10}$	$\frac{9}{9} \times \frac{3}{4}$ $\frac{3}{4}$
3.	$\frac{3}{5} \times \frac{1}{3}$ $\frac{1}{5}$	$\frac{2}{3} \times \frac{5}{6}$ $\frac{5}{9}$	$\frac{1}{2} \times \frac{2}{7}$ $\frac{1}{7}$	$\frac{2}{5} \times \frac{1}{4}$ $\frac{1}{10}$
4.	$\frac{5}{6} \times \frac{2}{3}$ $\frac{5}{9}$	$\frac{1}{4} \times \frac{2}{5}$ $\frac{1}{10}$	$\frac{1}{3} \times \frac{3}{5}$ $\frac{1}{5}$	$\frac{2}{7} \times \frac{1}{2}$ $\frac{1}{7}$

If you were hungry, would you rather have—

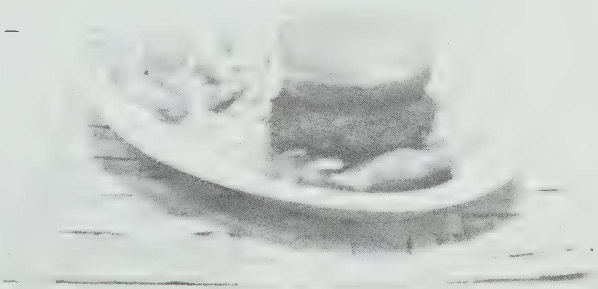
a $\frac{1}{4}$ of $\frac{1}{2}$ hamburger or $\frac{1}{2}$ of $\frac{1}{4}$?

b $\frac{1}{3}$ of $\frac{2}{3}$ milkshake or $\frac{2}{3}$ of $\frac{1}{3}$?

c $\frac{4}{5}$ of $\frac{1}{2}$ order of french fries or $\frac{1}{2}$ of $\frac{4}{5}$?

d $\frac{3}{4}$ of $\frac{1}{6}$ piece of cake or $\frac{1}{6}$ of $\frac{3}{4}$?

e $\frac{1}{8}$ of $\frac{1}{2}$ pizza or $\frac{1}{2}$ of $\frac{1}{8}$?



Either one. Both amounts are the same in all of these questions. (You may want some pupils to draw a picture if they get in trouble with the answers.)

173

See activity 12, page 175c.

lesson Pages 173, 174, 175

goal Practice in multiplying two fractions and renaming products when possible

page 173 More than just practice—there are patterns too! Something for everyone:

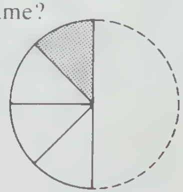
- Need practice with the multiplication algorithm? There are plenty of problems.
- Need practice renaming products? Have the pupil compute mentally (if possible) and then focus on renaming.
- Challenge the sharpies to find problems that require no computation to find the product in its simplest form

1d $\frac{1}{5} \times \frac{4}{4}$ **1h** $\frac{2}{5} \times \frac{4}{4}$ **2d** $\frac{9}{9} \times \frac{3}{4}$

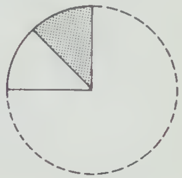
Why not? What do $\frac{4}{4}$ and $\frac{9}{9}$ equal? What is 1 times any number?

You'll want the pupils to share their answers for problem 5.

Changing the order of the factors does not change the product in computing. Yet if a picture were drawn for each problem situation, the pictures would not be identical. Would the quantity be the same?



$\frac{1}{4}$ of $\frac{1}{2}$



$\frac{1}{2}$ of $\frac{1}{4}$

goal Practice in multiplying fractions, and application of multiplying skills to problem situations

page 174 More practice - this time with more unusual fractions. Let pupils choose one of the first three questions to work on. Those who find the work a challenge may want to do more, but do not insist on this.

Questions 4 to 7 bring them back to real-world situations, and relate fractional amounts to real quantities of real things like eggs and boards. This helps to keep fractions from becoming mere numbers on paper.

Everyone should do questions 4 to 10 independently. Discussion may bring out difficulties in understanding the last three questions. Models may be helpful for those who have trouble with these problems.

EXTRA PRACTICE

Compute. Products should be named with their simplest nam

	a		b		c		d	
1.	$\frac{1}{3} \times \frac{2}{5}$	$\frac{2}{15}$	$\frac{2}{3} \times \frac{1}{7}$	$\frac{2}{21}$	$\frac{3}{7} \times \frac{2}{5}$	$\frac{6}{35}$	$\frac{5}{9} \times \frac{1}{3}$	$\frac{5}{27}$
2.	$\frac{1}{3} \times \frac{4}{5}$	$\frac{4}{15}$	$\frac{3}{5} \times \frac{2}{3}$	$\frac{2}{5}$	$\frac{2}{7} \times \frac{1}{4}$	$\frac{1}{14}$	$\frac{8}{9} \times \frac{1}{2}$	$\frac{4}{9}$
3.	$\frac{5}{6} \times \frac{1}{2}$	$\frac{5}{12}$	$\frac{5}{7} \times \frac{2}{3}$	$\frac{10}{21}$	$\frac{5}{8} \times \frac{3}{4}$	$\frac{15}{32}$	$\frac{5}{9} \times \frac{0}{5}$	$\left(\frac{0}{9}\right) = 0$

Try these word problems. Each problem needs two answers.

4. George had $\frac{3}{4}$ dozen eggs.
He used $\frac{1}{3}$ of the eggs for an omelette.
What part of a dozen did George use? $\frac{1}{4}$
How many eggs was that? 3 eggs

5. Monday, Bob's homework took $\frac{3}{4}$ of an ho
Tuesday it took $\frac{2}{3}$ of that time.
What part of an hour did it take Tuesday?
How many minutes was that? 30 minutes

6. Lynn had $\frac{3}{5}$ of a dollar.
She spent $\frac{1}{2}$ of her money for a pen.
What part of a dollar did she spend? $\frac{3}{10}$
How many cents was that? 30¢

7. Frank has to paint $\frac{2}{5}$ of a 100-board fence.
Alice has to paint only $\frac{1}{2}$ as much as Frank
What fraction of the job is Alice's? $\frac{1}{5}$
How many boards is that? 20 boards

Think about these problems. Write Yes or No for each answer

8. Is $\frac{1}{2}$ of $\frac{1}{3}$ the same as **a** $\frac{1}{3}$ of $\frac{1}{2}$? Yes **b** $\frac{1}{6}$ of

9. Is $\frac{1}{4}$ of $\frac{2}{3}$ the same as **a** $\frac{1}{4}$ of $\frac{5}{9}$? No **b** $\frac{2}{3}$ of

10. Is $\frac{1}{5}$ of $\frac{1}{7}$ the same as **a** $\frac{1}{5}$ of 7? No **b** $\frac{1}{7}$ of

174

CHECKOUT

Skills: Adding and subtracting fractions with unlike denominators; renaming answers

Add or subtract. Write the simplest name for your answer.

	a	b	c	d
1.	$4\frac{3}{8} + 2\frac{7}{8}$ $7\frac{1}{4}$	$2\frac{1}{3} + 1\frac{4}{9}$ $3\frac{7}{9}$	$7\frac{4}{5} - 3\frac{3}{5}$ $4\frac{1}{5}$	$3\frac{7}{8} - 1\frac{3}{4}$ $2\frac{1}{8}$

Skills: Multiplying fractions; renaming products

Multiply. Write the simplest name for your answer.

2. $2 \times \frac{1}{4}$ $\frac{1}{2}$ $\frac{2}{3} \times 4$ $2\frac{2}{3}$ $\frac{5}{6} \times \frac{1}{2}$ $\frac{5}{12}$ $\frac{3}{5} \times \frac{1}{3}$ $\frac{1}{5}$

SUPER SLEUTH

Look at this set of problems. The answer to the first problem became the top number of the next problem.

$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ $\frac{3}{4} - \frac{1}{8} = \frac{5}{8}$ $\frac{5}{8} + 1\frac{1}{4} = ?$ $1\frac{7}{8}$ $?$ $1\frac{7}{8}$ $- \frac{7}{8} = 1$

If you complete this series correctly, your last answer will match the one printed.

$5\frac{7}{12} + \frac{2}{3} = ?$ $5\frac{15}{12}\ (6\frac{3}{12})$

$?$ $5\frac{15}{12}\ (6\frac{3}{12})$ $- 1\frac{5}{12} = ?$ $4\frac{10}{12}\ (4\frac{5}{6})$

$?$ $4\frac{10}{12}\ (4\frac{5}{6})$ $- 2\frac{1}{3} = ?$ $2\frac{6}{12}\ (2\frac{1}{2})$

$?$ $2\frac{6}{12}\ (2\frac{1}{2})$ $- \frac{5}{8} = ?$ $1\frac{7}{8}$

$?$ $1\frac{7}{8}$ $+ 4\frac{1}{2} = 5\frac{11}{8}$ or $6\frac{3}{8}$

goal **Checkout** – adding and subtracting mixed numbers, multiplying fractions, and renaming answers

page 175 Watch for pupils who do not follow the addition and subtraction signs.

Note that one addition and one subtraction problem have like denominators while the other two problems require the youngsters to find the common denominator first. This difference will help you identify whether an error is one of computation or one of renaming to a common denominator.

Two of the multiplication problems involve a whole number factor and a fraction factor. Check for this error:

$$2 \cdot \tfrac{1}{4} = \tfrac{2}{8} \text{ or } \tfrac{2}{3} \cdot 4 = \tfrac{8}{12}$$

A youngster who makes this error has multiplied both the numerator **and** denominator.

Check carefully for answers that are computed correctly but have been renamed in simplest form incorrectly. This error identifies a youngster who needs to practice only the renaming step – not the computation.

Supersleuths should be operating independently.

STOP See activity 13, page 175c.

See activity 14, page 175c.

RESOURCES

another form of evaluation

for progress check–page 155

Compute. Watch out for the operation signs. You are to do both addition and subtraction. Rename when necessary.

1. $2\frac{1}{3}+\frac{1}{3}$ $2\frac{2}{3}$
2. $4\frac{1}{2}+\frac{1}{2}$ 5
3. $2\frac{5}{9}-\frac{2}{9}$ $2\frac{1}{3}$
4. $1\frac{1}{6}+\frac{5}{6}$ 2
5. $3\frac{2}{3}-\frac{2}{3}$ 3
6. $1\frac{3}{4}+\frac{1}{4}$ 2
7. $3\frac{3}{4}-\frac{1}{4}$ $3\frac{1}{2}$
8. $1\frac{1}{6}-\frac{0}{6}$ $1\frac{1}{6}$
9. $2\frac{1}{8}+\frac{3}{8}$ $2\frac{1}{2}$
10. $\frac{3}{4}+\frac{3}{4}$ $1\frac{1}{2}$
11. $\frac{3}{4}-\frac{3}{4}$ 0
12. $\frac{3}{4}+\frac{0}{4}$ $\frac{3}{4}$

for progress check–page 157

Subtract. Watch out for renaming.

1. $5\frac{4}{5} - 3\frac{1}{5}$ $2\frac{3}{5}$
2. $2\frac{1}{3} - 1\frac{2}{3}$ $\frac{2}{3}$
3. $6\frac{7}{10} - 4\frac{3}{10}$ $2\frac{2}{5}$
4. $4\frac{2}{9} - 2\frac{7}{9}$ $1\frac{4}{9}$
5. $4\frac{1}{6} - 1\frac{5}{6}$ $2\frac{1}{3}$
6. $2\frac{3}{5} - 1\frac{4}{5}$ $\frac{4}{5}$
7. $5\frac{1}{4} - 2\frac{3}{4}$ $2\frac{1}{2}$
8. $4\frac{2}{7} - 3\frac{5}{7}$ $\frac{4}{7}$
9. $6 - 3\frac{1}{2}$ $2\frac{1}{2}$
10. $5\frac{3}{5} - 2$ $3\frac{3}{5}$
11. $2\frac{3}{8} - 1\frac{1}{4}$ $1\frac{1}{8}$
12. $3\frac{5}{6} - 1\frac{2}{3}$ $2\frac{1}{6}$

for progress check–page 162

Find the sums, renaming when you need to. Rename your answers.

1. $\frac{1}{3}+\frac{2}{9}$ $\frac{5}{9}$
2. $\frac{2}{5}+\frac{1}{10}$ $\left(\frac{5}{10}\right)$ $\frac{1}{2}$
3. $\frac{1}{8}+\frac{1}{16}$ $\frac{3}{16}$
4. $\frac{2}{3}+\frac{1}{6}$ $\frac{5}{6}$
5. $\frac{3}{4}+\frac{1}{8}$ $\frac{7}{8}$
6. $\frac{5}{12}+\frac{1}{3}$ $\left(\frac{9}{12}\right)$ $\frac{3}{4}$
7. $\frac{1}{4}+\frac{7}{12}$ $\left(\frac{10}{12}\right)$ $\frac{5}{6}$
8. $\frac{3}{5}+\frac{3}{10}$ $\frac{9}{10}$
9. $\frac{4}{9}+\frac{2}{3}$ $\left(\frac{10}{9}\right)$ $1\frac{1}{9}$
10. $\frac{3}{25}+\frac{21}{50}$ $\frac{27}{50}$

*11. $\frac{9}{10}+\frac{3}{100}$ $\frac{93}{100}$

*12. $\frac{8}{10}+\frac{17}{100}$ $\frac{97}{100}$

for progress check–page 168

Compute.

1. $\frac{3}{4}\times 2$ $\left(\frac{6}{4}\right)$ $1\frac{1}{2}$
2. $\frac{1}{4}\times 8$ $\left(\frac{8}{4}\right)$ 2
3. $\frac{2}{9}\times 3$ $\left(\frac{6}{9}\right)$ $\frac{2}{3}$
4. $5\times\frac{5}{8}$ $\left(\frac{25}{8}\right)$ $3\frac{1}{8}$
5. $4\times\frac{5}{6}$ $\left(\frac{20}{6}\right)$ $3\frac{1}{3}$
6. $6\times\frac{1}{3}$ $\left(\frac{6}{3}\right)$ 2
7. $2\times\frac{3}{5}$ $\left(\frac{6}{5}\right)$ $1\frac{1}{5}$
8. $10\times\frac{3}{7}$ $\left(\frac{30}{7}\right)$ $4\frac{2}{7}$
9. $\frac{3}{10}\times 4$ $\left(\frac{12}{10}\right)$ $1\frac{1}{5}$

for progress check–page 172

Multiply.

1. $\frac{3}{4}\times\frac{1}{7}$ $\frac{3}{28}$
2. $\frac{2}{3}\times\frac{1}{6}$ $\left(\frac{2}{18}\right)$ $\frac{1}{9}$
3. $\frac{1}{3}\times\frac{5}{6}$ $\frac{5}{18}$
4. $\frac{1}{3}\times\frac{1}{3}$ $\frac{1}{9}$
5. $\frac{1}{2}\times\frac{3}{4}$ $\frac{3}{8}$
6. $\frac{3}{10}\times\frac{7}{10}$ $\frac{21}{100}$
7. $\frac{2}{3}\times\frac{2}{7}$ $\frac{4}{21}$
8. $\frac{0}{4}\times\frac{1}{3}$ $\left(\frac{0}{12}\right)$ 0
9. $\frac{4}{5}\times\frac{1}{2}$ $\left(\frac{4}{10}\right)$ $\frac{2}{5}$

for checkout–page 175

1. Add or subtract. Write the simplest name for your answer.

   (a) $2\frac{4}{9} + 4\frac{8}{9}$ $7\frac{1}{3}$
   (b) $3\frac{3}{4} + 1\frac{5}{8}$ $5\frac{3}{8}$
   (c) $4\frac{5}{6} - 1\frac{1}{6}$ $3\frac{2}{3}$
   (d) $6\frac{11}{12} - 2\frac{1}{2}$ $4\frac{5}{12}$

2. Multiply. Write the simplest name for your answer.

   $4\times\frac{3}{4}$ 3 $\quad$ $\frac{2}{5}\times 3$ $1\frac{1}{5}$ $\quad$ $\frac{2}{3}\times\frac{1}{7}$ $\frac{2}{21}$ $\quad$ $\frac{5}{6}\times\frac{4}{5}$ $\frac{2}{3}$

activities

1. things wood cubes

Write a mixed number or a fraction on each face of two cubes. For each pair of cubes make sure that the denominators are alike. For example:

$1\frac{2}{5}$	$\frac{3}{5}$	$3\frac{1}{5}$	$\frac{4}{5}$	$\frac{1}{5}$	$6\frac{1}{5}$
$2\frac{3}{5}$	$\frac{2}{5}$	$4\frac{1}{5}$	$3\frac{4}{5}$	$\frac{0}{5}$	$\frac{1}{5}$

Pair pupils. Players alternate rolling the two cubes. Points are earned as follows:

- Sum of the numbers that land faceup—1 point
- Difference of the numbers that land faceup—1 point

A player can earn 2 points per round. Players predetermine the number of points needed to win.

2. things spirit master or cards

Duplicate on a spirit master or activity card the following:

Which square is a magic square? (b)

a)

7	$2\frac{2}{3}$	$2\frac{1}{3}$	6
$5\frac{1}{3}$	5	$5\frac{1}{3}$	$4\frac{1}{3}$
$4\frac{2}{3}$	$3\frac{2}{3}$	4	$5\frac{2}{3}$
3	$6\frac{2}{3}$	$6\frac{1}{3}$	2

b)

$6\frac{3}{4}$	$4\frac{1}{4}$	$6\frac{1}{4}$	$\frac{3}{4}$
$1\frac{1}{4}$	$5\frac{3}{4}$	$3\frac{3}{4}$	$7\frac{1}{4}$
$1\frac{3}{4}$	$5\frac{1}{4}$	$3\frac{1}{4}$	$7\frac{3}{4}$
$8\frac{1}{4}$	$2\frac{3}{4}$	$4\frac{3}{4}$	$2\frac{1}{4}$

c)

8	$1\frac{2}{4}$	1	$6\frac{1}{2}$
$2\frac{1}{2}$	5	$5\frac{1}{2}$	4
$4\frac{4}{8}$	3	$\frac{1}{2}$	6
2	$7\frac{2}{4}$	7	$\frac{1}{2}$

d) Your turn. Make up your own magic square.

3. things wood cubes

Write a mixed number on each face of two cubes. Make sure the denominators are alike. Choose the numbers carefully to ensure renaming. For example:

$5\frac{1}{7}$ $8\frac{2}{7}$ $9\frac{3}{7}$ $6\frac{1}{7}$ $7\frac{2}{7}$ $10\frac{3}{7}$

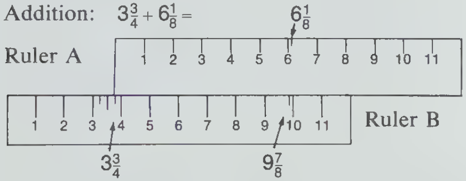
$1\frac{5}{7}$ $2\frac{4}{7}$ $3\frac{6}{7}$ $2\frac{6}{7}$ $3\frac{5}{7}$ $4\frac{4}{7}$

Pair pupils. Players alternate rolling the two cubes. One point is earned for finding the difference of the numbers that land faceup. Players predetermine the number of points needed to win.

4. things 2 rulers

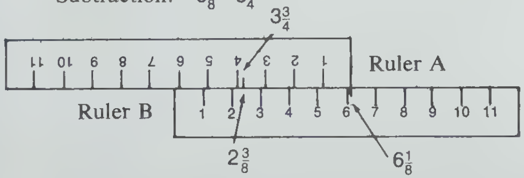
Use two rulers to form a slide rule for addition and subtraction.

Addition: $3\frac{3}{4} + 6\frac{1}{8} =$



Lay one ruler below the other. Locate $3\frac{3}{4}$ on ruler B. Slide ruler A so that it begins at this mark. Locate the $6\frac{1}{8}$ mark on ruler A. Follow this mark straight down to find the sum on ruler B.

Subtraction: $6\frac{1}{8} - 3\frac{3}{4} =$



Turn ruler A so that it begins at the right. Match this end of the ruler to $6\frac{1}{8}$ on ruler B. Locate $3\frac{3}{4}$ on ruler A. Follow this mark straight down to find the difference on ruler B.

5. things game board; 2 rubber jar rings; masking tape

Have the pupils prepare a 3-by-3 array game board. Write a fraction in each cell of the board. Make sure that all the denominators are multiples of a specific factor.

$\frac{3}{9}$	$\frac{5}{12}$	$\frac{1}{3}$
$\frac{7}{21}$	$\frac{2}{6}$	$\frac{5}{9}$
$\frac{3}{6}$	$\frac{4}{15}$	$\frac{4}{12}$

Tape the game board to the floor. Mark a line several feet in front of the board. Each player in turn stands behind this line and tosses 1 ring at a time. The sum of the 2 numbers hit is the player's score. The player with the highest score wins the round. Let the players determine a rule for when a ring touches 2 or 4 squares.

6. things sets of colored plastic sticks

Assign a fractional value to the sticks of each color. For each different set assign different values. **But** make sure that for each set the denominators are all multiples of a specific factor. For example:

	Set A	Set B
orange	$\frac{1}{3}$	$\frac{3}{4}$
red	$\frac{7}{3}$	$\frac{3}{8}$
green	$\frac{2}{6}$	$\frac{1}{4}$
blue	$\frac{3}{9}$	$\frac{0}{4}$
yellow	$\frac{5}{9}$	$\frac{4}{4}$
black	$\frac{5}{6}$	$\frac{5}{8}$

At first you will want to limit the number of sticks in a set.

Each player takes a turn picking up sticks with a "neutral" stick until he moves a stick he is not trying to pick up or has picked up all the sticks. The sum of the values of the sticks picked up is the player's score. Player with the highest score wins the round.

Variation: Subtract the value of the sticks moved during the player's turn.

7. things yarn, string, or ribbon

The commutative property belongs to an operation on abstracted numbers. Changing the order of the factors does not change the product, but it does change a real-life problem situation.

Think about 3 pieces of ribbon, each $\frac{1}{2}$ yard long. What is the sentence for this situation? ($3 \times \frac{1}{2} = ?$) Now think about $\frac{1}{2}$ of a 3-yard length of ribbon. What is the sentence now? ($\frac{1}{2} \times 3 = ?$) Indeed, the products are the same. But are the problem situations identical? A mental picture is probably too abstract for the youngsters, so enact the problems with pieces of yarn, string, or ribbon.

8. things pictures of racing cars; large sheet of paper; small cards; scissors

Have each pupil draw a midget racing car or find one pictured in a magazine. These pictures are cut out. Next have the group design its own racetrack on the sheet of paper. An appropriate multiplication sentence is written on one side of each card and the product on the back of the card. These cards are placed in stacks, sentence side up, around the track to serve as obstacles that the driver must clear to continue in the race. A car advances to the next obstacle only when the driver gives the correct product for the card on the top of the stack. Products are verified with the product given on the back of the card. Encourage the pupils to make up their own racing rules for situations that arise in playing the game.

9. things spirit master

Prepare a spirit master as shown.

a	b	c
d	e	f
g	h	i

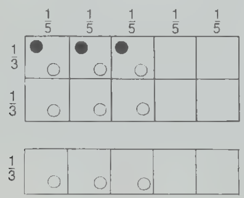
Record the products in the appropriate boxes.

a. $270 \times \frac{1}{9}$ (30) b. $36 \times \frac{1}{4}$ (9)
c. $48 \times \frac{1}{2}$ (24) d. $120 \times \frac{1}{8}$ (15)
e. $14 \times \frac{3}{2}$ (21) f. $81 \times \frac{1}{3}$ (27)
g. $\frac{1}{2} \times 36$ (18) h. $231 \times \frac{1}{7}$ (33)
i. $\frac{3}{8} \times 32$ (12)

Add the products across each row. What is the sum?
Add the products down each column. What is the sum?
Add the products on the diagonals. What is the sum?
Is this a magic square? (Yes)

10. things egg cartons; 2 kinds of dried beans

The denominators of the factors serve as clues to making an array with egg carton cups. To find $\frac{1}{3}$ of $\frac{3}{5}$ have the pupil show fifths vertically by making 5 columns of cups and show thirds horizontally by making 3 rows of cups.



The numerators of the fractions serve as clues for filling the cups with beans. A lima bean is placed in each cup for 3 columns. A dried red bean is placed in each cup already containing a lima bean for 1 row. The total number of cups tells the denominator of the product. The number of cups containing 2 beans tells the numerator of the product.

11. things 2-inch pieces of colored paper; small box

Write a part of a multiplication sentence on each of five pieces of paper of the same color.
For example: $\boxed{\frac{2}{3}}$ $\boxed{\times}$ $\boxed{\frac{3}{4}}$ $\boxed{=}$ $\boxed{\frac{1}{2}}$

Use paper of a different color for each sentence. The parts for several sentences can then be put into a small box.

The pupil sorts the pieces by color and arranges them to form sentences. You may wish to have an answer key available so that he can check his own work.

12. If you are a ham at heart, tell the youngsters you are going to send them answers to the following questions by mental telepathy. Have each youngster write down the message he receives.

- I'm thinking of another name for $\frac{1}{2}$. What name do you think it is?
- I'm thinking of the simplest name for $\frac{3}{2}$. What is that name?
- I'm thinking of a fraction name for $2\frac{1}{4}$. What fraction name am I thinking of?
- I'm thinking I would like to have $\frac{1}{2}$ of $\frac{1}{2}$ of a pecan pie right now. What fractional part of a pie am I thinking of?
- I'm thinking that you will know more about fractions in $\frac{1}{2}$ of $\frac{3}{4}$ of an hour. What fractional part of an hour will that be?

Share the answers you were thinking of. Several answers are possible for some of the questions. Who didn't get your message? He may have a correct answer, simply not **your** answer.

13. things wood cubes

To generate practice problems, write a fraction on each face of two cubes. The pupil rolls the cubes and finds the product for the two fractions that land faceup.

The game element may be added by pairing pupils and having each player add as his score the products he finds. Players predetermine the number of points needed to win.

14. things large sheet of paper; small cards

Have the pupils make a baseball diamond on a sheet of paper. Place a stack of cards, each card with a fraction written on it, facedown at home plate. Place a stack of cards, each card with a whole number written on it, facedown on the pitcher's mound. Place a stack of cards at each base, each card containing a multiplication sentence on one side, the product on the back. The base cards should focus on multiplying two fractions.

Players organize in two teams. Each team selects a pitcher, 3 basemen, and an umpire. The umpires are supplied with an answer key for multiplying the cards at the pitcher's mound and those at homeplate.

As each player comes to bat, the pitcher turns over a whole-number factor for the batter. The batter turns over a fraction factor from the home-plate stack. He multiplies the two factors and advances to first base if the correct product is given; otherwise he is out and the next batter is up. This decision is made by the umpires. Players on base must give the correct product for the top card in the stack at each base in order to advance to the next base. Basemen verify the products given with those stated on the back of the card. An incorrect product forces the player out. Three outs and the opposing team comes to bat. The team with more runs scored wins.

additional learning aids

operation – chapter objectives 1, 2, 3, 4, 5

SRA products

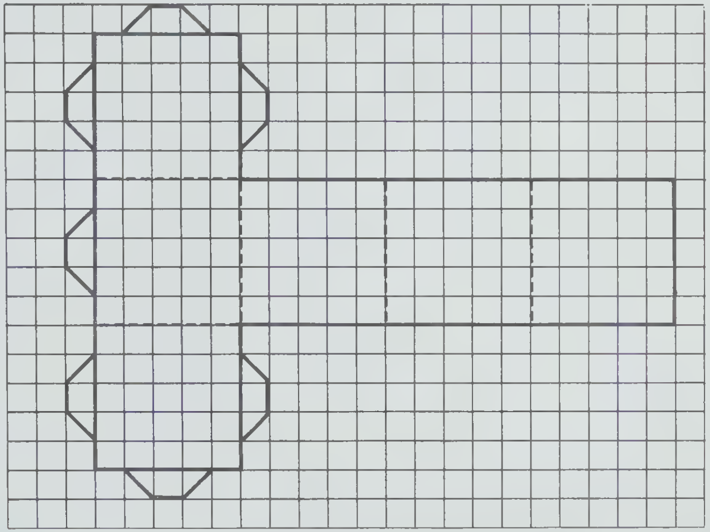
Mathematics Learning System, Activity Masters, level B, SRA (1974)
Spirit masters: F-4, 6, 7, 8, 9, 10, 11, 12, 14, 15; P-11
Computapes, SRA (1972)
Module 5, Lessons: Fr 11, 12, 13, 17, 18
Cross-Number Puzzles (Fractions), SRA (1966)
Addition cards: 1, 2, 3, 4
Subtraction cards: 1, 2
Mathematics Involvement Program, SRA (1971)
Cards: 264, 105, 56
Skill through Patterns, level 5, SRA (1974)
Spirit masters: 60, 61, 62, 66, 67, 70, 71, 72
Visual Approach to Mathematics (Rational Numbers), SRA (1967)
Visuals: 8, 9

other learning aids (described on page 216e) – Action Fraction Games, The Fat Fraction Game, Fraction Bars Student Activity Book (levels I and II), Fraction Dominoes, Good Time Mathematics, Mathimagination (Book D Fractions)

for the future

Fractions (chapter 7), decimals (chapter 8), and metric measure for capacity (chapter 9) can all be brought together with a model for a litre. It will be great if you have plastic models available. If you don't let, each child make his own over the next weeks.

The pupils should mark 1 cm squares on something like a shirt card board (light-weight tag stock). Then cut the flat shape needed to make a cube.



If the card board is less than 30 cm wide, make the "top" a separate 10 cm x 10 cm square.

Filling the litre with wood or plastic centimetre cubes will let the youngsters work with ideas such as:

1. One cm cube fills $\frac{1}{1000}$ (.001) of the space
2. One row of cm cubes fills $\frac{1}{100}$ (.01) of the space
3. One layer of cm cubes (10 in each row and 10 rows) fills $\frac{1}{10}$ (.10) of the space.
4. Using different colored cubes, any fractional part from .10 to .001 can be investigated
5. Learning about metric measure will be a bonus as you come to chapter 9. Maybe youngsters who have a pharmacist, chemist, nurse, or doctor in the family can share some information about how popular the measure of cubic centimetres (cc) is in the world of science and medicine.

8 DECIMALS

before this chapter the learner has—

1. Ordered a set of fractions having a denominator of 10 or 100
2. Located fractions on a number line
3. Used a place-value chart for whole numbers
4. Mastered estimating and finding the sum or difference of any two 4-digit numbers
5. Read and computed money notation

in chapter 8 the learner is—

1. Reading and writing decimals expressed in tenths or hundredths
2. Comparing and ordering fractions expressed in tenths or hundredths
3. Mastering finding the sum or difference of two decimals expressed in tenths or hundredths

in later chapters the learner will—

1. Master reading and telling the value of each digit of a decimal expressed in tenths, hundredths, or thousandths
2. Master finding the sum or difference of two decimals expressed in tenths or hundredths

Notes & Things

Thanks to the advertising in newspapers, magazines, billboards, and on TV, decimals have become part of everyone's world. Every child knows the notation for money and how to say a fraction such as $\frac{1}{10}$. The thrust of this decimals chapter is to tie the concept and the notation closely to the learner's own experiences. The emphasis that decimals are simply another way to write fractions is in the chapter, but it is overshadowed by the use of money problems. (You will have still another opportunity to apply this knowledge in the next chapter, which features the metric system.)

In order to minimize learning a bunch of new algorithms, the place-value development of decimals parallels the place-value work with whole numbers. This also allows the algorithms for both addition and subtraction of decimals to parallel those of whole numbers. Not much more need be said other than a special plea. Use ads, newspaper and magazine stories about business or world trade, the mail-order catalog, grocery slips, all kinds of sports records, and even public information about the school budget to make the notion of decimals real and vital to every child. There really are lots of reasons to know all about this set of numbers.

things

coin models (optional)
calculator (optional)
number line (optional)
place-value chart (optional)

For the extra activities you will want to have these things available:

- squares of oaktag
- newspapers and magazines
- overhead projector
- transparency film

goal Think about and explore ideas through a picture clue

page 176 Let this photograph motivate thoughts about the pupils' experiences with a grocery store. Who has had the responsibility for doing some grocery shopping for the family? Has everyone gone along on a grocery-shopping expedition? When a person shops for food, what things does the person think about? Do you have any comparison shoppers in your group? How much does a loaf of bread cost? A quart of milk? A pound of butter or margarine? Hamburger? Potatoes?

Many pupils are completely unaware of the cost of food. If this is true of your group, do take time to figure out how much food might cost for a simple meal for an average-size family. Let them plan their own menu. The meal should include more than a hamburger if you can convince the pupils that something else might be good. Then let them figure out how much it would cost to buy that food in the grocery store. They can use ads in the newspaper, go to the store themselves, or interview the person who buys the food at home.

Decimal notation is most frequently used in connection with money and measurement. Perhaps this independent assignment will yield experience with both uses.

fraction can be written in many ways. or example, $\frac{3}{4}$ of a dollar can be xpressed as $0.75.

ake time to look at decimal otation.

loney coins are something veryone knows about. Their value is ritten with decimal notation. It is a ractional part of one dollar.

$\frac{1}{10}$ of a dollar is the same as $0.10.

$\frac{1}{100}$ of a dollar is the same as $0.01.

Is there a difference between $10 and $0.10? Of course there is! $10 means ten dollars and $0.10 means ten cents. For safety's sake it's always a good idea to put a 0 to the left of the decimal point if there is no other digit there.

Write each amount of money with a $ and a decimal point.

1. If $\frac{1}{10}$ of a dollar is $0.10, what is $\frac{2}{10}$ of a dollar? $0.20
   What is $\frac{1}{5}$ of a dollar? $0.20
2. If $\frac{1}{10}$ of a dollar is $0.10, what is $\frac{5}{10}$ of a dollar? $0.50
   What is $\frac{1}{2}$ of a dollar? $0.50
3. If $\frac{1}{10}$ of a dollar is $0.10, what is $\frac{10}{10}$ of a dollar? $1.00
4. How would you write:
   a $1\frac{2}{10}$ dollars? $1.20
   b $2\frac{5}{10}$ dollars? $2.50
   c $3\frac{15}{100}$ dollars? $3.15
   d $4\frac{6}{10}$ dollars? $4.60
   e $\frac{12}{10}$ dollars? $1.20
5. What are some slang phrases you have heard used for coins? Try to find out how some phrases started. Since the 1600s *bit* has been used in numerous countries to refer to a small silver coin

GOAL

If you know a lot about money, this chapter will be easy. Your goal is to operate with decimal fractions.

177

lesson Page 177

goal **Survey**—ability to write common fractions in decimal notation

memo Learners who have completed level 4 of this program will have explored writing common fractions in money notation. This chapter is their first real introduction to decimal fractions.

page 177 The ability to record money in decimal notation will carry the pupil through most of this chapter. The page will help you identify pupils who are ready to go on and those who need to spend a little time learning to record dollars and cents before moving on to the chapter.

Independent readers should have no trouble. You may want to discuss the page with the others and have them jot down their replies to the questions.

Question 5 will provide an interesting research project for those pupils who are interested. If they cannot think of money slang themselves, start them off with two bits, four bits, six bits. They will be interested to learn that since the 1600s **bit** has been used in numerous countries to refer to a small silver coin.

lesson Pages 178, 179

goal Relating decimal notation to money notation

memo The understanding of place value and notation developed on pages 178 through 181 is prerequisite to success in computing with decimals. Nice and easy does it.

page 178 The focus is place value. For anyone who knows about money notation, decimal notation really is not too different. Talk about uses of decimal notation. Distances travelled by car or motorcycle are recorded in tenths. Most (but not all) youngsters are familiar with the automobile odometer. The mass of meat packages in a store is usually recorded in tenths.

You go shopping.
You get a piece of
paper like this:

01.33 MZ TB IT •
01.05 TB GR : •
00.29 GR : •
00.37 GR : •
02.15 MT : •
01.59 MT : •
00.17 TX : •
06.95 : TL •

THANK YOU
REGISTER NO 64

What is it?
What does it say?
A receipt: the money you must pay for certain items you are buying

Look at this figure from a cash register.

The 2 tells you how many ten-dollar units.
The 1 tells you how many one-dollar units.
The 4 tells how many ten-cent units.
What does the 5 tell? The number of pennies

1. Now consider \$34.68. How many ten-dollar units? 3 How many one-dollar units? 4 How many ten-cent units? 6 How many one-cent units? 8
2. A ten-dollar unit is worth _?_ dollars. 10 A one-dollar unit is worth _?_ 10¢ coins. 10 A 10¢ coin is worth _?_ cents. 10
3. A dollar is worth _?_ cents. 100 Ten dollars are worth _?_ 10¢ coins. 100

We use decimals for things other than money.
But they work the same way. Look at this decimal.

21.45

The 2 means 2 tens.
The 1 means 1 one.
The dot is called a decimal point.
The 4 means 4 *tenths*.
The 5 means 5 *hundredths*.

Compare this with the cash-register figure.

178

Decimal notation for a fraction is not limited to money.

Time of a race may be reported in tenths of a second.

The measure of a weight may be reported in tenths or hundredths of a kilogram.

The measure of a length may be reported in tenths or hundredths of a metre.

The place-value chart you have worked with can be expanded.

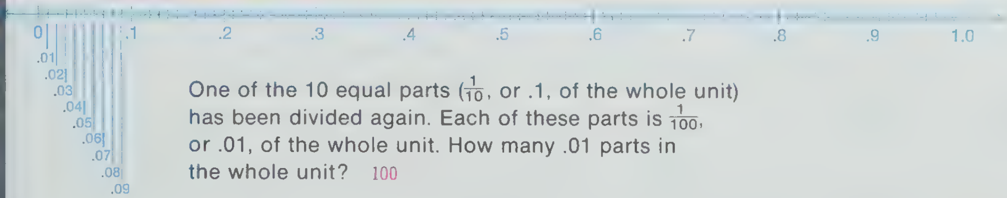
Decimals name fractional parts of 1.

.5 is the same as $\frac{5}{10}$.

.56 names the same number as $\frac{56}{100}$.

.567 names the same number as $\frac{567}{1000}$.

These fractions are not new to you. The number line below shows one unit divided into 10 equal parts. Each part is $\frac{1}{10}$, or .1, of the whole unit.



One of the 10 equal parts ($\frac{1}{10}$, or .1, of the whole unit) has been divided again. Each of these parts is $\frac{1}{100}$, or .01, of the whole unit. How many .01 parts in the whole unit? 100

goal Development of decimal fraction place-value positions

page 179 Examine the place-value chart with the group. Why are there no ones to the right of the decimal point? Challenge pupils to try to read the number shown. The digits shown name the numerator of the fraction. In decimal notation, how is the denominator shown? (By the place-value positions)

The number line should cause no trouble. Emphasize thinking one place to the right of the decimal point to indicate tenths, two places to indicate hundredths. Youngsters are accustomed to thinking two places for tens and three places for hundreds when working with whole numbers. Watch for confusion.

lesson Pages 180, 181, 182

goal Development of decimal place value to thousandths and examination of equivalent decimal fractions

page 180 You really need to have the group share ideas on this page. The numbers should be read aloud. There's a careful use of the digits 1 and 0 and place value. Without guidance, confusion could result.

Ideas to emphasize:

- In problem 4, the two numbers are not written alike, yet they are equivalent.

$$.1 = \frac{1}{10} \qquad .100 = \frac{100}{1000} \qquad \frac{1}{10} \times \frac{100}{100} = \frac{100}{1000}$$

- Annexing a zero to the right of a decimal fraction simply multiplies the number by ten tenths.

$$.1 = .10 = .100$$
$$\frac{1}{10} \times \frac{10}{10} = \frac{10}{100}$$
$$\frac{10}{100} \times \frac{10}{10} = \frac{100}{1000}$$

- Zeros between the decimal point and a digit do change the value.

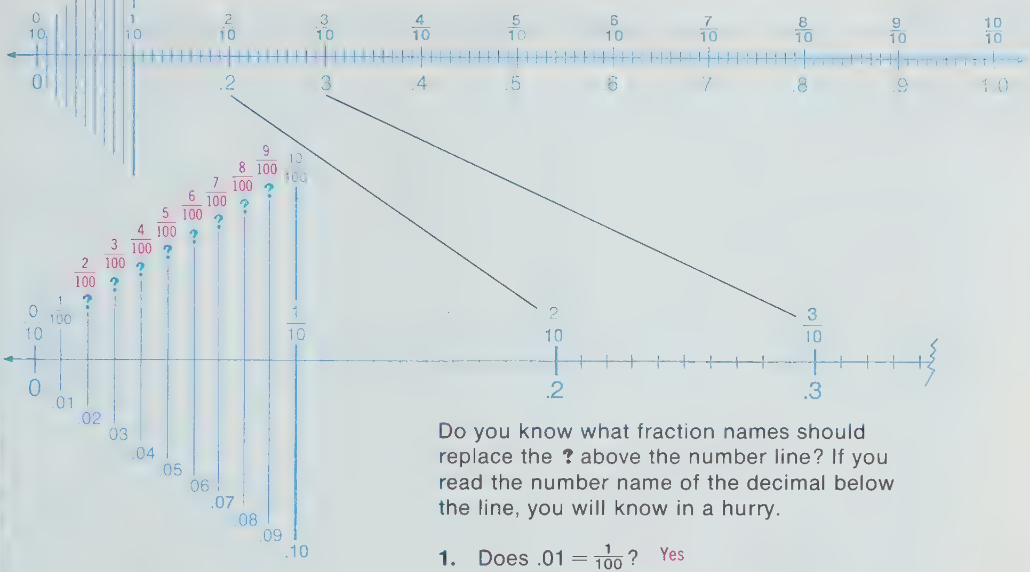
$$.1 \neq .01 \neq .001$$
$$\frac{1}{10} \neq \frac{1}{100} \neq \frac{1}{1000}$$

	hundred-thousands	ten-thousands	thousands	hundreds	tens	ones	tenths	hundredths	thousandths
one thousand			1	0	0	0	0	0	0
one hundred				1	0	0	0	0	0
ten					1	0	0	0	0
one						1	0	0	0
one tenth or one hundred thousandths							1	0	0
one hundredth or ten thousandths							0	1	0
one thousandth							0	0	1

1. Read each of the numbers on the chart.
2. Is 1000.000 the same number as 1000? Yes
3. Is 10.000 the same number as 10? Yes
4. Is .1 the same number as .100? Yes
5. Is .01 the same number as .010? Yes
6. Is 100 the same number as .001? No
7. Name the larger number.
   Use the chart for help if you need it.
   a 10 or 1 b .1 or 1
   c 10 or .01 d 100 or 10
   e .1 or .01 f 1 or .1
   g 1000 or 100 h .01 or .001
8. If you had .1 dollar, would you have the same amount as someone else who had .10 dollar? Y
9. If you had .80 dollar, would you have the same amount as someone else who had .8 dollar? Ye
10. Answer yes or no to these.
    a Does .10 = .100? Yes b Does .1 = .100? Y
    c Does 1.0 = 1.00? Yes d Does .400 = .40?
    e Does .1 = .11? No f Does .11 = .111?

180

You saw a line of this length before. Enlarge it.



Do you know what fraction names should replace the **?** above the number line? If you read the number name of the decimal below the line, you will know in a hurry.

1. Does $.01 = \frac{1}{100}$? Yes

a Does $.10 = \frac{10}{100}$? Yes **b** Does $\frac{1}{10} = .1$? Yes

2. Do .1, .10, $\frac{1}{10}$, and $\frac{10}{100}$ all name the same number? Yes

3. What is the place value of each digit in the following numbers?

a 12.56 — tens, ones, tenths, hundredths

b 100.01 — hundreds, tens, ones, tenths, hundredths

c 21.10 — tens, ones, tenths, hundredths

181

goal Practice in reading decimal notation and telling the value of each digit

page 181 The relationship of the digits in a decimal fraction to the numerator of a fraction is shown once more with the number line. Emphasize reading the decimal notation and place value rather than the common fraction. Common-fraction notation should be used only to clarify or give meaning. The goal is to avoid a dependence on this familiar notation and to put the emphasis on place value. Thus the pupil will be prepared to see the simplicity of operating with decimals.

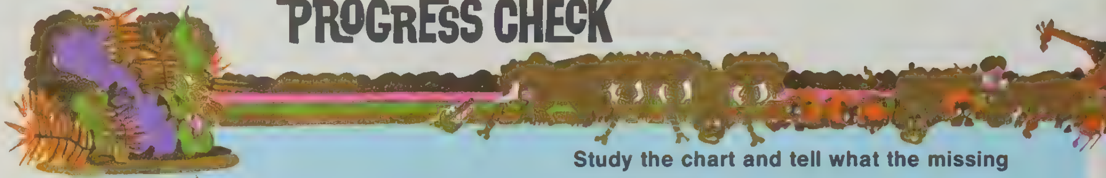
Encourage pupils who are unsure when working problem 3 to use a place-value chart.

goal **Progress Check**—telling the value of each digit in a decimal fraction

page 182 There's no sense in having each pupil copy the whole chart. Agree on how missing numbers are to be recorded. Then everyone works independently.

Have the pupil who made errors read the numerals orally so that you can hear where he is having difficulty. Check on whether he understands that the digits of a decimal name the numerator, while place value alone names the denominator.

PROGRESS CHECK



Skill: Telling the value of each digit in a decimal fraction

Decimals are really just another kind of fraction. In this chart the decimals in the first column are equal to the fractions in the second column. The third column tells how each decimal is read.

Study the chart and tell what the missing numbers are.

	Decimal	Fraction	How you read it
1.	.1	$\frac{1}{10}$	1 tenth
2.	.01	$\frac{1}{100}$	1 hundredth
3.	.001	$\frac{1}{1000}$	1 thousandth
4.	.6	$\frac{?}{10}$ 6	6 tenths
5.	.02	$\frac{2}{?}$ 100	2 hundredths
6.	.005	$\frac{5}{1000}$	5 ? thousandths
7.	.48	? $\frac{48}{100}$	48 hundredths
8.	.039	$\frac{39}{1000}$	39 ? thousandths
9.	.125	$\frac{125}{1000}$	125 ? thousandths
10.	.708	$\frac{708}{1000}$	708 ? thousandths

11. **What is the largest number in the chart?** .708

182

See activity 1, page 189a.

See activity 2, page 189a.

Now is the time to start adding with decimals.

Suppose you walk 0.3 km to a friend's house.
Then you walk 0.4 km to the park.
How far do you walk in all?

$$\begin{array}{r} 0.3 \\ +\ 0.4 \\ \hline 0.7 \end{array} \quad \left(\text{or } \frac{3}{10} + \frac{4}{10} = \frac{7}{10}\right)$$

You have already walked 0.7 km. Now you walk another 0.5 km going home. How far have you walked in all?

$$\begin{array}{r} 0.7 \\ +\ 0.5 \\ \hline ? \end{array} \quad \left(\text{or } \frac{7}{10} + \frac{5}{10} = \frac{12}{10}\right)$$

What is the answer this time? 1.2 km

If you remember the place value of the numbers you are adding, there are no new rules. It is the same as adding whole numbers.

Here is some practice. Add.

1. $\begin{array}{r} 0.6 \\ +\ 0.3 \\ \hline 0.9 \end{array}$ 2. $\begin{array}{r} 0.4 \\ +\ 0.5 \\ \hline 0.9 \end{array}$ 3. $\begin{array}{r} 0.8 \\ +\ 0.3 \\ \hline 1.1 \end{array}$ 4. $\begin{array}{r} 4.5 \\ +\ 0.2 \\ \hline 4.7 \end{array}$ 5. $\begin{array}{r} 0.4 \\ +\ 7.3 \\ \hline 7.7 \end{array}$

6. $\begin{array}{r} 6.1 \\ +\ 3.7 \\ \hline 9.8 \end{array}$ 7. $\begin{array}{r} 5.8 \\ +\ 0.6 \\ \hline 6.4 \end{array}$ 8. $\begin{array}{r} 0.9 \\ +\ 4.4 \\ \hline 5.3 \end{array}$ 9. $\begin{array}{r} 3.56 \\ +\ 2.73 \\ \hline 6.29 \end{array}$ 10. $\begin{array}{r} 9.81 \\ +\ 7.47 \\ \hline 17.28 \end{array}$

183

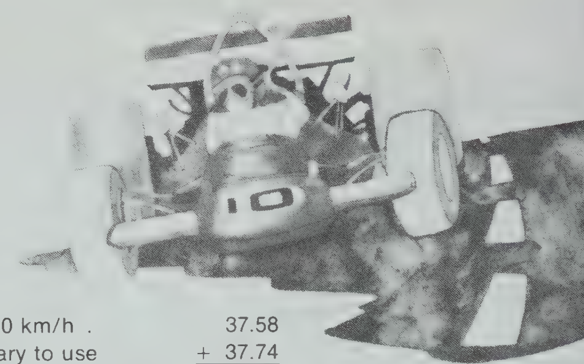
lesson Pages 183, 184, 185, 186

goal Introduction to adding decimals

page 183 Pupils should cheer as they realize how computation is simplified when they use decimal notation rather than common fractions. There are no new rules for adding decimals. The job is pretty much like adding whole numbers **if** the emphasis remains on **place value**—the location of the decimal point.

goal Introduction to subtracting decimals

page 184 Pupils who had no trouble with the preceding page should have no trouble here. Play with the same old subtraction rules for whole numbers, **but** emphasize place value.



You should be able to add and subtract decimals with hundredths because it is just like using money. Here is an example.

Formula I racing cars hit speeds around 250 km/h . With such speeds, officials found it necessary to use clocks that measure hundredths of a second. If a car turned one lap in 37.58 seconds (s) and another lap in 37.74 s, how many seconds did the car take to make the two laps?

$$\begin{array}{r} 37.58 \\ +\ 37.74 \\ \hline \blacksquare\blacksquare.\blacksquare\blacksquare \end{array}$$

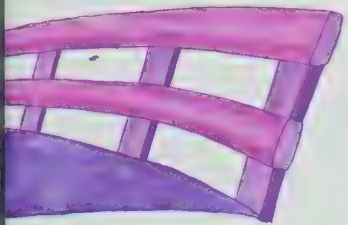
75.32

Jim ran the 100 m race in 12.4 s .
Betty ran it in 11.9 s .
How much faster than Jim did Betty run the 100 m? 0.5 second

Here are some practice exercises. Subtract.

1. $\begin{array}{r} 0.9 \\ -\ 0.7 \\ \hline \end{array}$ 0.2	**2.** $\begin{array}{r} 0.8 \\ -\ 0.3 \\ \hline \end{array}$ 0.5	**3.** $\begin{array}{r} 1.8 \\ -\ 0.4 \\ \hline \end{array}$ 1.4	**4.** $\begin{array}{r} 2.6 \\ -\ 0.5 \\ \hline \end{array}$ 2.1	**5.** $\begin{array}{r} 3.6 \\ -\ 0.7 \\ \hline \end{array}$ 2.9
6. $\begin{array}{r} 5.5 \\ -\ 0.9 \\ \hline \end{array}$ 4.6	**7.** $\begin{array}{r} 8.2 \\ -\ 3.5 \\ \hline \end{array}$ 4.7	**8.** $\begin{array}{r} 6.3 \\ -\ 5.4 \\ \hline \end{array}$ 0.9	**9.** $\begin{array}{r} 9.74 \\ -\ 4.63 \\ \hline \end{array}$ 5.11	**10.** $\begin{array}{r} 8.19 \\ -\ 7.41 \\ \hline \end{array}$ 0.78

184



You know how to add and subtract money.
Cents represent hundredths.

June earned \$1.25 on Monday.
She earned \$2.50 on Tuesday.
How much did she earn in the two days? \$3.75

She took \$2.50 with her to the store.
She bought only one thing. It cost \$.57 with tax. How much money did she have left? \$1.93

When you did those problems, you probably didn't even think about place value.
But start thinking about it now. You grouped ones, tens, hundreds, and thousands when you computed with whole numbers. You group tenths, hundredths, and thousandths when you compute with decimal fractions.

Addition

thousands	hundreds	tens	ones	tenths	hundredths	thousandths
		1				
	4	1	0	7	2	5
+		3	5	6	0	0
	4	4	6	3	2	5

Could the addition problem have been written in this way?
Yes

thousands	hundreds	tens	ones	tenths	hundredths	thousandths
	4	1	0	7	2	5
+		3	5	6		

Would the answer be different? Why?
No; the zeros in 35.600 do not change the value of 35.6.

Subtraction

thousands	hundreds	tens	ones	tenths	hundredths	thousandths
				6	9	1
		9	8	$\cancel{7}$	$\cancel{0}$	0
	−		6	2	5	5
		9	2	4	4	5

Could the subtraction problem have been written this way? Yes

thousands	hundreds	tens	ones	tenths	hundredths	thousandths
		9	8	7		
−			6	2	5	5

Could you still subtract? How?
Yes; rename 98.7 as 98.700.

goal Extension of addition and subtraction of decimals to thousandths

page 185 Adding without showing the zeros presents no problem—except perhaps one of aligning the place-value columns.

$$\begin{array}{r} 410.725 \\ +\ 35.6 \\ \hline 446.325 \end{array}$$

But watch out for subtraction!

$$\begin{array}{r} 98.7 \\ -\ 6.255 \\ \hline \end{array} \quad \text{might become} \quad \begin{array}{r} 98.7 \\ -\ 6.2\,55 \\ \hline 92.5\,55 \end{array}$$

This is a carry-over from addition.

Pupils who are able to compute this problem without showing the zeros and renaming should be encouraged to do so. Sometimes it's easier for them to annex zeros before they begin either operation. You know best on this issue.

goal Relating rounding and estimation skills to decimals; **Progress Check** — adding and subtracting decimals

page 186 The top of the page is for discussion only. Practice rounding several numbers. The youngsters may wish to check their answers to the Progress Check by estimation. This skill will be developed more fully in a later chapter.

The Progress Check should be completed independently. How should problems 5 through 9 be written? Stress aligning the decimal points.

When examining errors, look for computational errors not related to decimal notation. These errors indicate pupils who need additional help with the operations of addition and subtraction.

Watch also for pupils who compute correctly but are careless with placement of the decimal point. These youngsters require individual attention.

You have used estimation to help you before. It is especially useful when you are working with decimals. If you put a decimal point in the wrong place — ***WOW!*** Here is an example.

A department store bills you $5.87 for a shirt and $4.13 for a tie. The total comes to $100. Is this total reasonable?

Using estimation, you can see quickly that a mistake has been made. The total should be about $10, not $100.

$$\begin{array}{r} 5.87 \\ +\ 4.13 \\ \hline \end{array} \longrightarrow \begin{array}{r} 6 \\ +\ 4 \\ \hline 10 \end{array}$$

Probably you already have good estimation sense from working with money. But you should develop this same sense working with other decimals, too.

PROGRESS CHECK

Skills: Estimating; adding and subtracting decimals with renaming

Here is some mixed practice. Estimate. Then find exact answers. Make sure your answers are reasonable.

([illegible] answer in parentheses)

1. $\begin{array}{r} 3.5 \\ +\ 6.0 \\ \hline \end{array}$ (10) 9.5

2. $\begin{array}{r} 45.7 \\ -\ \ 8.9 \\ \hline \end{array}$

3. $\begin{array}{r} 5.0 \\ -\ 3.5 \\ \hline \end{array}$ (1) 1.5

4. $\begin{array}{r} 14.76 \\ +\ 23.14 \\ \hline \end{array}$

5. $5.2 + 3.8 = \underline{?}$

6. $17.3 - 9.5 = \underline{?}$ (7) 7.8

7. $\begin{array}{r} 54.71 \\ +\ \ 6.09 \\ \hline \end{array}$

8. $31.50 + 32.76 = \underline{?}$

9. $20.09 - 7.77 = \underline{?}$

10. $\begin{array}{r} 10.05 \\ -\ \ 7.99 \\ \hline \end{array}$ (2) 2.06

11. $\begin{array}{r} 15.00 \\ -\ \ 1.11 \\ \hline \end{array}$

12. $\begin{array}{r} 5.70 \\ +\ 3.58 \\ \hline \end{array}$ (10) 9.28

13. $\begin{array}{r} 15.76 \\ +\ 14.83 \\ \hline \end{array}$

186

See activity 3, page 189a.

See activity 4, page 189b.

Roundy Wheeler borrowed $37 from his dad to buy a bicycle. He paid him back at the rate of $1.75 a week for the first 5 weeks and $2.25 a week from the sixth week on until he paid it all back. Copy and complete the payment table. It's for the first ten weeks.

How many weeks did it take to pay off the loan? 18

How much was the last payment? $1.25

End of week	Amount paid	Balance owed after payment
1	$1.75	$35.25
2	$1.75	? $33.50
3	? $1.75	? $31.75
4	? $1.75	? $30.00
5	? $1.75	? $28.25
6	$2.25	? $26.00
7	? $2.25	? $23.75
8	? $2.25	? $21.50
9	? $2.25	? $19.25
10	? $2.25	? $17.00

SUPER SLEUTH

Place these coins in three straight lines so that the sum in each line is 20 cents.

5 5 10
5 5
10 5

187

lesson Pages 187, 188

goal Practice in computing with money

page 187 This is independent work. Roundy Wheeler's loan involves lots of computation. You may wish to group less capable pupils and complete the table as a group project.

Movable coin models may help the Supersleuths.

goal Practice in computing with money

page 188 A challenge for capable students. If anyone has a calculator, this is a good time to introduce everyone to the advantages of these wonderful little machines. Maybe you can borrow one from the school office.

For Experts Only

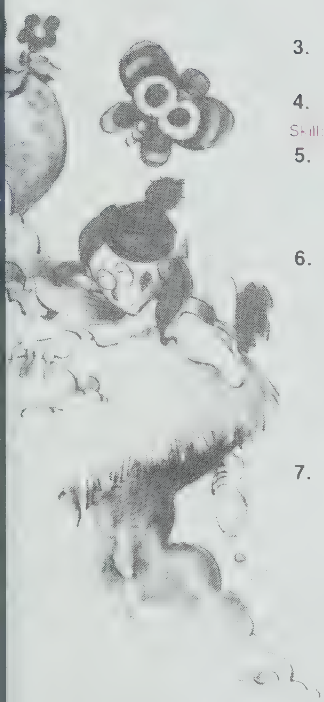
Bob delivers newspapers every morning. If he delivers a paper every day, the customer pays $0.80 a week. The cost for weekdays only is $0.50 a week, and for weekend delivery only it is $0.35 a week. Customers usually pay on Saturdays. The payments of 15 customers last Saturday are recorded below. Complete this table.

	Customer name	Weekly charge	Paid for this many weeks	Total paid	Paid with	Given this change
1.	Mrs. Seaver	$0.80	2	a $1.60	$2	b $0.40
2.	Mr. Lacher	$0.35	4	a $1.40	$2	b $0.60
3.	Miss Liere	$0.35	6	a $2.10	$5	b $2.90
4.	Miss Smith	$0.50	2	a $1.00	$1	b 0
5.	Mrs. Erickson	$0.80	2	a $1.60	$5	b $3.40
6.	Mrs. Humphrey	$0.80	3	a $2.40	$3	b $0.60
7.	Mrs. Dormal	$0.50	5	a $2.50	$3	b $0.50
8.	Mr. Stone	$0.50	3	a $1.50	$10	b $8.50
9.	Miss Snyder	$0.35	4	a $1.40	$5	b $3.60
10.	Mrs. Campbell	$0.35	5	a $1.75	$2	b $0.25
11.	Mr. Koslowski	$0.35	3	a $1.05	$2	b $0.95
12.	Mr. Jameson	$0.80	2	a $1.60	$5	b $3.40
13.	Mrs. Randall	$0.80	4	a $3.20	$5	b $1.80
14.	Mrs. Clark	$0.35	5	a $1.75	$10	b $8.25
15.	Miss Adams	$0.50	4	a $2.00	$2	b 0
16.	TOTAL			a $26.85	$62	b $35.15

This had better equal this plus this

or you are in trouble!

HECKOUT



Skill: Recognizing equivalent decimals

1. Do 0.1, 0.10, and 0.100 mark the same point on the number line? Yes

2. Do 0.3, 0.03, and 0.003 mark the same point on the number line? No

3. Do 4, 4.0, and 4.00 mark the same point on the number line? Yes

4. Do 0.006, 0.06, and 0.6 mark the same point on the number line? No

Skill: Ordering decimals

5. Order each set from smallest to largest.

a 1.0, 1.1, 10.1 1.0, 1.1, 10.1	**b** 0.01, 0.10, 1.1 0.01, 0.10, 1.1	**c** 0.11, 0.10, 0.01 0.01, 0.10, 0.11

6. Add. Skill: Adding decimals

a 0.65 and 0.34 0.99	**b** 0.50 and 0.50 1.00	**c** 0.49 and 0.52 1.01
d 0.82 and 0.88 1.70	**e** 3.75 and 2.25 6.00	**f** 1.37 and 2.72 4.09
g 3.5 and 6.75 10.25	**h** 10.1 and 9.9 20.0	**i** 0.99 and 0.01 1.00
j 50 and 0.05 50.05	**k** 0.02 and 20 20.02	**l** 9.2 and 0.8 10.0

7. Subtract. Skill: Subtracting decimals

a 0.75 – 0.23 0.52	**b** 0.94 – 0.58 0.36	**c** 0.80 – 0.37 0.43
d 1.52 – 0.61 0.91	**e** 10.30 – 0.71 9.59	**f** 1.00 – 0.31 0.69
g 1.05 – 0.37 0.68	**h** 1.5 – 0.7 0.8	**i** 1.8 – 0.95 0.85
j 40 – 0.09 39.91	**k** 0.98 – 0.09 0.89	**l** 3.70 – 3.69 0.01

189

See activity 5, page 189b.

See activity 6, page 189b.

lesson Page 189

goal **Checkout** – comparing and ordering decimals; adding and subtracting decimals

page 189 Everyone should complete problems 1 through 5. The addition and subtraction sets are really longer than is necessary for a check on skill. Why not have each pupil select five addition and five subtraction problems to compute? To facilitate checking, the problems will need to be numbered as they appear in the book. Then the youngsters can use the remaining problems for additional practice.

Youngsters who have trouble with problems 1 through 5 need additional work on the number line and with a place-value chart.

Possible causes for addition and subtraction errors are –

- Computational error
- Placement of the decimal point

For more practice with computation, use the remaining problems on the page. Give individual attention to the pupil who has trouble with the decimal point.

RESOURCES

another form of evaluation

for progress check–page 182

Write a fraction for each decimal.

1. .43 $\frac{43}{100}$	2. .020 $\frac{20}{1000}$	3. .3 $\frac{3}{10}$
4. .005 $\frac{5}{1000}$	5. .60 $\frac{60}{100}$	6. .375 $\frac{375}{1000}$

7. Read each decimal in problems 1 through 6.
8. Which is largest: .020, .002, .200?

for progress check–page 186

Estimate. Then find exact answers. Watch the operation signs. You are to do both addition and subtraction. Estimates in parentheses

1. 2.6 +3.1 (6)5.7	2. 4.78 −1.62 (3)3.16	3. 58.24 +21.83 (80)80.07	4. 25.0 − 6.3 (20)18.7
5. 68.84 −32.20 (40)36.64	6. 12.37 +48.51 (60)60.88	7. 6.78 +4.65 (12)11.43	8. 99.06 −34.42 (70)64.64

9. 5.8 + 7.3 = ? (13) 13.1
10. 14.8 − 6.5 = ? (8) 8.3
11. 46.83 − 21.76 = ? (30) 25.07
12. 52.47 + 35.58 = ? (90) 88.05

for checkout–page 189

1. Do .5, .50, and .500 mark the same point on the number line? Yes
2. Do 2.0, .20, and .200 mark the same point on the number line? No
3. Do .8, .08, and .008 mark the same point on the number line? No
4. Do 6, 6.0, and 6.00 mark the same point on the number line? Yes
5. Order each set from smallest to largest.
   a) .32, .46, .05 — .05, .32, .46
   b) 2.0, .02, .20 — .02, .20, 2.0
   c) .40, .05, .5 — .05, .40, .5
   d) 3.01, 3.10, 1.03 — 1.03, 3.01, 3.10
6. Add.
   a) .52 and .46 .98
   b) .25 and .48 .73
   c) .76 and .51 1.27
   d) .84 and .97 1.81
   e) 5.43 and 6.28 11.71
   f) 14.5 and 26.7 41.2
   g) 29.72 and 48.68 78.40
   h) 35.98 and 46.73 82.71
   i) 36 and .65 36.65
   j) 78.3 and 18.57 96.87
7. Subtract.
   a) .98 − .53 .45
   b) .82 − .47 .35
   c) 2.38 − .85 1.53
   d) 4.60 − 1.28 3.32
   e) 27.87 − 15.36 12.51
   f) 58.91 − 42.35 16.56
   g) 64.25 − 37.49 26.76
   h) 81.05 − 13.68 67.37
   i) 7.46 − 2.7 4.76
   j) 20 − .45 19.55

activities

1. things game board; 2-inch squares of oaktag

For each pair of pupils, prepare a 6-by-6 array game board consisting of 2-inch squares. On the 2-inch oaktag squares write equivalent decimal, fraction, or word names for a variety of fractions. Be sure to have at least 18 pairs of matching cards. For example:

.1	$\frac{1}{10}$	1 tenth

Rules:

- Mix all the cards and place them facedown on the game board.
- The first player turns over any 2 of the cards. If the cards match (name the same number), the player removes these cards and takes another turn.
- If the cards turned over do not match, the cards are again turned facedown and left on the board in play. The other player takes a turn.
- Play continues until all the cards have been removed from the game board.
- The player with more cards wins.
- Players predetermine a rule for what happens if an incorrect match is made.

2. things newspaper and magazines

Project: Find examples of numbers used in decimal notation, fractional notation, and word notation. Keep track of your finds in some kind of chart. Does one form seem to be used more often than the others? If so, which one? Can you think of any reasons why this might be so?

3. things 2 rubber jar rings; game board; masking tape

Have the pupils prepare a game board as shown and then tape it to the floor. Mark off a line a few feet before the game board. Each player in turn will stand behind the line and toss the rubber rings, one at a time. The sum of the 2 numbers on which the rings land is his score. Play continues until one player accumulates 10 points.

.6	.1	.7
.4	.9	.2
.8	.3	.5

Let the players predetermine a rule for when a ring touches 2 or 4 squares. They may also predetermine a different winning score, or decide that a player must reach the winning score exactly, not go over it, to win.

Variations:

1. To practice subtraction of decimals, each player begins with a score of 10 points (or any other score chosen). Players alternate throwing only 1 ring, each time subtracting the number the ring lands on from their score. First player to reach 0 is the winner.
2. More capable pupils can combine the rules of both games. For example: Toss 2 rings; add 2 numbers hit; then subtract the sum from a score of 10 (or any other number chosen). The first player to reach 0 is the winner.

4. Provide the pupil with these directions:

1. Make a 3-by-3 array of squares. Now arrange the numbers .1, .2, .3, .4, .5, .6, .7, .8, .9 so that the sum of each row, each column, and the two diagonals is the same. What is the magic sum? (1.5) What is the number in the center square?

A possible arrangement is

.6	.1	.8
.7	.5	.3
.2	.9	.4

2. Now try making a 3-by-3 magic square using the numbers .3, .6, .9, 1.2, 1.5, 1.8, 2.1, 2.4, and 2.7. What is the magic sum? What number is in the center square?
3. Try making your own magic square using decimal numbers. Then challenge your friends with your magic square.

5. things overhead projector; transparency film; 64 markers

Prepare an 8-by-8 grid. At random, write a decimal numeral in each of the 64 squares. Make a transparency of this grid. Before turning on the projector, cover each square on the transparency with an individual marker so that the numerals are not visible. Have the pupils form 2 teams.

The goal of this game is to remove markers from numbers on the grid whose **sum** or **difference** is a whole number.

Rules:

- Players from each team alternate playing by selecting 2 or more markers to remove from the grid and then adding or subtracting the numbers uncovered.
  NOTE: Initially, you may want to restrict the number of markers that may be removed, since the player will be confronted with the fact that subtraction is not associative.
- If the sum or difference of the numbers selected is a whole number, the player keeps the markers for his team and takes another turn.
- If the sum or difference of the numbers selected is not a whole number, the markers are replaced and play passes to the opposing team.
- Play continues until all possible squares have been uncovered, and play is no longer possible. The team with more markers is the winner.

6. The goal is to search for uses of decimals in the real world. The reward is points assigned from a scale predetermined by the pupils. The highest score wins.

Applications of decimals require certain skills. Points can be assigned on the basis of recognizing what skill is used in the application. The pupil need not perform the computation—only recognize the skill used. Here are some suggested point values, but these should be decided on by the youngsters.

1.	Ability to read a decimal	1 point
2.	Ability to write a decimal	1 point
3.	Ability to add decimals	2 points
4.	Ability to subtract decimals	2 points
5.	Ability to multiply decimals	2 points
6.	Ability to divide decimals	2 points

additional learning aids

notation—chapter objectives 1, 2

SRA products

Mathematics Learning System, Activity Masters, level B, SRA (1974)
Spirit masters: P-12; F-19

Computapes, SRA (1972)
Module 6, Lessons: DP-1, 2

Mathematics Involvement Program, SRA (1971)
Card: 365

other learning aids (described on page 216e)—Chip Trading, Decimal Fraction Dominoes, Decimal/Fraction Matching Cards

operation—chapter objective 3

SRA products

Mathematics Learning System, Activity Masters, level B, SRA (1974)
Spirit masters: F-13, 19

Computapes, SRA (1972)
Module 6, Lessons: DP-4, 5

Mathematics Involvement Program, SRA (1971)
Card: 316

Visual Approach to Mathematics (Rational Numbers), SRA (1967)
Visuals: 14, 15

9 MEASUREMENT

before this chapter the learner has—

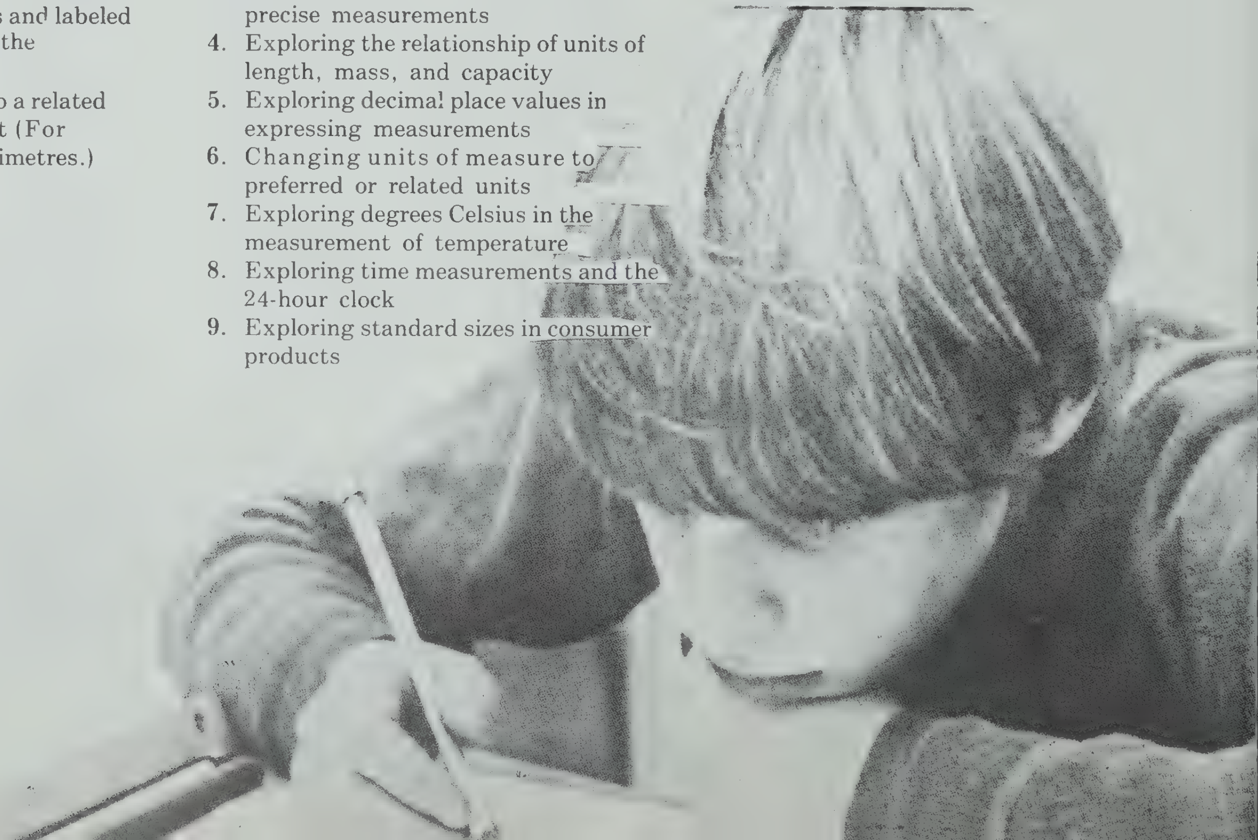
1. Experienced measuring objects with metric units of measure
2. Selected an appropriate unit of measure for measuring an object
3. Added or subtracted two measurements in like units and labeled the sum or difference with the appropriate unit
4. Changed a measurement to a related equivalent measurement (For example: 1 metre = ? centimetres.)

in chapter 9 the learner is—

1. Mastering selecting an appropriate unit of measure
2. Examining symbols and prefixes for metric units of measure
3. Distinguishing between estimates and precise measurements
4. Exploring the relationship of units of length, mass, and capacity
5. Exploring decimal place values in expressing measurements
6. Changing units of measure to preferred or related units
7. Exploring degrees Celsius in the measurement of temperature
8. Exploring time measurements and the 24-hour clock
9. Exploring standard sizes in consumer products

in later chapters the learner will—

1. Multiply and divide measurements by a 1-digit number
2. Tell whether a metric unit with a prefix is larger or smaller than a related unit.



Notes & Things

This is a "talk and do" chapter. It is intended to develop the concept of measure and measurement with emphasis on SI units.

It is an obligation to emphasize the metric system of measurement—not because it is easier to compute but because it is the system that each pupil will be using for the rest of his life. It is up to you whether or not you want to find out which countries are using the metric system, examine its use in North America, or enter the controversy about the problems created and solved by switching to it. These would, however, make great research projects for your more able pupils.

The metric units of length, mass, and capacity are all reviewed. For the first time, the prefixes of the root words are analysed. The child should come to know that the prefix indicates the size (number) and the root gives the clue as to the type of measure (length, mass, or capacity).

From the beginning the children should learn the internationally accepted spelling of metric terms. Most countries have adopted the **re** spellings for **metre, litre,** and their derivatives. In any case the children will more often see the symbol for each unit of measure, rather than the whole word.

The pupils should remember that no measurement is absolutely accurate. It will always be limited by the precision of the measuring device that is being used. There is no need to make much of this. But do be aware that when someone says that the length of a book is exactly equal to 25 cm, for example, he is really stretching the mathematical definition of equal. Avoid this situation whenever possible.

Temperature is briefly presented near the middle of the chapter. The name Celsius is used in honor of Andres Celsius (1707-1744), the Swedish astronomer who invented the centigrade thermometer.

Many labels on food products have SI units of measure clearly marked. Recycle the containers to mathematics class first so that you can investigate the marked units. A field trip to a local drug store or supermarket can be a treasure house of metric information.

things

books
samples of tape, ribbons, plywood, lumber
metrestick
standard masses and scales, if available
centimetre cubes
litre containers
digital clock and stop watch, if available

For the extra activities, you will want to have these things available:

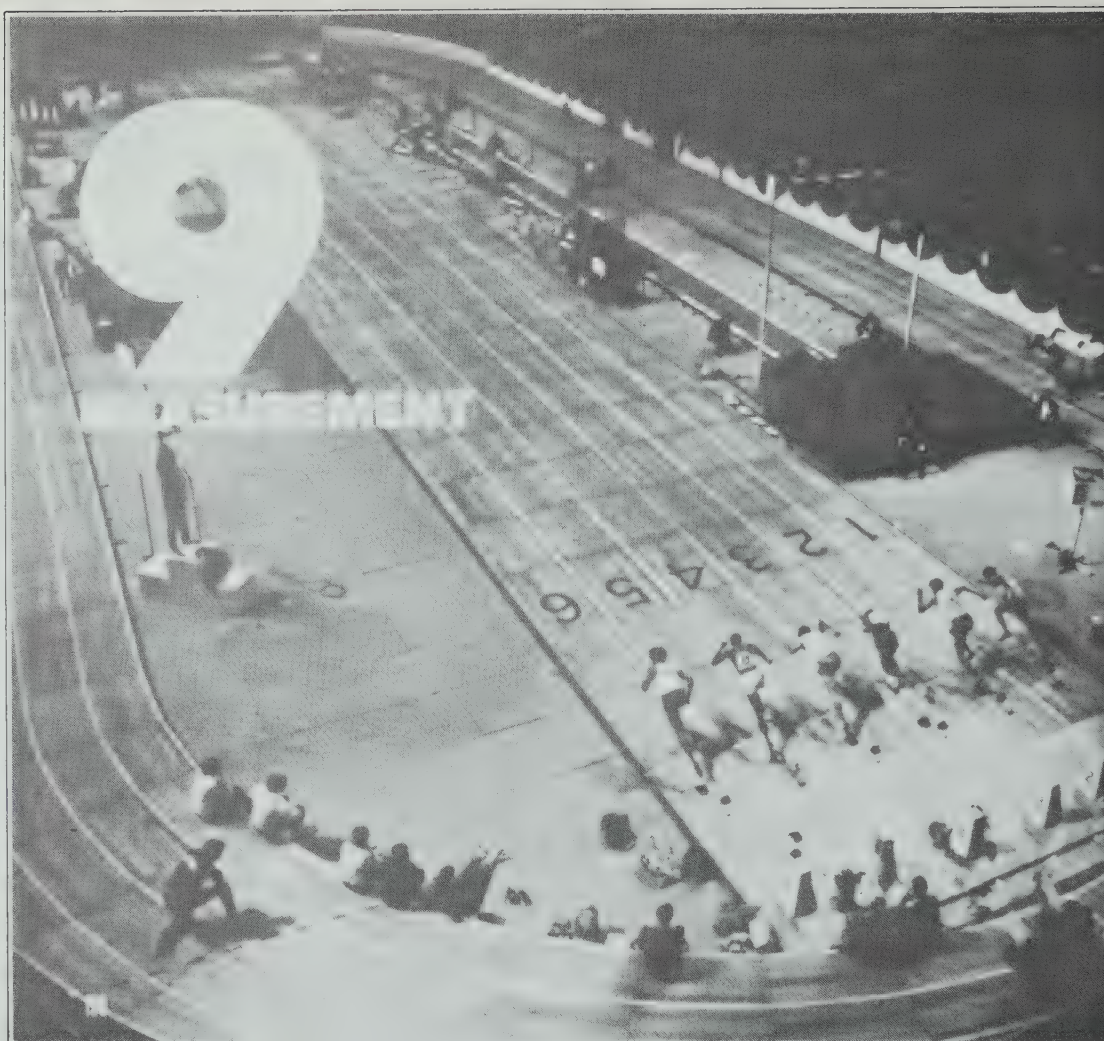
metric rule (See page 216d)
scale
1-cm cubes
5-cent coins

goal Think about and explore ideas through a picture clue

page 190 This photograph of an international track meet in Toronto, Ontario, should motivate discussion about important concepts: the need for accurate measurement and for internationally accepted standards of measure, and the importance of measurement in everyday situations as well as in international competitions. Many children will be interested enough to carry out research on Olympic records and report on some events.

Ask questions to stimulate discussion: What do we mean by an official record? Do we need to measure school athletic events as precisely as Olympic events? When do we need to be as precise as possible? When will an estimate be good enough? How accurate should an estimate be?

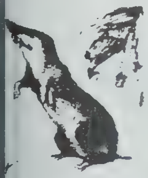
The **types** of measurement suggested by the photograph are length (distance) and time. This chapter deals with these three types and others: mass, capacity, and temperature. It also deals extensively with the sizes and relationships of the units used for the five types of measurement, and with the importance of the decimal system in measurement.



ID YOU KNOW?

The fastest land animal in the world, the cheetah, can cover a kilometre in about half a minute. The human record for the 1000 m run, established in 1974, is 2 min 13.9 s .

kangaroo can jump as far as 12 m in a ngle leap. The human record for the long mp, set in 1968, is 8.90 m .



The smallest mammal in the world, the shrew, has a mass of only a few grams. One kind of shrew has an average adult mass of only 2.5 g .

A camel can go for days without water, but when it does drink, it gulps more than 120 ℓ in a few minutes. The average adult human being needs a minimum of 2500 ml of water daily.

OUR GOAL is to become more skilful at measuring.

earn to estimate accurately and measure precisely.

Jse measurements to suit your needs.

lesson Page 191

goal Survey—applications of common units of measurement in approximations and precise measurements

page 191 This is a discussion page. It reviews what pupils already know of topics to be developed in the chapter:
1. measurement symbols for units of four of the five types of measurement;
2. decimal notation in measurement;
3. possible relationships between types of measurement (distance and time, capacity and time);
4. the difference between estimates (approximations) and precise measurement (for example: "a kilometre in about half a minute"—"1000 m run—2 min 13.9 s").

Pupils should classify the data in the descriptions as approximations or precise measurements, and decide whether they are satisfactory. (Don't expect an exact understanding of *average;* it is enough if pupils realize that "2.5 g" for the mass of a shrew is more precise than "only a few" grams. If necessary, explain the meaning of *minimum*.)

Ask pupils to suggest situations that require only estimates, and others that demand accurate measurements.

Thorough discussion of this page will help pupils towards an important skill: the ability to decide **when** some degree of precision is needed in measurement.

goal Reviewing symbols, prefixes, and common units in our system of measure

memo Wherever possible, pupils should use only symbols and numerals. This not only saves time (and spelling problems) but ensures SI usage in all measurement work. Correct use of symbols is especially important, to avoid ambiguities and inaccuracies.

page 192 Spelling and pronunciation may vary the world over, but SI symbols do not vary significantly. The rules for use of symbols are simple, and this review page should cause few problems. One item in everyday life that will cause problems, however, is labeling on commercial goods.

This is a good time to point out that even grown-ups and "experts" make mistakes—we're all human. An examination of labels collected from grocery items, or copied from commercial labels, will lead your keen-eyed pupils to spot obvious mistakes. The exercise provided on this page is only a start. Encourage the pupils to bring in collections of correct and incorrect labels for display and discussion.

You may be surprised at their editorial expertise! In addition, some pupils may wish to examine the difference between "hard" and "soft" metric conversion—problems presently facing many commercial suppliers. Giving dual measures on some packages seems at present to be a necessary evil. Pupils may wish to discuss the economic (and ecological) problems of introducing new containers, such as the one-litre bottle.

SYMBOLS

Like our number system, our measurement system is international.
Like number symbols, our measurement symbols are understood everywhere.
Measurement symbols should always be used with numerals.

Here are some examples.

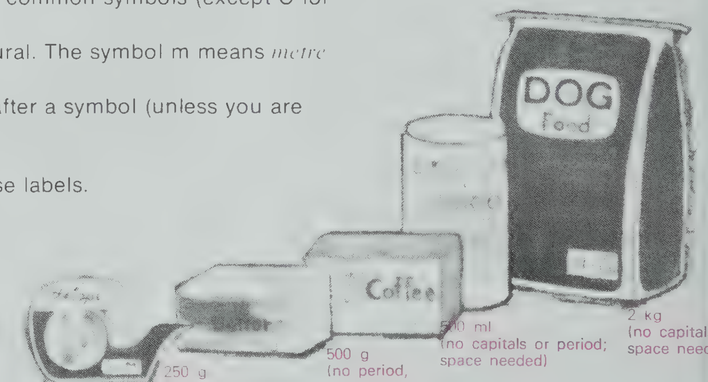
0.75 kg   24.06 m   3525 ℓ   8 500 000 km   30°C

You know the rules about writing numbers and about using decimal notation for a fractional part of a measurement.

There are rules for writing measurement symbols too.
Here are four rules to remember.

1. You leave a space between a numeral and a symbol (except in temperature readings).
2. You use small letters for common symbols (except C for Celsius).
3. You don't add *s* for a plural. The symbol m means *metre* or *metres*.
4. You don't use a period after a symbol (unless you are ending a sentence).

There are some errors on these labels.
Find the errors.
Write correct measurements.



192

YOU'VE COME A LONG WAY!

You know about place value and decimals.
You know about common units of measurement.
You know about symbols.

Do you know about PREFIXES?

Prefixes are word parts added before other words.
Prefixes are used in names for our measurements.

A metric prefix always has the same meaning.
It always uses the same letter (or letters) as part of a symbol.

For example:

The prefix *kilo-* means *one thousand*, and uses the letter *k* when it forms part of a symbol.
A *kilo*metre is 1000 m, or 1 km; a *kilo*gram is 1000 g, or 1 kg.

The prefix *milli-* means *one thousandth*, and uses the letter *m* when it forms part of a symbol.
A *milli*metre is 0.001 m, or 1 mm; a *milli*litre is 0.001 ℓ or 1 ml.

The chart below looks like a place-value chart — and for a good reason. Our system of measurement is related to the decimal system. The headings give the prefixes with their symbols and meanings. The words to which the prefixes are added take the place of the ones column in a place-value chart.

Some words and symbols are missing. Copy the chart and fill in the missing items.

Prefix Symbol Meaning	kilo- k thousand (1000)	hecto- h hundred (100)	deca da ten (10)		deci- d tenth (0.1)	centi- c hundredth (0.01)	milli- m thousandth (0.001)
Units of Length	kilometre km	hectometre hm	decametre dam	metre m	decimetre dm	centimetre cm	millimetre mm
Units of Mass	kilogram kg	hectogram hg	decagram dag	gram g	decigram dg	centigram cg	milligram mg
Units of Capacity	kilolitre kl	hectolitre hl	decalitre dal	litre ℓ	decilitre dl	centilitre cl	millilitre ml

See activity 4, page 216b.

lesson Page 193

goal Relating prefixes to place value

memo Not all prefixes given here are in common use for measurement, and some metric prefixes are not even mentioned (unfamiliar prefixes, such as *pica-* and *giga-*, that are used in specialized fields like computer science).

page 193 Review the ease and usefulness of the decimal system. (Place-value charts start on page 179.) Review measurement units pupils already know. You may ask pupils to list on the board all the units they can remember. Then ask them to organize the units by type of measurement (length, mass, and capacity), and to list the units for each in order of size. A column arrangement could be used in contrast to the horizontal presentation in the text. Have pupils note the common element for each type (metre, gram, litre), and explain the meaning of the related units.

Discuss the statements in the text to make sure that pupils understand their meaning. Ask which prefixes in the chart are not used in the lists on the board (likely *hecto-*, *deca-*, and *deci-*), and why the units they name are needed.

Don't expect mastery of all the units of measurement given here. The important point is the decimal nature of the metric system, which will be reviewed again throughout the chapter.

goal Review and practice with common units of measurement

page 194 Since our concern with measurement is practical, we need not stress rarely used units. They are important for a basic understanding of the metric system, but only a few units are in common use. With complete conversion to the system, however, the list given here may change. The decimetre, for example, may prove to be a useful unit.

Before pupils begin independent work, ask them to suggest items usually measured in the units listed here. Milligrams are used in such medicines as antibiotics, and in lists of vitamin and mineral content on cereal boxes.

Exercise 3 is a review of the previous two pages. Don't expect mastery; some pupils may have to refer to the chart on page 193 to complete exercise 3. Have them try the activity given at the foot of this page.

Most people don't need to use all the units of measure.
Here are the common units that are used most often.

LENGTH	kilometre	metre	centimetre	millimetre
CAPACITY	kilolitre	litre	millilitre	
MASS	tonne	kilogram	gram	milligram

Remember *tonne* (t) is the common unit used for a mass of 1000 kg

Choose your answers from the list of common units.

1. **a** Which is the largest unit of length? The smallest? kilometre millimetre
   b Which is the largest unit of capacity? The smallest? kilolitre millilitre
   c Which is the largest unit of mass? The smallest? kilogram milligram

2. Write the measurement that would be commonly used to describe each item.
   a The capacity of a large pail: 0.1 hl 1 dal 10 ℓ
   b The distance of an Olympic sprint: 1 hm 1 dam 100 m
   c The length of a long walk: 5 km 50 hm 500 dam
   d The mass of a thick slice of bread: 4 dag 40 g 400 dg
   e The capacity of an oil drum: 2 hl 200 ℓ 2000 dl
   f The mass of a postage stamp: 0.2 dg 2 cg 20 mg

Try these without looking at any list.

3. Give the missing numbers.

a ? m = 1 km (1000)	**b** ? m = 1 hm (100)	**c** 1 m = ? cm (100)	**d** ? mm = 1 m (1000)
e ? ℓ = 1 dal (10)	**f** ? ℓ = 1 kl (1000)	**g** 1 ℓ = ? ml (1000)	**h** 100 ℓ = ? hl (1)
i ? g = 1 kg (1000)	**j** 1 g = ? mg (1000)	**k** 1 t = ? kg (1000)	**l** 10 dg = ? g (1)

things index cards or slips of paper

Write Length, Mass, Capacity, and the various unit names on cards or slips of paper. Mix the cards. The pupil first finds the three cards giving type of measure, and places them in a column. Then he sorts the unit cards, placing them in rows according to type. Next he arranges the unit cards from left to right, beginning with the largest unit. He may compare his arrangement with the chart on page 193.

Now work with the pupil. Ask him what the units in each column measure (length, mass, capacity) and what numerical value the prefix of each unit indicates.

EN YOU'RE TALKING ABOUT MEASUREMENT . . .

You may need to give only an *estimate*.
Nan drives about 8 km to her office.
The game will last about two hours.

You may need a *precise* or *close measurement*.
The window pane is 480 mm by 312 mm .
The winner's time was 11.8 s .

1. Write Yes for each item for which an estimate is likely to be close enough.
Write No if an estimate will not do.

a Length of a kite string Yes
b Clearance for vehicles under a bridge No
c Safety limit for a boatload No
d Time you will take to read 100 pages Yes
e Correct dosage of a medicine No
f Amount of water you drink daily Yes
g Dimensions of a lawn you cut Yes
h Dimensions of an Olympic-sized pool No
i Government report on rainfall No
j Mass of your desk Yes

2. If you want to give a precise or close measurement, you need to choose a suitable unit.
For each item, name the unit that seems most suitable for the measurement asked for.

a Distance to the moon km
b Length of a pin mm
c Mass of a cent g
d Mass of a railway car t
e Capacity of a tank car kl
f Capacity of an eye-dropper ml

3. Each item (**a** to **f**) gives three possible measurements for the same thing. Some measurements have been made to the nearest whole number. Some are made to the nearest tenth. Some are to the nearest hundredth, and some to the nearest thousandth.
For each item, write the measurement that comes closest to a precise measurement.

a	2 km	1.9 km	1.87 km	b	3.86 kg	4 kg	3.9 kg
c	3.1 ℓ	3.09 ℓ	3.088 ℓ	d	21.5 s	22 s	21.49 s
e	1.8 m	1.76 m	1.758 m	f	9.486 kl	9.5 kl	9.49 kl

195

lesson Pages 195, 196, 197

goal Examining precision in measurement

memo Each exercise here deals with a specific aspect of precise measurement: deciding whether it is needed, choosing an appropriate unit, and relating decimal notation to precise expression. Most pupils at this level are not ready to grasp fully the idea that there is no such thing as an **exact** measurement with the instruments we use.

page 195 The examples introduce precise measurements, which are often expressed with decimal fractions, in contrast with estimates. Ask pupils for more examples measuring length and time, and others measuring mass and capacity. Exercise 1 may help them to make suggestions.

Exercise 2 may also stimulate ideas: When would you use kilometres rather than millimetres? Which unit would give a closer measurement? If pupils are ready, ask whether either unit would give an **exact** measurement.

With their knowledge of decimal notation, pupils should be able to make the correct choice in exercise 3. Lead them to generalize: When the **same unit** is used, a measurement to three decimal places (thousandths) is more precise than a measurement to two places (hundredths). Ask them if they can see any relationship between the three choices in each part.

goal Estimating and measuring in millimetres

memo Children usually do better than adults at this type of exercise!

things books
samples of tape and ribbons
plywood and lumber samples
other small items to be measured

page 196 The pupils have met this type of exercise before; the first three exercises provide review and reinforcement—an opportunity to improve on those important estimation skills. Have the pupils leave ample space on their charts for additional samples, particularly if they wish to try their estimation skills with the items listed above.

Exercises 4 and 5 will be more of a challenge, but will remind pupils that our measurement system is a decimal system. Give help where needed. There is more work on conversion later in the chapter (pages 210-215).

How well do you know millimetres?

You can find out by estimating and then measuring.
If you really know millimetres, your estimate will be very close to the measurement.

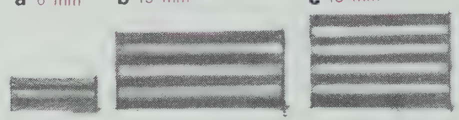
First make a copy of this table for your records.

Exercise	Estimate	Measure	Difference
1. a	mm	mm	mm
b	mm	mm	mm
c	mm	mm	mm

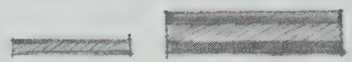
1. Estimate in millimetres, and then use a rule to check. Record your findings in the table.
   - **a** How wide is your little finger?
   - **b** How thick is the cover of this book? 3 mm
   - **c** How thick is the whole book? Accept 22 - 25 mm

2. Add to your table for records on the thickness of the plywood pieces shown below. Estimate first, and then use your rule.

a 6 mm **b** 15 mm **c** 18 mm

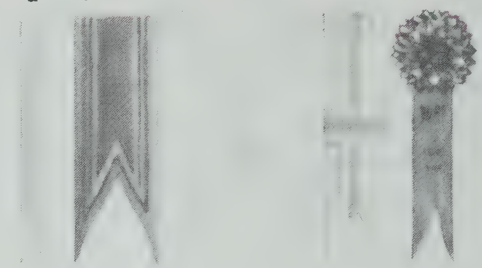


d 3 mm **e** 8 mm



3. Add to the table your estimate and then the measurement of the width of each ribbon shown below.

5 mm **a** 16 mm **b** 22 mm **c** 12 mm **d** 8 mm **e**



If you really know millimetres, you should know how to express them as centimetres. Remember, a millimetre is a *tenth* of a centimetre. Our measurement system is a decimal system.
So 54 mm = 5.4 cm .

4. Use your table, and write in centimetres the measurement you recorded for each part of exercise 2. 0.6, 1.5, 1.8, 0.3, 0.8

5. Write in centimetres the measurement you recorded for each part of exercise 3.
0.5, 1.6, 2.2, 1.2, 0.8

How well do you know centimetres?

ny small things must be measured in millimetres. Centimetre measurement is exact, but it may be all that you need. You can find out how well you know timetres — again by estimating and measuring.

have two "handy rules" for making estimates in centimetres: your fingers and r handspan.

the rule at the side of this page to find which of your fingers is about 1 cm wide. n find how many fingers measure 5 cm.

/ use the rule to measure your handspan. It is the number of centimetres you reach with your fingers spread as far apart as possible.

v a 4-column table like that on page 196. Change the symbol to cm, and give mates and measurements to the nearest centimetre.

Use your fingers for estimating. Complete the records on your table.

How long is **a** your hand? **b** your foot? **c** your pencil?

How wide is **d** this book? 21 cm **e** your workbook? Answers will vary

Use your handspan for estimating. Complete the records on your table.

How long is **a** your arm? **b** your leg? **c** the longer side of your desk?

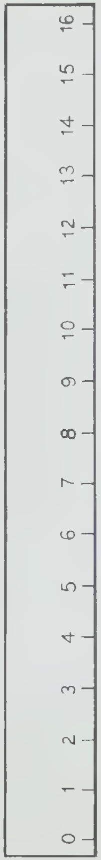
What is **d** your height? **e** the height of your desk? Answers will vary.

timetre measurements may be expressed as metres. A *centi*metre is a *hundredth* metre. You must have two digits to the right of the decimal point when you ress centimetres as metres.

9 cm = 0.09 m   19 cm = 0.19 m   109 cm = 1.09 m

Write in metres the measurement you recorded for each part of exercise 1.

Write in metres the measurement you recorded for each part of exercise 2.



197

goal Estimating and measuring in centimetres

page 197 This page is a natural extension of page 196. A key statement is in paragraph 1: "Centimetre measurement is less exact, but it may be all that you need." Ask pupils if they would prefer to remember their heights in centimetres or millimetres. What are the advantages and disadvantages of each measurement? For everyday purposes, do we really **need** to know our heights in millimetres?

At this level, as in previous levels, it is important that pupils be able to relate parts of the body to common measures. Knowing the widths of one, two, or three fingers and of a handspan is an aid in measuring. Knowing one's height is helpful in judging heights of other people or of objects in and out of school.

Exercises 3 and 4 are similar to the last exercises on the preceding page. Some pupils may need help here, but the practical exercise of conversion paves the way for more detailed practice later.

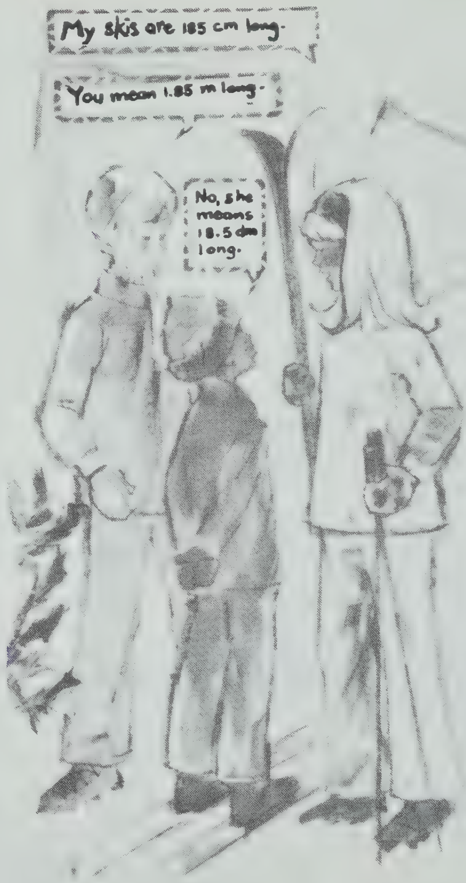
See activity 3, pages 216a-216b.

lesson Page 198

goal Practice with preferred units of measurement

memo This page provides extra practice with conversion from one unit to another. It also introduces a concept for discussion: preferred units. Don't expect mastery here, since preferred units are those that evolve over many years of standard practice. In general, however, we do not apply all common units to everyday items. Camera film and photographic measurements are normally in millimetres. Small amounts of liquid are normally measured in millilitres. Heights of children and adults are normally expressed in centimetres. Despite the fact that our measurement system is a decimal system, we often avoid the use of decimals in everyday situations.

page 198 This is primarily a discussion page. In particular, discuss **why** some units are preferred to others. Some pupils may wish to do further research, using labels, catalogues, and so on, to find out what units are preferred for certain types of measurement.



Who is correct? They all are, of course. But 182 cm is t
preferred measurement.
The centimetre is preferred for measuring body sizes, lengths of such objects as skis and belts, and dimensio of furniture. The millimetre is preferred for smaller obje

You'll have no problems changing metres to centimetre centimetres to millimetres if you are using whole numb But what if the measurement given has fractional parts

You know 24 mm = 2.4 cm . So 2.4 cm = 24 m
1.5 cm must equal 15 mm, and 7.6 cm equal
76 mm .

You know 134 cm = 1.34 m . So 1.34 m = 134
2.56 m must equal 256 cm, and 4.75 m equa
475 cm .

Change each measurement given with a decimal fracti to a preferred unit given with a whole number.

1. Alice is 1.68 m tall. 168 cm
2. I have some 3.5
3. My skis are 1.90 m long. 190 cm
4. The snow is 1.15
5. The ski is 5.5 cm wide. 55 mm
6. That belt is 2.8 c

198

ow important is the metre? It's *very* important!

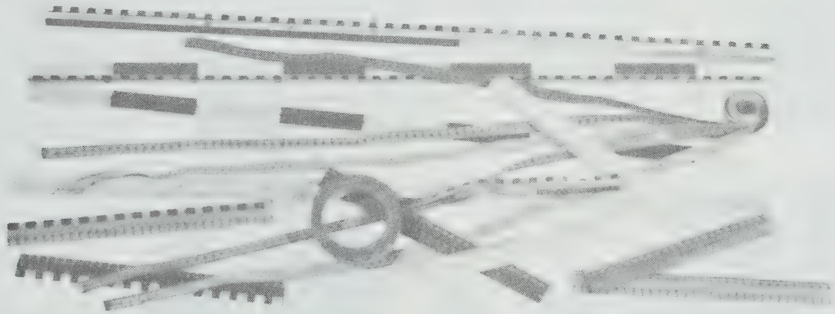
invention was the beginning of the **METRIC SYSTEM** of measurement.

About two hundred years ago, some French scientists decided to invent a system of measurement that would suit our decimal (base-ten) number system. The metre was the first standard unit they chose. It gave the new system its name.

ne metre is the **BASE UNIT** of length in the metric system.

All other units for measuring length are based on the metre. Larger units measure the length of a metre multiplied by 10, 100, 1000, and so on. Smaller units measure the length of a metre divided by 10, 100, 1000, and so on. You already know the prefixes that show the numerical relationships of these other units to the metre.

ne metre is the **PREFERRED UNIT** for many measurements of length.



Measurements given in metres include room dimensions, distances on golf courses, depths of lakes and rivers, heights of buildings and mountains, distances in many Olympic events, and lengths of carpeting and other materials.

199

See activities 1 and 2, page 216a.

lesson Pages 199, 200, 201

goal Exploring the International System of Units

page 199 The **base units** of SI (from the French name for the system) are the metre, kilogram, second, ampere, kelvin, mole, and candela. Only the first three are discussed in this chapter. Other units treated—litre, tonne, minute, hour, day, and degree Celsius—are non-SI units permitted for use with SI. The significant concept is that SI is a **system** (S)—and therefore easy to learn.

This is a discussion page, and pupils should be encouraged to volunteer opinions and information, and to ask questions. Some pupils may wish to examine other base units in our system; others may want to do some research on the historical development of SI; still others will want to find out more about preferred units.

The key to understanding SI is in paragraph 2: the relationship between our measurement system and the decimal system.

goal Exploring units of mass

things standard masses and scales, if available. (A litre container filled with water will give pupils a direct way of getting the "feel" of a kilogram as well as reinforcing the relationships in SI between length, mass, and capacity.)

page 200 This is another discussion page. Again, you will have an opportunity to emphasize the value of our system of prefixes: the same prefixes apply to mass, length, and capacity.

(The distinction between mass and weight can be a complex issue, especially for pupils just becoming familiar with SI. Whether or not you wish to discuss the distinction now will depend on your pupils and on local curriculum objectives.)

A useful exercise is to have the pupils construct a cubic metre frame. This could be made from metresticks, taped at the corners, or from commercially available rods. Your pupils may suggest alternatives, such as square metres of cardboard taped at the edges.

If a tank this size (one cubic metre) were filled with water, its capacity would be about one kilolitre and its mass would be about one tonne. Pupils who construct such a device will have a firm visual image of these large measures.

Similarly, a litre container could be improvised from some milk cartons.

How about the gram?

The scientists who invented the metre had to choose a new standard unit for measuring *mass* — the amount of matter in an object. They decided to relate it to their new units for measuring length.

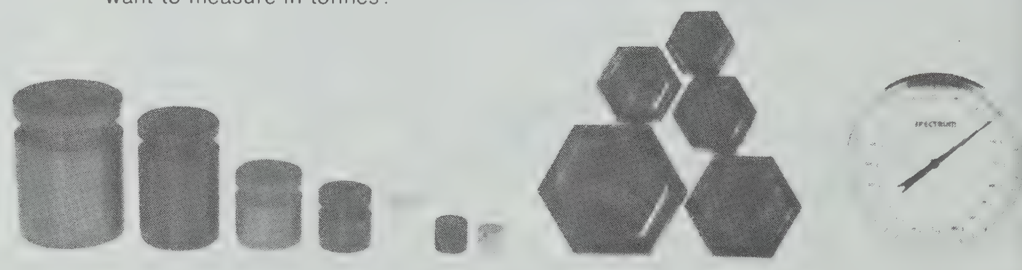
Imagine a cube of water 1 cm long, 1 cm wide, and 1 cm high. The mass of such a cube became the new standard unit for mass — the gram.

All other units for measuring mass are related to the gram. The metric prefixes used for units of length are used for the units of mass. The prefixes show the relationships of the other units of mass to the gram.

Because the gram itself is a small measurement, it is not the base unit for mass. The base unit is the mass of 1000 cm cubes of water — the **kilogram**.

Imagine one million centimetre cubes of water: the mass of water that would fill a tank 1 m long, 1 m wide, and 1 m high. This mass of 1000 kg has a special name — tonne.

A small car has a mass of about 1 t . What are some other things you might want to measure in tonnes?

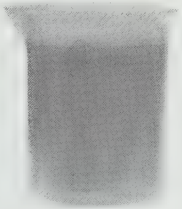


200

See activity 5, pages 216b.

ɔw about the litre?

other measurement the scientists had to consider was *capacity* — the ount a container will hold. Again they decided to relate their new standard capacity to the new units for measuring length.

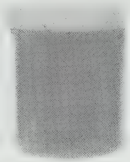


agine a container that holds 1000 cm cubes of water. The capacity of that ntainer became the new standard unit for capacity — the litre.

e litre is not called a base unit in the metric system, but all other measures capacity are related to it. The metric prefixes show the relationships of other ts of capacity to the litre.

tres, grams, and litres are all important units in our system of measurement. r measures of length, mass, and capacity are related to each other.

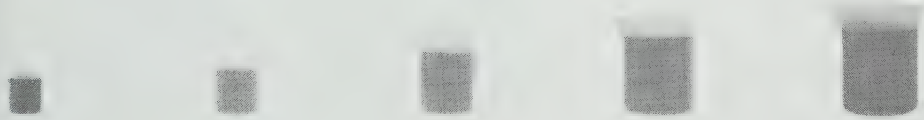
A 1 cm cube holds 1 ml of water that has a mass of 1 g .
1000 cm cubes hold 1 ℓ of water that has a mass of 1 kg .
1 000 000 cm cubes hold 1 kl of water that has a mass of 1 t .



e official name for the system is the International System of Units — SI for ɔrt. SI symbols mean the same in all languages. Most countries today are either ng SI or changing to it.

y is an international system like SI useful?

st a litre container always be in the form shown on this page? No
at other shapes can you think of?



201

See activity 6, page 216b.

goal Exploring units of capacity and the International System of Units

things centimetre cubes
litre containers
(commercial or improvised)

memo Care has been taken throughout this mathematics series to avoid confusion between **capacity** (the amount of fluid a container will hold) and **volume** (the amount of space taken up by an object). This is an important distinction, since capacity is measured in litres and volume is measured in cubic units (see the Tables of Measures).

page 201 The discussion on this page rounds out our review of SI. Examination of the litre and its relationship to length and mass stresses the importance of the base unit of length: the metre.

Once again, this is an opportunity to review the prefixes and point out their usefulness: Regardless of the unit to which they are applied, they always have the same meaning.

Discussions of SI can be related to current events, since the change to SI is going on at different rates in many countries. No doubt your pupils will be able to quote from the media on the progress of SI conversion in various places, and discuss the reactions of various segments of the population to the changes that are taking place.

goal Exploring temperature measurement

things if available, a collection of thermometers, including indoor-outdoor thermometers

memo Although the base unit of temperature measurement is the kelvin, the degree Celsius is the unit chosen for everyday use. The kelvin is used mainly in scientific work. A temperature change of one degree kelvin is the same as a change of 1°C.

page 202 This discussion page has been included because temperature is a form of measurement. However, more common applications will probably be found in other areas—graphing, for example, or science and social studies.

Review the relationship between our number system and our measurement system—the key number is 10. The Celsius scale is based on this number system, and so temperature readings are very easy. More precise thermometers, such as those used in hospitals, will register readings to within 0.1°C.

It is unlikely that pupils will have difficulty with the exercise, although some pupils will want to know more about "normal" body temperature (37°C), and will want to research variations in body temperatures.

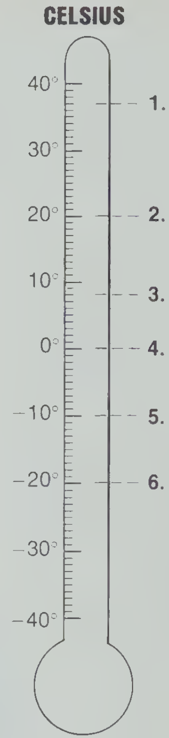
Hands-on experience with a variety of thermometers will enhance student understanding of the Celsius scale, although this activity may well be reserved for a science lesson.

TEMPERATURE MEASUREMENTS — SI

The common SI units for temperature measurements were invented about 200 years ago by a Swedish astronomer named Anders Celsius. He set his scale by fixing the freezing point of water at 0°, and the boiling point at 100°. The hundred equal divisions between these two points and other equal divisions above and below them are known as degrees Celsius (symbol °C).

Temperatures below 0°C are called minus temperatures and written with a minus sign: −4°C (minus 4 degrees) means 4 degrees below zero, the freezing point.

Fractional parts in Celsius readings, like other SI measurements, are given as decimal fractions. A reading halfway between 20°C and 21°C is 20.5°C .



Read the thermometer to give the temperature that matches each description.

1. Normal body temperature 37°C
2. Normal room temperature 20°C
3. A mild spring day 8°C
4. A day on which ice forms 0°C
5. Good skating weather -10°C
6. A very cold day -20°C

202

See activity 4, page 216b.

EXTRA PRACTICE

No rules or any other kind of measuring device allowed for this page!

Which measurement best describes the object?

1. a Mass of a small car: 20 000 g 500 kg 1 t
   b Width of a small car: 5 m 2 m 1 m
   c Mass of a bag of sugar: 5 kg 10 g 100 g
   d Capacity of a carton of milk: 10 ℓ 1 ℓ 100 ml
   e Height of a large dog: 10 cm 50 cm 150 cm

PROGRESS CHECK

Write the symbol for:

1. a ten kilograms 10 kg b twenty-five centimetres 25 cm c three hundred millilitres 300 ml
   d five metres 5 m e fifty grams 50 g

Write the preferred unit measure for the following:

2. a 0.001 ℓ 1 ml b one thousand grams 1 kg c one thousandth of a metre 1 mm
   d 0.01 m 1 cm e 1000 m 1 km

203

goal **Progress Check**—writing SI symbols

page 203 The EXTRA PRACTICE at the top of this page is optional, but may be found useful for pupils who need to review reading symbols and selecting appropriate measures.

Those who have difficulty with item 1 in the Progress Check should be provided with additional review and reinforcement of symbolic notation. Those who have problems with item 2 will need extra help with conversion from one unit to another; additional practice on this topic is provided later in the chapter (pages 210-215). An alternate Progress Check appears on page 216a.

lesson Pages 204, 205

goal Exploring standard sizes in consumer products

memo Before starting this discussion, make sure that the pupils are aware of the meanings of such words as **standard, consumer, producer,** and **manufacturer.**

page 204 This is a discussion page that can lead to an on-going research project. Not all items come in standard sizes, and many items will never be needed in such sizes.

Standardization of packages has many advantages for the consumer—for example, comparison shopping is made easier. Are there disadvantages for the consumer? Are there disadvantages for the manufacturer?

What are some of the difficulties faced by people who want to standardize packages? How would you package apples, for example—by mass, by number, or by bag size?

How should the sizes be determined? Would it make sense for toothpaste producers to produce 250 ml, 500 ml, and 1 ℓ containers for their products?

CONSUMER PACKAGING

When you go shopping, look for items in *standard sizes*.

For example, have you ever noticed the measurement on a tube of toothpaste?

It comes only in sizes such as 25 ml, 50 ml, 100 ml, and 150 ml . The manufacturers of toothpaste have agreed to make their brands in the very same quantities.

Now compare these prices with the amounts in the container.

Brand A shampoo 83 ml for 69¢	Brand X toothpaste 50 ml for 47¢
Brand B shampoo 71 ml for 59¢	Brand Y toothpaste 50 ml for 46¢

When manufacturers make things in standard sizes, what advantages are there for the buyer? for the manufacturer? Can you think of items for which standard sizes wouldn't matter? Discuss all answers.

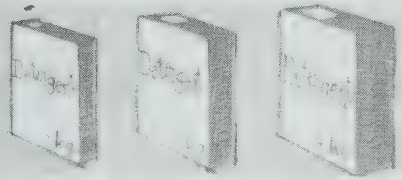
In what quantity does your mother or father buy the following items? (Check when you get home if you're not sure.)

COFFEE	**SALT**	**DETERGENT**
SUGAR	**TEA**	**FRUIT JUICE**

204

Possible projects for researchers:
- The metric system as it is used today
- Specialized measuring instruments (micrometer, balance scale, barometer)
- Interviews with people who use measurement in their work (pharmacist, carpenter, engineer, hardware clerk, grocery clerk, butcher, sportsman, car repairman, science teacher, school sports coach) The focus of the interview should be on how the person uses measurement and what units of measure are used most.
- Advantages and disadvantages of using the metric system

hich of these is the best buy? Why?



They can save on packaging.

hy do manufacturers offer better prices on larger sizes?
you can save money by buying the largest package, why
people buy the smaller packages? To use less space; the product may spoil before you use it all.

andard sizes of some items are set by an agreement
etween their manufacturers. They agree that their
istomers will always use a particular size.

hoose the items that you think should have standard
zes.

SWIMMING POOLS BOOKS TIRES

ENVELOPES BALLOONS

GOLF BALLS HOUSES CARPETS

SHIRTS FOOTBALLS Discuss all answers.

hould these items always be sold in standard sizes?
hy? Can you think of other things that have standard
zes set by agreement? Discuss all answers.

andard sizes make it easier to decide which is the
etter buy" — but you can't always decide by price
one.

hat other things might make you decide on a "better
y"? Quality; special needs

CAN YOU SAVE MONEY?

Choose the better buy. (It's not *always* the larger size!) Tell how much you would save.

1. Toothpaste
   - a 25 ml — 44¢ *or* 50 ml — 89¢ 1¢
   - b 50 ml — 89¢ *or* 150 ml — $2.09 58¢
2. Dog food
   - a 1 kg — 89¢ *or* 3 kg — $2.57 10¢
   - b 500 g — 47¢ *or* 1 kg — 95¢ 1¢
3. Paint
   - a 1 ℓ — $1.29 *or* 5 ℓ — $6.20 25¢
   - b 500 ml — 75¢ *or* 1 ℓ — $1.35 15¢
4. Masking tape

   5 m — 39¢ *or* 15 m — 99¢ 18¢
5. 20 mm tape

   5 m — 89¢ *or* 10 m — $1.59 19¢

205

goal Exploring advantages of standard sizes

page 205 Here is an opportunity to examine samples of misleading advertising. It is also an ideal place to discuss other factors in purchasing besides price and quantity. For example, would there be an advantage in buying 25 ℓ of paint when you want to paint only one room? Should books, houses, and swimming pools all come in standard sizes?

One important factor (not mentioned on the page) that should come from the discussion is **quality**. Is the cheaper paint really a better buy?

The exercise on this page may be a challenge for those pupils who still need help with unit conversions—let them have the help, since the topic is treated again in this chapter.

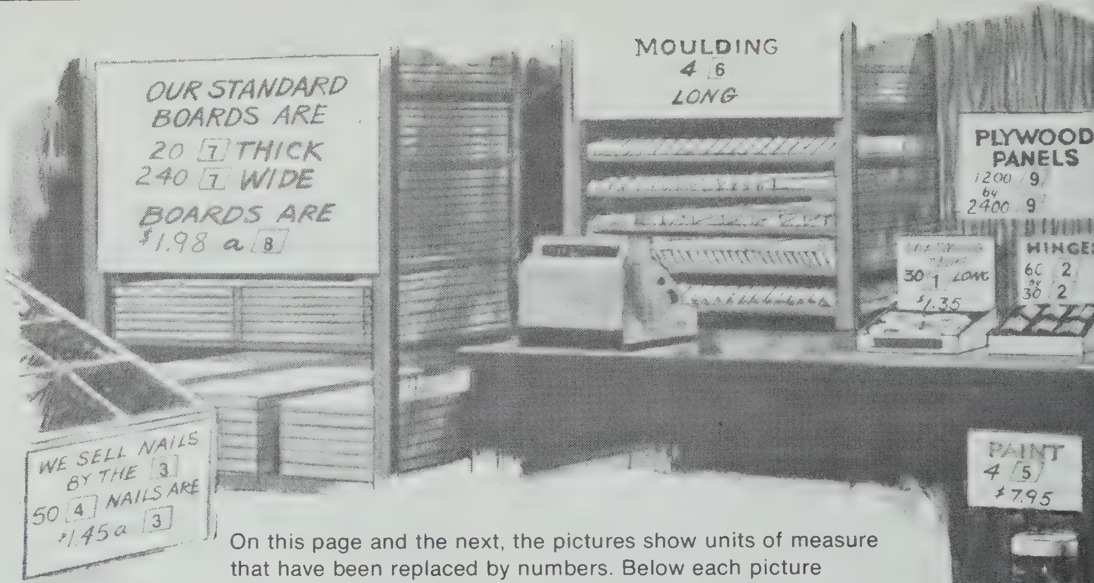
lesson Pages 206, 207

goal Selecting an appropriate unit of measure

memo The directions for pages 206 and 207 are given once, on page 206.

page 206 The pupils should have few difficulties in completing this exercise independently. For each problem there are three choices: two units for the type of measure required and one unit for an inappropriate type. The pupils are therefore required first to eliminate obviously inappropriate units (length, mass, or capacity), and second, to choose a preferred measure that makes sense. For instance, nails would not be sold by the milligram, and nail lengths would normally be measured in millimetres. Let the pupils discover these two steps for themselves.

The independent activity here gives you an opportunity to help pupils who are having special problems.



On this page and the next, the pictures show units of measure that have been replaced by numbers. Below each picture these numbers are listed, together with a choice of three units of measure. Choose the unit that fits the sign in the picture.

1 gram, metre, millimetre

2 metre, milligram, millimetre

3 kilogram, kilometre, milligram

4 centimetre, decagram, millimetre

5 kilolitre, litre, metre

6 centimetre, decilitre, metre

7 centimetre, litre, millimetre

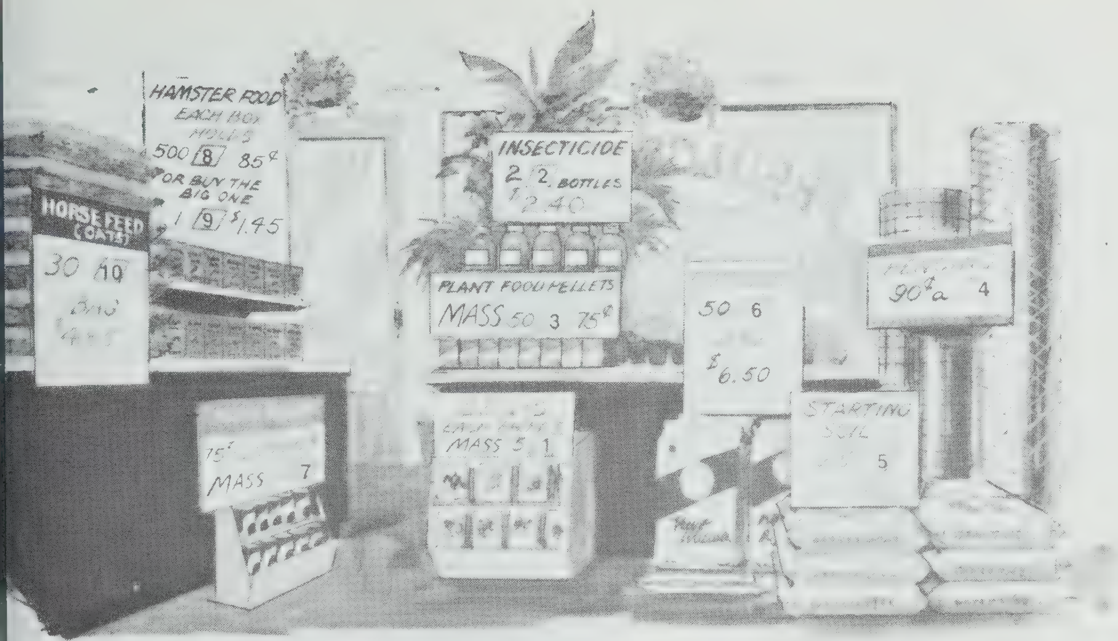
8 decametre, kilogram, metre

9 kilogram, metre, millimetre

206

Possible projects for researchers:

- The metric system as it is used today
- Specialized measuring instruments (micrometer, balance scale, barometer)
- Interviews with people who use measurement in their work (pharmacist, carpenter, engineer, hardware clerk, grocery clerk, butcher, sportsman, car repairman, science teacher, school sports coach) The focus of the interview should be on how the person uses measurement and what units of measure are used most.
- Advantages and disadvantages of using the metric system



1 decilitre, gram, milligram

2 decalitre, kilogram, litre

3 decimetre, gram, hectogram

4 decagram, kilometre, metre

5 kilogram, kilolitre, tonne

6 gram, kilogram, kilolitre

7 centigram, gram, millilitre

8 centimetre, gram, milligram

9 decagram, decalitre, kilogram

10 decigram, kilogram, kilolitre

goal Selecting an appropriate unit of measure

page 207 Once again, this is an independent activity, similar to the previous page, involving real-world examples. Another opportunity to give some of your valuable time to those pupils who have problems that only a teacher can solve.

lesson Pages 208, 209

goal Exploring time measurement with the 24-hour clock

memo Although the 24-hour clock system for measuring time is not essential to an understanding of SI, this system does appear with increasing frequency, particularly in airline and railway schedules. Please note, however, that most digital clocks are **not** 24-hour clocks. Not all 24-hour clocks are digital clocks, for that matter; some hospital clocks have been converted simply by adding a second circle of numerals to the circular dial. You might wish to consider doing this with your classroom clock.

page 208 This is mainly a discussion page. Don't expect mastery in the exercise, although most pupils will not find it difficult. The exercise relates the 24-hour system to everyday events. Your more ambitious pupils may want to collect samples of airline and train schedules, and other schedules based on the 24-hour system.

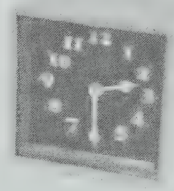
It is important to point out that the 24-hour digital system is **not** a decimal system. Decimals and time measurement are dealt with on the next page.

TIME TO THINK ABOUT TIME

You already know about common units for measuring time: seconds, minutes, hours, days, months, and years. Units such as *decades*, *centuries*, and *millennia* are used for measurements of longer periods of time. (Try to find out more about these.)

With the customary method of measuring time, we use a.m. and p.m. with numerals to show whether times are before or after noon.

Time expressed by 24-hour digital clocks does not need the usual abbreviations. A 24-hour clock tells time by showing four digits. Midnight is 00 00. Noon is 12 00. After 23 59 (one minute before midnight), the clock goes on to 00 00 again. The first two numbers show how many hours have passed since midnight. The other two show how many minutes have passed in the hour.



The photographs show an ordinary clock and a 24-hour digital clock. Do they give the same time? How would you write the times they show?

Where can you find 24-hour digital clocks? Where is this method of showing time used most?

WRITE THE TIME THAT MATCHES

a	Getting out of bed	07 30	08 30
b	Going to school	08 30	13 00
c	Lunchtime	12 00	15 30
d	End of lunchtime	13 00	20 30
e	End of school day	15 30	12 00
f	Eating dinner	18 10	07 30
g	Going to bed	20 30	23 59
h	Sound asleep	23 59	18 10

BEING PRECISE ABOUT TIME

There are special clocks and watches that can give time measurements more precisely than the ordinary watch or the 24-hour digital clock can. We use the decimal system to express fractional parts of a second.

1. A stopwatch measures time to the nearest tenth of a second (0.1 s).

   a When he began practice, Peter ran the 200 m distance in a time of 23.4 s . The next day, his time for the distance was 23.1 s . By how much had he improved? 0.3 s

   b On the day of the competition, he ran the 200 m race in 22.9 s . By how much had he improved since his first practice run? 0.5 s

2. Officials timing Formula I racing cars use clocks that measure hundredths of a second.

   a A car turned one lap in 37.54 s and another lap in 37.13 s . How many seconds did it take for the two laps? 74.67 s

   b What was the difference in time between the two laps? 0.41 s

3. Scientists and engineers may need to measure time to the nearest millisecond.

   a What do you think a millisecond is? (Hint: Remember your prefixes.) A thousandth of a second (0.001 s)

   b How many digits to the right of the decimal point would millisecond measurements show? 3 digits (0.001)

goal Using decimals in measurements of time

things stopwatch or similar decimal timer

memo Note that a regular watch measures parts of a minute (seconds) but does **not** measure decimal parts of a minute.

page 209 Discuss the idea of measuring decimal parts of a second. When might we need to be that precise in measurement? (The pupils who examined Olympic records will be able to make a contribution here.)

Items 1 and 2 should pose no problems for your pupils. Item 3, however, may be treated as a discussion and research question.

Individual activity

Just for fun. Suppose that the prefixes used for other measurement units were used with our units to measure time.
* Would you like it better if your school day were a deciday long? (Deciday = 2.4 h = 2 h 24 min) Why?

* Would you like to study mathematics for only a milliday each school day? (1 milliday = 0.024 h or about 1½ minutes) Why or why not?

lesson Page 210

goal Relating decimal place value to measurement

memo By now, most pupils will have an appreciation of the importance of decimals in our measurement system, especially after the discussions of pages 199-201. This page begins a formal presentation of this relationship.

page 210 For pupils at this stage, decimal notation should be a breeze—they have been dealing with money problems for years, and they have just completed a chapter on decimals. If necessary, refer to the charts on pages 179 and 193.

The key to success on this page is remembering the **meanings** of decimal places and prefixes. Note that questions 1 g and h require a O place-holder, and that all questions require a zero before the decimal point when there is no whole number. The step from exercise 1 to exercise 2 should pose no problems, but watch for pupils who have difficulty. They need to be reminded of the **meanings** of place value and prefixes, and of the importance of the zero as a place-holder.

What is the chart on the left? on the right? ARE THEY RELATED?

thousands	hundreds	tens	ones	tenths	hundredths	thousandths

kilo-	hecto-	deca-	metric unit	deci-	centi-	milli-

Decimals and measurement go together, thanks to the people who invented our metric system of measurement.

You know that 25 cm is 25 hundredths of a metre.
You can write this as a decimal fraction: 0.25 m .

You know that 225 ml is 225 thousandths of a litre.
You can write this in decimal form: 0.225 ℓ .

1. Replace the ? by a decimal fraction.

a 15 cm = ? m 0.15 **b** 600 ml = ? ℓ 0.600 **c** 755 mm = ? m 0.755 **d** 250 mg = ? g 0.250

e 455 ml = ? ℓ 0.455 **f** 85 cm = ? m 0.85 **g** 5 cm = ? m 0.05 **h** 5 ml = ? ℓ 0.005

Did you remember that 5 hundredths is written 0.05?
that 5 thousandths is written 0.005?

Here are more problems that may be tricky.

2. Give a decimal fraction for each answer.

a 5 mm = ? m 0.005 **b** 55 ml = ? ℓ 0.055 **c** 3 cm = ? m 0.03 **d** 25 mg = ? g 0.025

e 50 ml = ? ℓ 0.050 **f** 1 mm = ? m 0.001 **g** 10 cm = ? m 0.10 **h** 1 cm = ? m 0.01

Our measurement system makes it easy to change a measurement to a related unit.
You can change to a smaller unit by multiplying by 10, 100, 1000, and so on.
You can change to a larger unit by dividing by 10, 100, 1000, and so on.

Copy and complete these tables. Watch for the pattern in each. IT IS SIMPLE!

1.

cm	mm
5.5	55
2.1	? 21
17.5	? 175
20.8	? 208
36.0	? 360

2.

m	cm
2.50	250
1.25	? 125
10.65	? 1065
20.01	? 2001
36.10	? 3610

3.

ℓ	ml
1.550	1550
2.125	? 2125
3.300	? 3300
5.675	? 5675
8.080	? 8080

4.

m	mm
1.695	1695
1.990	? 1990
2.625	? 2625
5.000	? 5000
7.333	? 7333

5.

cm	m
105	1.05
230	? 2.30
475	? 4.75
709	? 7.09
965	? 9.65

6.

mm	m
35	0.035
75	? 0.075
175	? 0.175
300	? 0.300
2400	? 2.400

7.

ml	ℓ
60	0.060
95	? 0.095
305	? 0.305
500	? 0.500
7575	? 7.575

8.

mm	cm
35	3.5
60	? 6.0
128	? 12.8
410	? 41.0
809	? 80.9

In the tables above, you kept the zeros or added zeros (when there were no ones or no tenths).

Tables 9 to 12 have some disappearing zeros. Watch for them in the patterns as you copy and complete the tables.

9.

cm	mm
0.2	2
0.8	? 8
0.9	? 9
1.2	? 12
10.5	? 105

10.

m	cm
0.01	1
0.65	? 65
0.90	? 90
5.50	? 550
12.05	? 1205

11.

ℓ	ml
0.010	10
0.055	? 55
0.070	? 70
0.404	? 404
3.095	? 3095

12.

g	mg
0.003	3
0.007	? 7
0.070	? 70
0.190	? 190
6.705	? 6705

Did *you* discover when the zeros disappeared?

211

lesson Pages 211, 212

goal Practice in applying the decimal system to equivalent measurements

memo A pattern, started on page 210, is developed on these next two pages—sliding the decimal point. Some pupils will already have discovered this pattern; for those who haven't, let them read and work ahead. The "sliding decimal point" is treated more formally on page 213.

page 211 The first four exercises involve conversion from a larger unit to a smaller unit, while exercises 5 to 8 reverse the process. Stress the importance of zero as a place-holder, including the zero before the decimal point.

Exercises 9 to 12 involve dropping zeros in some instances. This should pose no problems for the pupils—they may drop extra zeros to the left of the decimal. However, zeros to the right of the decimal point **are** important: they indicate a degree of precision in measurement. But don't worry—these exercises have been "rigged" to avoid this issue; the topic is treated in more detail in the Level 6 text.

goal Practice in applying the decimal system to equivalent measurements

memo By now, most pupils will have caught on to the pattern, but extra practice is provided here. The idea of "sliding the decimal point" is treated on page 213.

page 212 Exercises 1 to 8 provide mixed practice for the pupils, in the familiar and easy tabular form.

Exercise 9 is trickier—here the pupils will have to stop and think each time, and remind themselves of the meanings of the prefixes and the various place-value positions.

The problems on page 211 gave you a chance to find patterns when you change measurements.
You were working with the smaller units of measurement.
You have probably made discoveries that will help in changing measurements given in larger units.

Copy and complete the tables. Watch for the pattern in each. (There may be some disappearing zeros!)

1.

km	m
1.500	1500
3.650	? 3650
7.885	? 7885
0.025	? 25
0.002	? 2

2.

kg	g
0.625	625
2.000	? 2000
0.400	? 400
1.055	? 1055
0.075	? 75

3.

kl	ℓ
1.200	1200
0.500	? 500
0.100	? 100
0.005	? 5
1.000	? 1000

4.

t	kg
1.010	1010
5.250	? 5250
0.800	? 800
3.500	? 3500
6.550	? 6550

5.

g	kg
25	0.025
250	? 0.250
100	? 0.100
1250	? 1.250
275	? 0.275

6.

m	km
450	0.450
25	? 0.025
625	? 0.625
2600	? 2.600
5	? 0.005

7.

ℓ	kl
1220	1.220
200	? 0.200
30	? 0.030
484	? 0.484
3908	? 3.908

8.

kg	t
5000	5.000
3000	? 3.000
303	? 0.303
6809	? 6.809
7075	? 7.075

Replace the ? by the correct numeral. (You will need to remember all the patterns.)

9. **a** 1.750 km = ? m 1750 **b** 205 m = ? km 0.205 **c** 350 g = ? kg 0.350 **d** 0.045 t = ? kg 45

e 1.209 kl = ? ℓ 1209 **f** 500 g = ? kg 0.500 **g** 175 cm = ? m 1.75 **h** 225 ml = ? ℓ 0.225

i 6.75 m = ? cm 675 **j** 0.350 km = ? m 350 **k** 4040 kg = ? t 4.040 **l** 3030 mg = ? g 3.030

THE SLIPPERY SLIDING DECIMAL POINT

Dennis Digit had problems with problems in changing measurements.
The decimal point always seemed to slip through his fingers.
His sister Dorothy showed him how to handle the slippery decimal point.
Perhaps you've already discovered the patterns Dennis didn't know.
Would you explain them the way Dorothy did?

OF COURSE YOU NEED TO KNOW YOUR MEASUREMENT UNITS.

You decide whether to multiply or divide, and by what number.

Multiply to change to a smaller unit. Divide to change to a larger unit.

MULTIPLY	*By 10*	24.5 cm = ? mm (245)	*By 100*	99.5 m = ? cm (9950)	*By 1000*	5.890 t = ? kg (5890)
DIVIDE	*By 10*	9 mm = ? cm (0.9)	*By 100*	405 cm = ? m (4.05)	*By 1000*	65 ℓ = ? kl (0.065)

Slide the decimal point: right, in multiplying; left, in dividing.

	One place for 10	*Two places for 100*	*Three places for 1000*
RIGHT	24.5 cm = 245 mm (The . isn't needed now.)	99.5 m = 9950 cm (You need to add 0.)	5.890 t = 5890 kg (0 was already there.)
LEFT	9 mm = 0.9 cm (0 is needed in the ones place.)	405 cm = 4.05 m	65 ℓ = 0.065 kl (0 is needed for ones and tenths.)

You don't need zero at the left of a whole number after you multiply.

0.8 cm = 8 mm   0.07 m = 7 cm   0.009 t = 9 kg

ARE YOU READY FOR THE PROBLEMS DENNIS LEARNED TO DO?

Try page 214.

lesson Pages 213, 214

goal Applying the decimal system to measurement problems

memo This page summarizes in a formal manner the pattern that has been developed in the preceding three pages. While the "sliding decimal" **is** one of the conveniences of our number and measurement system, it is still important that the pupils understand the relationships between prefixes and place value reinforced by pages 210 to 212.

page 213 Once the pupils' understanding is assured, the idea of the "sliding decimal point" is a simple one. Only two major decisions have to be made—do we move the decimal point to the left or to the right, and how many places do we move it?

Extra zeros to the left of the decimal point may be dropped, but at least one zero should be kept, if there is no whole number.

For pupils who fail to see the reason for this, remind them that there is a **big** difference between \$10 and \$0.10. If we write \$.10, it's very easy to miss that tiny decimal point!

goal More practice with the decimal system in measurement

memo This page of mixed practice reinforces the work of the previous four pages. Not all pupils will need the extra work, and some will be ready to tackle the problems on page 215.

page 214 The first exercise reinforces the pattern developed in the previous pages, and will be particulary useful for pupils who are still having some difficulty.

Exercises 2 to 9 provide mixed practice and involve conversions from larger to smaller and from smaller to larger units. Watch for those pupils who are still having trouble deciding when to add or drop zeros.

Copy and complete the tables.

1.

Problem	× or ÷	10, 100, 1000?	Right or left?	How many places?	Answer
0.095 km = ? m	×	1000	Right	3	95 m
35 mm = ? cm	÷	10	Left	1	3.5 cm
995 ℓ = ? kl	÷	1000	Left	3	0.995 kl
6.40 m = ? cm	x	100	Right	2	640 cm
8.775 t = ? kg	x	1000	Right	3	8775 kg
2425 ml = ? ℓ	÷	1000	Left	3	2.425 ℓ
35 cm = ? mm	x	10	Right	1	350 mm
16 g = ? mg	x	1000	Right	3	16 000 mg

2.

mm	cm
48	? 4.8
7	? 0.7
120 ?	12
558	? 55.8
25 ?	2.5

3.

cm	m
8.5 ?	0.085
90 ?	0.9
679	? 6.79
280 ?	2.8
5445	? 54.45

4.

m	km
1900 ?	1.9
78	? 0.078
495	? 0.495
8860 ?	8.86
9406 ?	9.406

5.

mg	g
95	? 0.09
750 ?	0.750
2708	? 2.70
65	? 0.06
3877 ?	3.877

6.

g	kg
5956	? 5.956
56	? 0.056
508 ?	0.508
384	? 0.384
1758 ?	1.758

7.

kg	t
2004 ?	2.004
522	? 0.522
7888	? 7.888
79 ?	0.079
5700 ?	5.7

8.

ml	ℓ
49 ?	0.049
4490 ?	4.49
799	? 0.799
4675	? 4.675
645 ?	0.645

9.

ℓ	kl
614	? 0.61
6982 ?	6.982
86	? 0.08
7790 ?	7.79
9673	? 9.67

Here are some word problems in which you have to change measurements.

A restaurant served 200 g steaks.
Six people at a table ordered steaks.
How many grams of meat was that? 1200 g
How many kilograms? 1.2 kg

2. Beth had a roll of ribbon 10 m long.
She cut the ribbon into 20 cm lengths.
How many lengths did she cut from 1 m? 5
How many from the roll? 50

3. There was a sale on toothpaste.
The shopper bought 10 150 ml tubes, 5 100 ml tubes, and 5 50 ml tubes. 1500 + 500 + 250 = 2250 ml
How many millilitres of toothpaste was that?
How many litres? 2.25 ℓ

A heavy rainfall may measure 50 mm.
A heavy snowfall may measure 30 cm.
What is the difference between the two in millimetres? in centimetres?
300 − 50 = 250 mm   30 − 5 = 25 cm

5. In 1972, a skater from the Netherlands set new speed records for 3000 m and 5000 m.
How many metres did the events total? 8000 m
How many kilometres? 8 km

6. The mass of a single telephone is about 2 kg.
The receiver alone has a mass of about 300 g.
What is the mass in grams of the other parts? 2000 − 300 = 1700 g

7. The mass of a carton of groceries was about 9 kg.
There were tinned foods and 5 packages of chips.
Each package of chips had a mass of 500 g.
How many grams of chips were there? 2500 g
About how many grams of tinned foods? 9000 − 2500 = 6500 g

lesson Page 215

goal Applications of decimal notation and measurement conversions

page 215 Most pupils will be able to handle the first six problems with little difficulty. The mathematics involved is not really new; it is similar to the money problems they have handled in the past. If necessary, have the pupils write number sentences to clarify each problem.

Problem 7 is optional and may be assigned to your faster pupils.

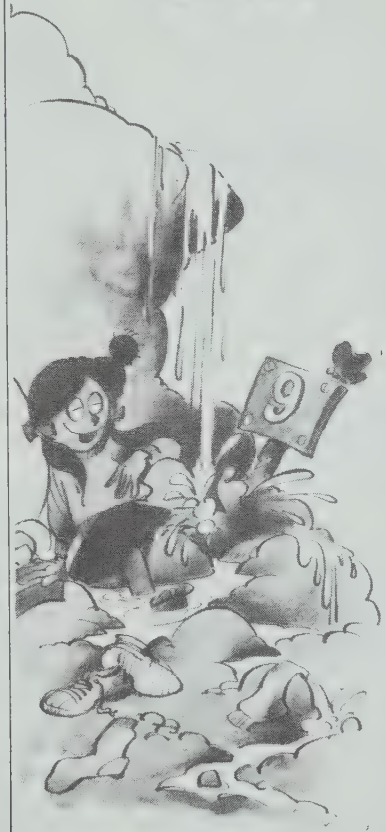
goal Checkout—relating symbols, prefixes, and decimal notation, and selecting appropriate units of measure

page 216 In exercise 1, don't look for mastery of the less common units. Not all students need to complete 1c and 1d, although most will find these fun. All pupils should be able to handle exercise 2 proficiently, and to make reasonable judgments for questions 3 to 6.

Those who make errors may need more practice with common symbols, common prefixes, or common conversions, or more real experience with metric measurements. This page will help you to identify the cause of their difficulties.

The Resource section (pages 216a-216d) provides additional practice and activities related to SI measurement.

CHECKOUT



1. a Give the place-value meaning of each prefix.
   kilo- (thousands) milli- (thousandths) deci- (tenths) deca- (tens) centi- (hundredths) hecto- (hundreds)

   b Write the word that each symbol stands for.
   dag (decagram) km (kilometre) t (tonne) hl (hectolitre) mg (milligram) dal (decalitre) cm (centimetre) ml (millilitre) mm (millimetre)

   c If we used metric prefixes with the word *dollar*, what word would be used to mean a thousand dollars? a hundred dollars? ten cents? one cent?
   kilodollar hectodollar decidollar centidollar

   d If the symbol for *dollar* were d, what would the symbol dad mean?
   decadollar (ten dollars)

2. Copy and complete the tables.

a

cm	m
39 ?	0.39
59	? 0.59
259	? 2.59
650 ?	6.50

b

mm	cm
6	? 0.6
9 ?	0.9
165 ?	16.5
398	? 39.8

c

g	kg
234	? 0.234
755 ?	0.755
1234	? 1.234
6905 ?	6.905

Tell which of the units of measure is probably the best one.

3. A friend of mine is 165 ? (centimetres or decimetres) tall.
4. A sign reads: "Speed limit, 90 ? (metres or kilometres) an hour."
5. A roll of transparent tape is about 19 ? (centimetres or millimetres) wide.
6. One of the events at the Winter Olympics was ski jumping from a 70 ? (metre or kilometre) hill.

216

See activity 7, page 216c.

See activity 8, page 216c.

RESOURCES

another form of evaluation

for progress check—page 203

1. Write the symbol for:
   a) fifteen centimetres 15 cm
   b) six kilograms 6 kg
   c) ninety-five metres 95 m
   d) eight kilolitres 8 kl
   e) six hundred twenty-five grams 625 g

2. Write the preferred unit measure for:
   a) 0.001 m mm b) 1000 g kg c) 0.001 g mg
   d) one hundredth of a metre 1 cm
   e) one thousand litres 1 kl

for checkout—page 216

1. Circle the best units of measure.
   John and his family are going camping on their holiday. They will travel 456 (m or km) to the campground. Their 6-man tent is 2 (cm or m) high and has a mass of 15 (mg or kg). Some of the food they'll take with them is 7 (g or kg) of meat, 5 (ml or ℓ) of milk, and 300 (g or kg) of marshmallows. While camping, John hopes to catch a fish at least 25 (cm or m) long.

2. Copy and complete the tables.

m	cm
0.92	92
0.78	78
3.20	320
4.95	495

cm	mm
1.9	19
6.3	63
29.6	296
26.7	267

ℓ	ml
0.225	225
0.750	750
1.225	1225
3.500	3500

kg	g
0.338	338
0.523	523
6.459	6459
7.288	7288

activities

1. things masking tape; metrestick

Independent activity for pairs of pupils (Provide each pair of pupils with the following directions.)

Follow the instructions below and complete the table.

1. Estimate each of the distances in column 1 by placing two pieces of masking tape that distance apart on a flat surface—desk, table, floor, wall, and so on.
2. Check your estimates by measuring to the nearest unit, using the unit listed in column 2.

	Estimated Distance	Measured Length
a)	1 cm	____ cm
b)	1 dm	____ cm
c)	1 m	____ m
d)	25 cm	____ cm
e)	Between 2 and 3 m and closer to 3 m	____ m
f)	5 m	____ m
g)	60 cm	____ cm
h)	6 dm	____ cm

Think

- Did you find some distances that could be used as guides for estimating distances in metric units? If so, what were they?
- What makes an estimation guide useful? (It must be easily available, easy to compute with, and have definite endpoints.)
- Describe any methods you used in estimating distances.

2. things centimetre rule, tape, or metrestick

Independent activity for pairs of pupils (Provide each pair of pupils with the following directions.)

Follow the instructions below and record your results in a table. Your table should include a description of each distance measured, your estimate of each distance, and the measured length of each distance.

1. Select six distances to measure in the vicinity of your classroom.
2. Estimate each distance, using a suitable unit of length.
3. Check the closeness of your estimates by measuring each distance with a centimetre rule, tape, or metrestick.
4. How did you select an appropriate unit of length for each distance?

3. Individual activity (Provide the pupil with copies of line segments—as many as you wish and any length you want). A five-cent coin is about 2 cm wide. Suppose you need to measure a length in centimetres and do not have a centimetre rule. An estimate could be obtained by "stepping off" the length in coin widths and converting the result to centimetres.

Measure each segment and several other distances in coin widths. Use 1 coin width ≈ 2 cm to contain your estimates ($\approx$ is read "about the same as"). Check the closeness of your estimates by measuring each distance to the nearest centimetre with a rule and computing the difference between your measurement and estimate. Record your results in the table.

Distance measured	Length in 5-cent widths	Estimate (cm)	Measured length (cm)	Difference
Segment *AB*				
Segment *CD*				
Segment *EF*				
Segment *GH*				
Width of desk				
Length of your foot				

Think

- Did the use of the coin widths result in a good estimate of the measured length?
- Could this method of estimation be useful in estimating longer distances such as the length of a classroom? Why or why not?
- What would be some advantages of using the width of a 5-cent coin as a guide for estimating short distances? What would be some disadvantages?

4. **things** dictionary

Individual activity (Provide the pupil with the following directions.)

The prefixes used in the common units of measurement are used with other units in the metric system, and with other words that do not belong to SI. For example, **hecto-** means 100 and a **hectograph** is a device for making many copies of a page.

Use a dictionary to find and write a definition for each word listed. How is the meaning of each word related to the prefix that is part of it? Did you find other words that use these prefixes? If so, share them with the rest of the class.

1. *centi*pede
2. *deca*gon
3. *deca*hedron
4. *Deca*logue
5. *deca*pod
6. *deca*syllable
7. *deca*thlon
8. *deci*bel
9. *hect*are
10. *kilo*coulomb
11. *kilo*cycle
12. *kilo*hertz
13. *kilo*joule
14. *mega*cycle
15. *mega*death
16. *mega*phone
17. *micro*be
18. *micro* film
19. *micro*phone
20. *micro*scrope
21. *milli*ampere
22. *milli*pede

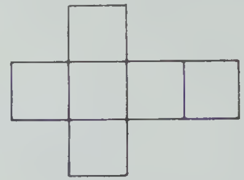
5. **things** objects of varying masses; scale

The idea of mass is not meaningful until the individual has developed a "feel" for various units of mass. This skill is acquired by the hands-on experience: holding an object in one hand and estimating its mass; then holding a known equivalent mass in the other hand and comparing the feel.

One error children frequently make is to assume that because something is larger, it has greater mass. If any of your pupils have no "sense" of mass, you will want to borrow a scale so that they can practise estimating the masses of all sorts of things and then verify their estimates.

6. **things** spirit master; transparent tape; scissors; 1-cm cubes

Enlarge and prepare a spirit master of this pattern for a cubic litre. The sides of each square should measure 10 cm. Have the youngsters cut out the pattern, fold along the lines, and tape the figure into shape.



If you have 1-cm cubes in your classroom, you can go one more step. Yes, a thousand 1-cm cubes will fit inside.

7. Individual activity (Provide the pupil with the following open sentences.)

Choose and circle the most appropriate unit to complete each sentence.

1. An economy passenger on an international air flight is allowed 20 (t or ht or kg) of baggage.
2. Niagara Falls is 50 (km or dam or m) high.
3. An egg is about 6 (dm or cm or mm) long.
4. The sizes of toothpaste tubes are 25, 50, 100, and 150 (dl or cl or ml).
5. A railway passenger coach has a mass of about 60 (t or kg or hg).
6. A giraffe measures about 5 (dam or m or dm) in height.
7. The usual postage stamp has a mass of about 20 (g or dg or mg).
8. Storage capacity in a refrigerator is often about 300 (kl or l or dl).
9. The measurement across a man's fist is about 10 (dm or cm or mm).
10. The fuel tank of a Volkswagen will hold about 45 (kl or dal or l).
11. A measurement of 1 (t or kg or hg) is fairly close for the mass of a Volkswagen.
12. Many small cars get about 12 (km or hm or m) on a litre of fuel.
13. A stick of gum is 20 (dm or cm or mm) wide.
14. A medium raisin has a mass of 1 (dag or g or dg).

8. Individual activity (Provide the pupil with the following open sentences.)

Select the most appropriate number to complete each sentence.

1. A new pencil measures about 19 cm in length.
   a) 19 b) 1.9 c) 0.19
2. The mass of a slice of bread is about 25.0 g.
   a) 250 b) 25.0 c) 2.50
3. A paper clip is about 32 mm in length.
   a) 32 b) 3.2 c) 0.32
4. The height of your school desk or table is about 70.0 cm.
   a) 700 b) 70.0 c) 7.00
5. A measurement of 250.00 kg is fairly close for the mass of a large motorcycle.
   a) 25 000 b) 2500.0 c) 250.00
6. The usual height of a bedroom door is 2.0 m .
   a) 20 b) 2.0 c) 0.20
7. The mass of a full-sized automobile is about 1800 kg .
   a) 1800 b) 180.0 c) 18.00

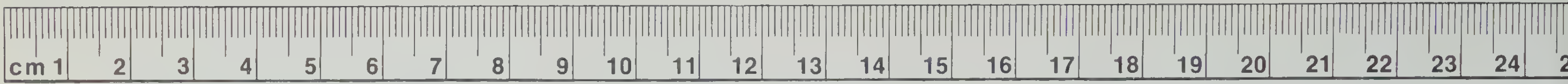
additional learning aids

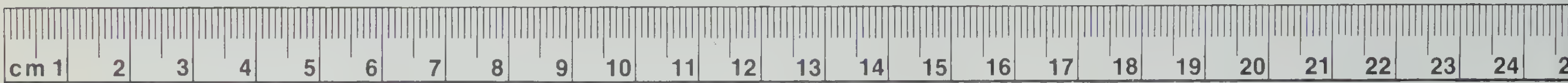
measurement – chapter objectives 1, 2, 3, 4, 5, 6, 7

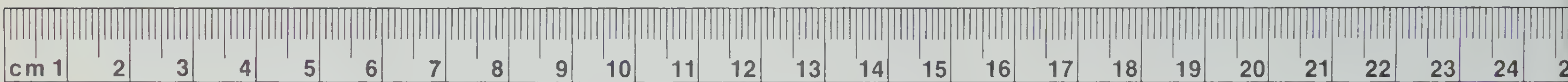
SRA products

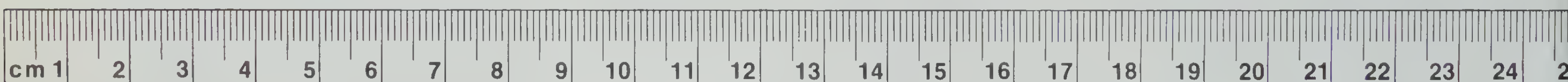
Mathematics Learning System, Activity Masters, level B, SRA (1974)
  Spirit masters: P-4, 5, 12, 13; M-1, 2, 5, 6
diagnosis: an instructional aid— Mathematics Level B, SRA (1972)
  Probes: M-25, 28
Math Applications Kit, SRA (1971)
  Appeteasers card: 7
  Science cards: 4, 5, 12, 20, 21, 28
  Sports and Games card: 27
  Occupations cards: 20, 38
  Everyday Things cards: 6, 9, 10, 11, 25, 28
Mathematics Involvement Program, SRA (1971)
  Cards: 325, 356
Skill through Patterns, level 5, SRA (1974)
  Spirit masters: 13, 56, 68, 69

other learning aids (described on page 216e)— The Fatal Foot, Good Time Mathematics, The Ghastly Gallon, Introducing the Metric System with Activities, Learning about Measurement, Metric Place Value Chart, The Perilous Pound


cm 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24


cm 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24


cm 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24


cm 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24


cm 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24

10 FRACTIONS

before this chapter the learner has—

1. Practiced renaming fractions
2. Mastered the addition and subtraction of fractions and mixed numbers with like denominators
3. Renamed appropriate fractions as whole or mixed numbers and renamed whole or mixed numbers as fractions
4. Had some experience with finding a common denominator for two fractions with unlike denominators
5. Explored the addition and subtraction of fractions and mixed numbers with unlike denominators
6. Explored the multiplication of a fraction and a whole number as well as two fractions

in chapter 10 the learner is—

1. Mastering finding a common denominator for two fractions
2. Mastering renaming appropriate fractions as mixed numbers and mixed numbers as fractions
3. Mastering the addition and subtraction of fractions and mixed numbers with unlike denominators
4. Mastering finding the product of two fractions or a fraction and a whole number

in later levels the learner will—

1. Master finding the product of two mixed numbers
2. Find the quotient for a division sentence having a fraction as the divisor

Notes & Things

This is it! This is the chapter on fractions that brings all the learner's work with fractions throughout the last four levels to a climax. The youngsters who make it through this chapter with flying colors will never need to worry about their future success with fractions. Those who have trouble will need a great deal of help, patience, and encouragement.

Renaming is the key. The learner should already know how to do the following renaming tasks:

- Use multiplication to find an equivalent fraction. For example:
  $$\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4} \text{ or } \frac{4}{5} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15}$$
- Use division to find an equivalent fraction. For example:
  $$\frac{3}{6} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2} \text{ or } \frac{14}{16} = \frac{14 \div 2}{16 \div 2} = \frac{7}{8}$$
- Rename fractions that name whole or mixed numbers. For example:
  $$\frac{5}{5} = 1,\ \frac{4}{3} = 1\frac{1}{3}$$
  And this one too: $\frac{8}{6} = 1\frac{2}{6} = 1\frac{1}{3}$

In this chapter, the learner will strive to master the last two big renaming jobs.

- Find a common denominator. Here is an example of one denominator being a factor of the second:
  $$\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4}$$
  This one will be new:
  $$\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12}$$
  And it will extend to include
  $$\frac{2}{3} + \frac{4}{5} \text{ as well as } \frac{5}{6} + \frac{3}{4}.$$
- Rename a mixed number as a fraction.
  For example: $1\frac{1}{2} = \frac{3}{2}$ or $2\frac{3}{5} = \frac{13}{5}$

The operations of addition, subtraction, and multiplication will again be covered. Both fractions and mixed numbers will be computed.

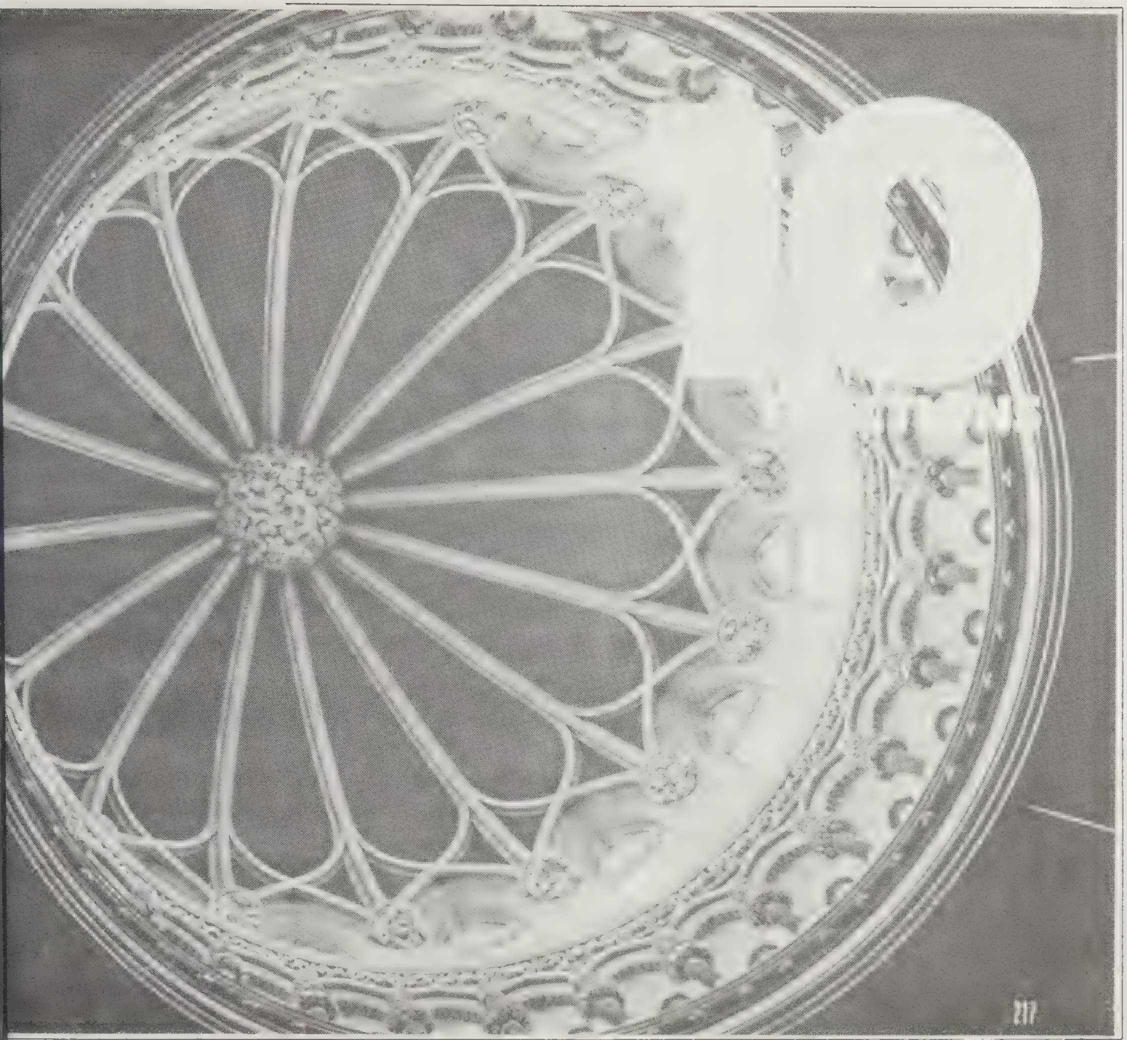
The chapter is carefully sequenced, and each set of problems is carefully controlled so that the learner will not be caught off guard. Please be especially watchful of individual progress in this chapter. Many pupils don't realize that they are off the track until after their faulty thinking has been reinforced by doing lots of problems the wrong way. Unlearning is a difficult job. Please consider enlisting the help of an able peer tutor at the earliest sign of trouble. This chapter can make or break a child's spirit as well as his skills.

things

fractional-part manipulatives (optional)
multiplication fact cards (optional)
(see page 288c)
multiplication drill kits (optional)

For the extra activities you will want to have these things available:

- muffin pan
- 6 small erasers
- paper bags
- 1-inch square cards



goal Talk about and explore ideas through a picture clue

page 217 Like the photograph on page 145, this photograph of the dome of the Library of Parliament in Ottawa can stimulate a discussion of the use of fractional parts in design. The discussion may deal with the number of equal parts in the dome (16) and the ways in which their separate identity is emphasized (for example, by windows). The discussion is unlikely to lead to considerations of structural strength, but pupils may be aware of the beauty of this design.

Direct the discussion to the classroom itself:

- The windows are what fractional part of the outer walls?
- The desks occupy what fractional part of the floor space?
- The boards take up what fractional part of the inner wall(s)?

Pupils may carry their observations on outside the classroom. In many modern buildings, the windows take ¾ of the side of the building. Columns may break up the front of some buildings into fractional parts.

lesson Page 218

goal **Survey** – renaming fractions; adding and subtracting fractions with like and unlike denominators; multiplying fractions; adding and subtracting decimals

memo Decimals are included in the survey to give the student a complete picture of the fraction skills he has developed thus far. The chapter will, however, focus on common fractions only.

page 218 Results from this page will help you identify the pupils who have retained these previously developed skills and those who need some review and practice. No need to take the time now to practice addition and subtraction with decimals. But do make a note of which pupils need further help in this area.

Everyone is not expected to compute the last four problems correctly. They identify the learning goal of the chapter.

Ready or not – it is fraction time again.

You have learned so much about fractions. It doesn't seem possible there is still more ground to cover.

You know how to –

Name equivalent fractions for $\frac{1}{2}$: $\frac{1 \times 2}{2 \times 2} = \underline{?}$ $\frac{2}{4}$ $\frac{1 \times 3}{2 \times 3} = ?$ $\frac{3}{6}$

Rename fractions so that they have their simplest name. $\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = ?$ $\frac{3}{4}$ $\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \underline{?}$ $\frac{2}{3}$

Add fractions with common denominators: $\frac{2}{5} + \frac{1}{5} = \frac{2+1}{5} = ?$ $\frac{3}{5}$

and mixed numbers too: $\begin{array}{r} 4\frac{4}{7} \\ +\ 2\frac{1}{7} \\ \hline \underline{?} \end{array}$ $6\frac{5}{7}$ $\begin{array}{r} 2\frac{3}{8} \\ +\ 1\frac{5}{8} \\ \hline \underline{?} \end{array}$ $3\frac{8}{8}$ or 4

and decimals: $\begin{array}{r} 25.67 \\ +\ \ 2.12 \\ \hline \underline{?} \end{array}$ 27.79 $\begin{array}{r} 3.06 \\ +\ 1.94 \\ \hline \underline{?} \end{array}$ 5.00

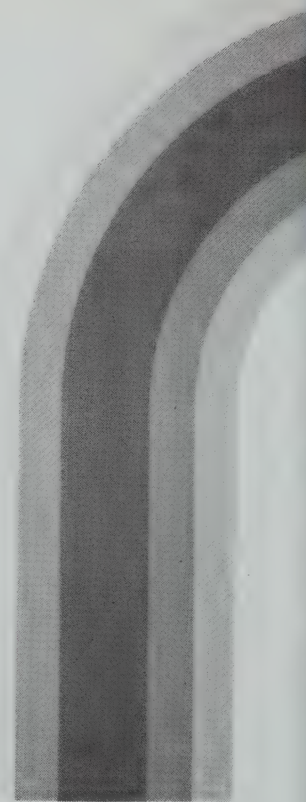
Subtract fractions with common denominators: $\frac{7}{8} - \frac{1}{8} = \frac{7-1}{8} = \underline{?}$ $\frac{6}{8}$ or $\frac{3}{4}$

and mixed numbers too. $\begin{array}{r} 5\frac{4}{5} \\ -\ 3\frac{2}{5} \\ \hline \underline{?} \end{array}$ $2\frac{2}{5}$ $\begin{array}{r} 10\frac{7}{8} \\ -\ \ 9\frac{7}{8} \\ \hline \underline{?} \end{array}$ $1(\frac{0}{8})$ 1 etc.

and decimals: $\begin{array}{r} 16.7 \\ -\ \ 5.6 \\ \hline \underline{?} \end{array}$ 11.1 $\begin{array}{r} 102.56 \\ -\ \ \ \ 7.69 \\ \hline \underline{?} \end{array}$ 94.87 etc.

You know a little about problems like this:

$\frac{7}{8} + \frac{1}{4} = \underline{?}$ $\frac{9}{8}$ $\frac{3}{4} - \frac{3}{5} = \underline{?}$ $\frac{3}{20}$ $4\frac{3}{4} + 1\frac{2}{3} = \underline{?}$ $6\frac{5}{12}$ $1\frac{3}{4} \times 2 = \underline{?}$ $3\frac{1}{2}$



Your goal is to know a lot about problems like that.

218

One of your last BIG problems with fractions was finding common denominators.

$\frac{5}{6} + \frac{1}{3} = ?$

A method of renaming sixths and thirds was explored.
You thought: Can $\frac{5}{6}$ be renamed as $\frac{?}{3}$? You said NO.
Then you thought: Can $\frac{1}{3}$ be renamed as $\frac{?}{6}$?
You said YES and you got the job done.
You found an equivalent fraction for $\frac{1}{3}$.

Focus just on the denominators 6 and 3.
You know that 3 is a factor of 6.
You know 6 is *not* a factor of 3; it is a multiple.
So save time. Look at the denominators only.

$\frac{1}{4} + \frac{1}{2} = ?$

Aha! 2 is a factor of 4. And of course 4 is a multiple of 2. Save time. You know you can find an equivalent for $\frac{1}{2}$. The equivalent must have the denominator 4.

$\frac{1}{3} + \frac{1}{9}$

Is 3 a factor of 9? Yes
Which fraction will be renamed? Do it. $\frac{1}{3}$ as $\frac{3}{9}$.
Rewrite the problem so that it has common denominators. $\frac{3}{9} + \frac{1}{9}$

Your turn. Rewrite each problem so that it has common denominators. Don't find the sum.

	a	b	c	d
1.	$\frac{1}{8} + \frac{1}{2}$ $\frac{1}{8} + \frac{4}{8}$	$\frac{1}{4} + \frac{1}{12}$ $\frac{3}{12} + \frac{1}{12}$	$\frac{1}{5} + \frac{1}{10}$ $\frac{2}{10} + \frac{1}{10}$	$\frac{1}{9} + \frac{1}{3}$ $\frac{1}{9} + \frac{3}{9}$
2.	$\frac{5}{6} + \frac{5}{12}$ $\frac{10}{12} + \frac{5}{12}$	$\frac{5}{6} + \frac{2}{3}$ $\frac{5}{6} + \frac{4}{6}$	$\frac{3}{4} + \frac{7}{8}$ $\frac{6}{8} + \frac{7}{8}$	$\frac{1}{3} + \frac{7}{12}$ $\frac{4}{12} + \frac{7}{12}$

219

lesson Pages 219, 220, 221

goal Development of a method for finding a common denominator

page 219 The focus is on finding a common denominator when one denominator is a multiple of the other denominator. One step at a time. Other possibilities will come up later.

The complete development of the method is on the page. Pupils who are not completely confident of their multiplication facts may be slow in catching on. You may want to point out that **multiple** is simply another name for **product.**

Warn everyone to read the directions for problems 1 and 2 carefully.

goal Practice in adding fractions with unlike denominators and renaming the sums when possible

page 220 The emphasis is on writing no more than is necessary to compute the problem correctly. The pupil's choice is his to make. Some will need to write very little; others will need to record every step.

No emphasis is placed on renaming the sums until after the computation is completed. Pupils are in the process of building skills.

You should be ready for practice.
Addition problems can be written like this: $\frac{1}{2} + \frac{3}{4}$

Rename $\frac{2}{4}$ ← That should become a **Think** step.

Rewrite $\frac{2}{4} + \frac{3}{4}$ or $\frac{2+3}{4} = \underline{?}$ $\frac{5}{4}$ or $1\frac{1}{4}$

Decide which of those forms helps you the most.
Save time. Don't write them both. Take your pick.

1. **a** $\frac{1}{4} + \frac{3}{8}$ Write the problem.
   Think the renaming.
   $\frac{2}{8} + \frac{3}{8}$ Then rewrite. Then add. $\frac{5}{8}$

   b $\frac{1}{12} + \frac{3}{4}$ Write the problem.
   Think the renaming.
   $\frac{?}{?} + \frac{?}{?}$ Then rewrite and add. $\frac{1}{12} + \frac{9}{12} = \frac{10}{12}$ or $\frac{5}{6}$

You are really on your own now. Find the sums. Renamed answers are for problem 4.

2. **a** $\frac{1}{2} + \frac{3}{8}$ $\frac{7}{8}$ **b** $\frac{3}{4} + \frac{1}{12}$ $\frac{10}{12}$; $\frac{5}{6}$ **c** $\frac{3}{10} + \frac{2}{5}$ $\frac{7}{10}$ **d** $\frac{1}{9} + \frac{2}{3}$ $\frac{7}{9}$ **e** $\frac{1}{6} + \frac{5}{12}$ $\frac{7}{12}$ **f** $\frac{1}{8} + \frac{3}{4}$ $\frac{7}{8}$

3. **a** $\frac{1}{2} + \frac{5}{6}$ $\frac{8}{6}$; $1\frac{1}{3}$ **b** $\frac{2}{3} + \frac{5}{6}$ $\frac{9}{6}$; $1\frac{1}{2}$ **c** $\frac{3}{4} + \frac{7}{8}$ $\frac{13}{8}$; $1\frac{5}{8}$ **d** $\frac{2}{3} + \frac{7}{12}$ $\frac{15}{12}$; $1\frac{1}{4}$ **e** $\frac{5}{8} + \frac{3}{4}$ $\frac{11}{8}$; $1\frac{3}{8}$ **f** $\frac{1}{2} + \frac{7}{10}$ $\frac{12}{10}$; $1\frac{1}{5}$

4. Look back. Find the sums that need to be renamed in simplest form. Get that job done now.

5. The sums in row three were all mixed numbers.
   Sometimes this renaming takes the longest time.
   Practice now on that skill.
   Rename these as mixed numbers.

   a $\frac{5}{4}$ $1\frac{1}{4}$ **b** $\frac{6}{4}$ $1\frac{1}{2}$ **c** $\frac{8}{4}$ 2 **d** $\frac{11}{4}$ $2\frac{3}{4}$ **e** $\frac{14}{4}$ $3\frac{1}{2}$ **f** $\frac{8}{6}$ $1\frac{1}{3}$ **g** $\frac{13}{6}$ $2\frac{1}{6}$ **h** $\frac{18}{6}$ 3

Try to find a shortcut for renaming fractions as mixed numbers.

$\frac{10}{3}$ can be renamed by thinking subtraction.

How many $\frac{3}{3}$s in $\frac{10}{3}$?

$\frac{10}{3} - \frac{3}{3} = \frac{7}{3}$ (1) $\qquad \frac{7}{3} - \frac{3}{3} = \frac{4}{3}$ (2) $\qquad \frac{4}{3} - \frac{3}{3} = \frac{1}{3}$ (3)

There are 3 and $\frac{1}{3}$ more, or $3\frac{1}{3}$.

Why not think division? $\frac{10}{3}$ is the same as $3\overline{)10}$.
How many threes in 10?
There are 3 threes and one part of three remaining.
Using numerals, that is $3\frac{1}{3}$.

Practice some more

Rename these fractions as mixed numbers in simplest form.

1. a $\frac{5}{2}$ $2\frac{1}{2}$ b $\frac{7}{3}$ $2\frac{1}{3}$ c $\frac{9}{4}$ $2\frac{1}{4}$ d $\frac{12}{5}$ $2\frac{2}{5}$

2. a $\frac{15}{6}$ $2\frac{1}{2}$ b $\frac{20}{7}$ $2\frac{6}{7}$ c $\frac{17}{8}$ $2\frac{1}{8}$ d $\frac{16}{9}$ $1\frac{7}{9}$

The next ones take more time.

3. a $\frac{10}{4}$ $2\frac{1}{2}$ b $\frac{14}{6}$ $2\frac{1}{3}$ c $\frac{18}{8}$ $2\frac{1}{4}$ d $\frac{22}{10}$ $2\frac{1}{5}$

4. a $\frac{9}{6}$ $1\frac{1}{2}$ b $\frac{12}{8}$ $1\frac{1}{2}$ c $\frac{15}{10}$ $1\frac{1}{2}$ d $\frac{30}{12}$ $2\frac{1}{2}$

Practice addition again.
Renamed answers only

5. a $\frac{1}{2} + \frac{3}{4}$ $1\frac{1}{4}$ b $\frac{5}{6} + \frac{5}{12}$ $1\frac{1}{4}$ c $\frac{5}{8} + \frac{3}{4}$ $1\frac{3}{8}$ d $\frac{2}{3} + \frac{5}{9}$ $1\frac{2}{9}$

goal Development of a shortcut for renaming fractions as mixed numbers

page 221 Some pupils will accept the use of division to rename a fraction as a mixed or whole number – no questions asked. This method is easier and shorter. Others will want proof. It's time for those manipulative fractional parts – 10 of size $\frac{1}{3}$. How many whole circular regions can be made? How much of another region is there also?

Youngsters generally do not need to practice renaming fractions as whole numbers. But you might want to challenge them with the following examples:

$$\frac{35}{5} \quad \frac{24}{6} \quad \frac{27}{4} \quad \frac{18}{3}$$

Have them look for a pattern. When is the fraction renamed as a mixed number? (When numerator **is not** a multiple of the denominator) When is it renamed as a whole number? (When numerator **is** a multiple of the denominator)

The problems in rows 3 and 4 require using two types of renaming:

- Renaming the fraction as a mixed number
- Renaming the fraction of the mixed number in simplest form

goal Adding mixed numbers with unlike denominators

page 222 The pupil should be able to add fractions having unlike denominators with a degree of confidence before extending this skill to mixed numbers. The whole numbers distract the youngster's attention from these prerequisite skills:

- Finding a common denominator
- Renaming with the common denominator

The vertical algorithm is not new. Pupils worked with it in a previous chapter.

Problem **1b** may need some special attention. Why was 6 chosen as the common denominator? Are there any other problems where one denominator is not a multiple of the other denominator? (**1c** and **1d**) What is the common denominator for problem **1c**? (20) for problem **1d**? (10)

You know this:

$\frac{1}{3} + \frac{5}{6}$

$\frac{2}{6} + \frac{5}{6}$ or $\frac{2+5}{6} = \frac{7}{6}$, or $1\frac{1}{6}$

You could have written it in vertical form like this:

$$\begin{array}{rcl} \frac{1}{3} & \longrightarrow & \frac{2}{6} \\ + \frac{5}{6} & \longrightarrow & \frac{5}{6} \\ \hline & & \frac{7}{6}, \text{ or } 1\frac{1}{6} \end{array}$$

Many people like the vertical form best. Do you? If so, use it. The vertical form is almost a must for mixed numbers.

$$\begin{array}{rcl} 4\frac{1}{3} & \xrightarrow{\textit{Rename}} & 4\frac{2}{6} \\ + 3\frac{5}{6} & \xrightarrow{\textit{Rewrite}} & 3\frac{5}{6} \quad \text{Now add.} \\ \hline & & 7\frac{7}{6} \end{array}$$

$\frac{7}{6} = 1\frac{1}{6}$

$7 + 1\frac{1}{6}$, or $8\frac{1}{6}$

Here's another example to study:

$$\begin{array}{rcl} 7\frac{2}{3} & \xrightarrow{\textit{Rename}} & 7\frac{8}{12} \\ + 3\frac{3}{4} & \xrightarrow{\textit{Rename}} & 3\frac{9}{12} \quad \text{Now add.} \\ \hline & & 10\frac{17}{12} \end{array}$$

$10 + 1\frac{5}{12}$, or $11\frac{5}{12}$

1. You try some.

a $2\frac{3}{8} \rightarrow 2\frac{?}{8}$ 3; $+ 1\frac{1}{4} \rightarrow 1\frac{?}{8}$ 2; $3\frac{5}{8}$

b $3\frac{1}{3} \rightarrow 3\frac{?}{6}$ 2; $+ 2\frac{1}{2} \rightarrow 2\frac{?}{6}$ 3; $5\frac{5}{6}$

c $3\frac{1}{4} \rightarrow 3\frac{?}{20}$ 5; $+ 2\frac{4}{5} \rightarrow 2\frac{?}{20}$ 16; $5\frac{21}{20}$ or $6\frac{1}{20}$

d $3\frac{1}{5} + 2\frac{1}{2}$; $5\frac{7}{10}$

e $6\frac{1}{2} + 2\frac{5}{8}$; $8\frac{9}{8}$ or $9\frac{1}{8}$

f $4\frac{1}{5} + 2\frac{3}{10}$; $6\frac{1}{2}$

222

Bill worked $1\frac{1}{4}$ hours on Monday and $1\frac{1}{2}$ hours on Friday. He had to know how much time he worked before he could get paid. He added.

$$\begin{array}{r} 1\frac{1}{4} \\ +\ 1\frac{2}{4} \\ \hline 2\frac{3}{4} \end{array}$$

← Where did the $\frac{2}{4}$ come from?

$\frac{2}{4}$ is another name for $\frac{1}{2}$.

Bill worked another $1\frac{1}{2}$ hours on Saturday. How much time now?

$$\begin{array}{r} 2\frac{3}{4} \\ +\ 1\frac{2}{4} \\ \hline 3\frac{5}{4} \end{array}$$

← Is $1\frac{2}{4}$ another name for $1\frac{1}{2}$? Yes

← Now what? You finish it. Rename; $4\frac{1}{4}$

Try this one on your own.

$$\begin{array}{r} 1\frac{3}{4} \\ +\ 1\frac{3}{8} \\ \hline ? \end{array}$$

← Which number must be renamed? $1\frac{3}{4}$

You need a common denominator.

$$\begin{array}{r} 1\frac{6}{8} \\ +\ 1\frac{3}{8} \\ \hline \blacksquare\frac{\blacksquare}{\blacksquare} \end{array}$$ $2\frac{9}{8}$

And you need to rename again. → $3\frac{1}{\blacksquare}$

1. Find a common denominator and then add.

a $\begin{array}{r} 1\frac{1}{2} \\ +\ 1\frac{1}{4} \\ \hline ? \end{array}$ *Rename* → $\begin{array}{r} 1\frac{2}{4} \\ +\ 1\frac{1}{4} \\ \hline 2\frac{3}{4} \end{array}$

b $\begin{array}{r} 1\frac{2}{3} \\ +\ 2\frac{1}{6} \\ \hline ? \end{array}$ *Rename* → $\begin{array}{r} 1\frac{?}{6} \\ +\ 2\frac{1}{6} \\ \hline 3\frac{5}{6} \end{array}$ 4

2. You're really on your own this time.

a $\begin{array}{r} 3\frac{3}{10} \\ +\ 1\frac{1}{5} \\ \hline \end{array}$ $4\frac{5}{10}$ or $4\frac{1}{2}$

b $\begin{array}{r} 2\frac{3}{4} \\ +\ 1\frac{1}{8} \\ \hline \end{array}$ $3\frac{7}{8}$

c $\begin{array}{r} 1\frac{1}{3} \\ +\ 2\frac{5}{6} \\ \hline \end{array}$ $3\frac{7}{6}$ or $4\frac{1}{6}$

d $\begin{array}{r} 2\frac{1}{3} \\ +\ 3\frac{5}{6} \\ \hline \end{array}$ $5\frac{7}{6}$ or $6\frac{1}{6}$

e $\begin{array}{r} 4\frac{5}{8} \\ +\ 2\frac{1}{4} \\ \hline \end{array}$ $6\frac{7}{8}$

223

goal Practice in adding mixed numbers with unlike denominators

memo Use this page only with pupils who need additional practice.

page 223 This page is designed for the youngster who needs additional guided help. He is led step by step right up to the last four problems—where he is expected to try on his own.

goal Subtracting mixed numbers that require renaming

page 224 That curve ball—having to rename **before** subtraction is possible—is back again. Much help is given on the page in an effort to avoid a common error:

$$\begin{array}{r} 3\frac{1}{3} \\ -1\frac{2}{3} \\ \hline \end{array} \longrightarrow \begin{array}{r} 3\frac{4}{3} \\ -1\frac{2}{3} \\ \hline \end{array}$$

The youngster actually has added 1 when making this renaming error.

Problems 7 through 10 will provide the opportunity for this error to be made:

$$\begin{array}{r} 6 \\ -2\frac{3}{4} \\ \hline 4\frac{3}{4} \end{array}$$

The fraction is simply brought down; then subtraction of whole numbers is done.

An easy way to operate. Unfortunately it does not yield the correct answer.

Are you ready? Remember that curve ball in the subtraction game. Start with problems that have common denominators.

Try this one.

$$\begin{array}{r} 3\frac{1}{3} \\ -1\frac{2}{3} \\ \hline \end{array}$$

Can you subtract the fractions? No

When this problem comes up in whole numbers, you rename—a ten as ten ones, for example. Can $3\frac{1}{3}$ be renamed? Rename one of the whole numbers as $\frac{3}{3}$. How many thirds then? Yes; 4

$$\begin{array}{r} 3\frac{1}{3} \\ -1\frac{2}{3} \\ \hline \end{array} \longrightarrow \begin{array}{r} 2\frac{1}{3} + \frac{3}{3} \\ -1\frac{2}{3} \\ \hline \end{array} \quad \textit{Rewrite} \quad \begin{array}{r} 2\frac{4}{3} \\ -1\frac{2}{3} \\ \hline 1\frac{2}{3} \end{array}$$

O.K. now Yes

Try these. You'll have to rename.

LOOK

1. $\begin{array}{r} 6\frac{1}{4} \\ -2\frac{3}{4} \\ \hline \end{array} \longrightarrow \begin{array}{r} 5\frac{5}{4} \\ -2\frac{3}{4} \\ \hline \end{array}$

NO! O.K. now. $3\frac{2}{4}$ or $3\frac{1}{2}$

2. $\begin{array}{r} 8\frac{2}{5} \\ -1\frac{4}{5} \\ \hline \end{array} \longrightarrow \begin{array}{r} 7\frac{7}{5} \\ -1\frac{4}{5} \\ \hline 6\frac{3}{5} \end{array}$

3. $\begin{array}{r} 3\frac{4}{7} \\ -1\frac{5}{7} \\ \hline 1\frac{6}{7} \end{array}$

4. $\begin{array}{r} 6\frac{1}{5} \\ -2\frac{4}{5} \\ \hline 3\frac{2}{5} \end{array}$

5. $\begin{array}{r} 8\frac{1}{4} \\ -2\frac{3}{4} \\ \hline \end{array}$ $5\frac{2}{4}$ or $5\frac{1}{2}$

6. $\begin{array}{r} 2\frac{1}{6} \\ -1\frac{5}{6} \\ \hline \end{array}$ $\frac{2}{6}$ or $\frac{1}{3}$

Ask about these if you have forgotten what to do.

7. $6 - 2\frac{3}{4}$ $3\frac{1}{4}$ **8.** $5 - 3\frac{2}{3}$ $1\frac{1}{3}$ **9.** $5 - 3\frac{1}{3}$ $1\frac{2}{3}$ **10.** $4 - 3\frac{4}{5}$ $\frac{1}{5}$

If you thought that last curve ball was a good one, just wait. This is the last pitch in the subtraction game. Don't worry. You will win the game.

$$\begin{array}{r} 7\frac{1}{2} \\ -\,3\frac{3}{4} \\ \hline \end{array} \xrightarrow{\textit{Rename}} \begin{array}{r} 7\frac{2}{4} \\ -\,3\frac{3}{4} \\ \hline \end{array} \xrightarrow{\textit{Rename again}} \begin{array}{r} 6\frac{6}{4} \\ -\,3\frac{3}{4} \\ \hline \end{array}$$

NO — NO, again! — At last! $3\frac{3}{4}$

First you had to rename to find a common denominator. Then you had to rename the fraction part of the mixed number so that you could subtract.

Take it one step at a time.

1. Find a common denominator and then subtract.

a $\begin{array}{r} 4\frac{2}{3} \\ -\,2\frac{1}{6} \\ \hline ? \end{array} \xrightarrow{\textit{Rename}} \begin{array}{r} 4\frac{4}{6} \\ -\,2\frac{1}{6} \\ \hline \end{array}$ You do it. $2\frac{3}{6}$ or $2\frac{1}{2}$

b $\begin{array}{r} 5\frac{3}{4} \\ -\,2\frac{1}{2} \\ \hline ? \end{array} \xrightarrow{\textit{Rename}} \begin{array}{r} 5\frac{3}{4} \\ -\,2\frac{2}{4} \\ \hline \end{array}$ You do it. $3\frac{1}{4}$

2. You're on your own. You will have to rename at least two fractions.

a $\begin{array}{r} 2\frac{1}{4} \\ -\,1\frac{2}{3} \\ \hline \frac{7}{12} \end{array}$ **b** $\begin{array}{r} 4\frac{3}{8} \\ -\,2\frac{3}{4} \\ \hline 1\frac{5}{8} \end{array}$ ***c** $\begin{array}{r} 1\frac{1}{12} \\ -\,\frac{3}{4} \\ \hline \frac{4}{12} \text{ or } \frac{1}{3} \end{array}$ ***d** $\begin{array}{r} 2\frac{1}{3} \\ -\,\frac{7}{10} \\ \hline 1\frac{19}{30} \end{array}$

***3.** You probably won't find many times in the real world when you need to do computation like this, except in measurement situations. What fractions might be used if you were computing with yards and fractional parts of a yard? Halves, thirds, fourths, sixths, ninths, twelfths

goal Subtracting mixed numbers with unlike denominators

page 225 Renaming all over the place! Patience and perseverance are necessary. Stress taking one step at a time. Careful of problem **2a** as well as **2d**. Reasoning that thirds and fourths must be renamed as twelfths is not nearly as difficult as reasoning that thirds and tenths must be renamed as thirtieths.

Talk about problem 3. Reasoning what fractional parts 36 inches could be divided into will help develop background for finding common denominators. Why not a fifth of a yard? or a seventh? or an eighth?

lesson Pages 226, 227

goal Finding a common denominator by the trial-and-error method

page 226 No fair providing your pupils with a quicker method right off the bat—trial-and-error first. This probing will help them see the advantages of an organized method. Read those directions carefully before beginning the problems in rows 1 and 2!

For your information only—these problems are rigged. For each problem, the lowest common denominator is found by simply multiplying the two denominators in the problem.

Here is a simple-looking problem: $\frac{1}{3} + \frac{1}{4}$ ← The troublemakers again.

Can $\frac{1}{3}$ be renamed as $\frac{?}{4}$? Can $\frac{1}{4}$ be renamed as $\frac{?}{3}$? No; no

It Can't Be Done! Or Can It?

Both fractions must be renamed.
Can both $\frac{1}{3}$ and $\frac{1}{4}$ be renamed as $\frac{?}{6}$? as $\frac{?}{8}$? as $\frac{?}{9}$? as $\frac{?}{12}$? No; no; no; yes

Here is the problem again: $\frac{1}{3} + \frac{1}{4} = \underline{?}$

Rename and *rewrite*. $\frac{4}{12} + \frac{3}{12} = \frac{?}{12}$ 7
Use the common denominator and add.

Look at another example: $\frac{2}{5} + \frac{1}{2} = \underline{?}$

Can both be renamed as $\frac{?}{2}$? as $\frac{?}{5}$? as $\frac{?}{10}$? No; no; yes

Here's the problem again: $\frac{2}{5} + \frac{1}{2} = \underline{?}$

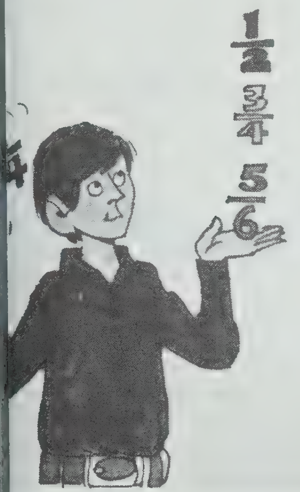
Rename and *rewrite*. $\frac{4}{10} + \frac{5}{10} = \underline{?}$ $\frac{9}{10}$
Use the common denominator and add.

Try some on your own. It won't hurt a bit. Go trial-and-error method.
Find the common denominator. Rewrite the problem. Do *not* find the sums.

	a	b	c	d
1.	$\frac{1}{4} + \frac{1}{5}$ $\frac{5}{20} + \frac{4}{20}$	$\frac{1}{3} + \frac{1}{2}$ $\frac{2}{6} + \frac{3}{6}$	$\frac{1}{2} + \frac{1}{7}$ $\frac{7}{14} + \frac{2}{14}$	$\frac{1}{3} + \frac{1}{5}$ $\frac{5}{15} + \frac{3}{15}$
2.	$\frac{1}{5} + \frac{1}{6}$ $\frac{6}{30} + \frac{5}{30}$	$\frac{1}{7} + \frac{1}{3}$ $\frac{3}{21} + \frac{7}{21}$	$\frac{1}{8} + \frac{1}{3}$ $\frac{3}{24} + \frac{8}{24}$	$\frac{1}{4} + \frac{1}{7}$ $\frac{7}{28} + \frac{4}{28}$

Maybe you had good luck on these. But you don't have to depend on luck.
Go on to the next page.

Let's Get Organized



$\frac{1}{4} + \frac{1}{7} = $? What's a common denominator? (There is more than one.) 28, 56, 84, . . .

$\frac{1}{4}$ can't be renamed $\frac{?}{7}$.
And $\frac{1}{7}$ can't be renamed $\frac{?}{4}$.

You can write equivalent fractions for each.

$\frac{1}{4} = \frac{2}{8} \quad \frac{3}{12} \quad \frac{4}{16} \quad \frac{5}{20} \quad \frac{6}{24} \quad \frac{7}{28}$

$\frac{1}{7} = \frac{2}{14} \quad \frac{3}{21} \quad \frac{4}{28}$ You can stop here.
Use 28 as the common denominator.

The problem: $\frac{1}{4} + \frac{1}{7}$

Rename and *rewrite*. $\frac{7}{28} + \frac{4}{28}$ And now you can find the sum. $\frac{11}{28}$

There is another way. Just multiply the denominators.

$\frac{1}{4} + \frac{1}{7}$ ($\times$) You know that 4 will be a factor of 28.
And 7 is a factor of 28.

1. Go back to the problems in rows 1 and 2 on the page before. Compare this method with the one you used. Did you come up with the same common denominator? Yes

2. Use any method you like to find common denominators. Add. Make sure each sum has its simplest name.

a $\frac{2}{8} + \frac{1}{2}$ $\frac{3}{4}$	**b** $\frac{3}{4} + \frac{1}{5}$ $\frac{19}{20}$	**c** $\frac{1}{4} + \frac{2}{3}$ $\frac{11}{12}$	**d** $\frac{4}{5} + \frac{1}{2}$ $1\frac{3}{10}$
e $\frac{5}{6} + \frac{1}{2}$ $1\frac{1}{3}$	**f** $\frac{2}{5} + \frac{1}{3}$ $\frac{11}{15}$	**g** $\frac{1}{2} + \frac{3}{7}$ $\frac{13}{14}$	**h** $\frac{4}{5} + \frac{2}{3}$ $1\frac{7}{15}$

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goal Development of organized methods for finding a common denominator

page 227 Two methods for finding a common denominator are presented. The first is time-consuming, but has some advantages.

- The renaming is already done.
- The pupil is more likely to find the least common denominator immediately.

With more experience, pupils tend to become selective and use this method only for pairs of denominators such as 6 and 9 or 4 and 6 – denominators that do not have the factor-multiple relationship but do have a common factor.

The second method will guarantee finding a common denominator immediately – although the denominator found may not be the lowest common denominator. When the denominators share no common factor, the second is the quicker method.

For the present, let the pupil use the method he prefers. He will be more selective when he is ready. What is important now is being able to use both methods correctly.

lesson Pages 228, 229

goal Practice in subtracting fractions with unlike denominators

page 228 The focus is on using the methods developed on the previous page to find a common denominator. The renaming steps are old stuff—no new skills. They're just all being put together.

The word problems should add a little reality. These crazy fractions that require so much work really are useful.

There are no new rules for subtraction.
Find the common denominator and **GO!**

	a		b		c		d	
1.	$\frac{3}{4} - \frac{1}{5}$	$\frac{11}{20}$	$\frac{2}{3} - \frac{1}{2}$	$\frac{1}{6}$	$\frac{4}{5} - \frac{1}{3}$	$\frac{7}{15}$	$\frac{5}{6} - \frac{1}{5}$	$\frac{19}{30}$
2.	$\frac{7}{8} - \frac{2}{3}$	$\frac{5}{24}$	$\frac{6}{7} - \frac{1}{4}$	$\frac{17}{28}$	$\frac{1}{2} - \frac{2}{7}$	$\frac{3}{14}$	$\frac{5}{6} - \frac{2}{5}$	$\frac{13}{30}$

Maybe word problems would be more fun. They will all require subtraction.

Tim the Timer went wild with the stopwatch.

3. He was supposed to time a runner for $1\frac{1}{3}$ laps of the track. He started talking and did not stop the watch until the runner had covered $1\frac{1}{2}$ laps. How much too far did the runner go? $\frac{1}{6}$ of a lap

4. Tim was supposed to time 3 horses as they crossed the finish line. He became excited and stopped the watch while the horses were still running. The first horse beat the third horse by $1\frac{1}{4}$ lengths. The second horse beat the third horse by $\frac{1}{2}$ a length. By how much did the first horse beat the second horse? $\frac{3}{4}$ of a length

5. Tim worked for $5\frac{1}{2}$ hours that day. He spent $2\frac{3}{4}$ hours at the track. How many hours did he spend timing other sports? $2\frac{3}{4}$ hours

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Choose a common denominator from the list.
Rename the fractions and compute.

a $\frac{1}{2} + \frac{1}{3}$ 3, 6, 9, 12 $\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

b $\frac{1}{5} + \frac{1}{2}$ 5, 10, 15, 20 $\frac{2}{10} + \frac{5}{10} = \frac{7}{10}$

c $\frac{3}{4} - \frac{1}{3}$ 4, 8, 12, 16 $\frac{9}{12} - \frac{4}{12} = \frac{5}{12}$

d $\frac{5}{6} - \frac{1}{4}$ 6, 12, 18, 24 $\frac{10}{12} - \frac{3}{12} = \frac{7}{12}$

e $\frac{2}{3} + \frac{5}{8}$ 8, 16, 24, 32 $\frac{16}{24} + \frac{15}{24} = \frac{31}{24}$ or $1\frac{7}{24}$

f $\frac{1}{3} - \frac{1}{4}$ 4, 8, 12, 16 $\frac{4}{12} - \frac{3}{12} = \frac{1}{12}$

g $\frac{3}{10} + \frac{3}{4}$ 10, 20, 30, 40 $\frac{6}{20} + \frac{15}{20} = \frac{21}{20}$ or $1\frac{1}{20}$

h $\frac{3}{4} - \frac{1}{5}$ 5, 10, 15, 20 $\frac{15}{20} - \frac{4}{20} = \frac{11}{20}$

i $\frac{5}{6} + \frac{2}{9}$ 9, 18, 27, 36 $\frac{15}{18} + \frac{4}{18} = \frac{19}{18} = 1\frac{1}{18}$

What is the source of the numbers on each list?
They are multiples of the greatest denominator of the fractions in each problem.

PROGRESS CHECK

Skill: Adding fractions and mixed numbers

Add. Rename the sum as a mixed number if you can.

1. $\frac{3}{5} + \frac{4}{5}$ $\left(\frac{7}{5}\right)$ $1\frac{2}{5}$
2. $\frac{1}{2} + \frac{5}{4}$ $\left(\frac{7}{4}\right)$ $1\frac{3}{4}$
3. $2\frac{1}{8} + 1\frac{1}{2}$ $3\frac{5}{8}$
4. $\frac{2}{5} + \frac{1}{2}$ $\frac{9}{10}$
5. $\frac{1}{4} + \frac{5}{6}$ $\left(\frac{13}{12}\right)$ $1\frac{1}{12}$
6. $3\frac{2}{3} + 1\frac{3}{4}$ $4\frac{17}{12}$ $5\frac{5}{12}$

Skill: Subtracting fractions and mixed numbers

Subtract.

7. $\frac{9}{10} - \frac{7}{10}$ $\left(\frac{2}{10}\right)$ $\frac{1}{5}$
8. $\frac{5}{6} - \frac{2}{3}$ $\frac{1}{6}$
9. $3\frac{1}{2} - 1\frac{1}{6}$ $2\frac{2}{6}$ $2\frac{1}{3}$
10. $\frac{4}{5} - \frac{2}{3}$ $\frac{2}{15}$
11. $\frac{5}{6} - \frac{2}{9}$ $\frac{11}{18}$

*12. $4\frac{1}{2} - 2\frac{1}{7}$ $2\frac{5}{14}$

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goal Practice in adding and subtracting fractions with unlike denominators; **Progress Check** – adding and subtracting fractions and mixed numbers with like and unlike denominators

page 229 You may want to go directly to the Progress Check with pupils who are operating confidently. If anyone's performance indicates more practice is needed, then use of the top half of the page is advisable.

It's O.K. if you use two days to complete the page. Take one for practice and one for the Progress Check.

Youngsters who can answer problem 2 at the top of the page will have a clue to another method for finding a common denominator. Simply name the multiples of the denominator having the greatest number and stop at the first multiple that is common to both denominators. There is no one right method to use. The emphasis should be on using the method that works best, and the method chosen may vary with the problem.

You will want to examine errors on the Progress Check to see if they are caused by –
- Finding a common denominator
- Renaming with the common denominator
- Computational error
- Renaming the answer

This diagnosis will help you determine the types of additional help that are necessary.

STOP

See activity 1, page 239a.

CHALLENGE No two denominators can be alike. Can you find the solution for this math sentence?

$\frac{1}{\square} + \frac{1}{\square} + \frac{1}{\square} = 1$ $\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1\right)$

lesson Page 230

goal Multiplying a fraction and a whole number

page 230 The earlier development of multiplication as repeated addition is reviewed here. Some pupils do not need to write all the steps. Those who are able to compute mentally should record only their products and then rename with a simpler name, if possible.

The questions in **1m** should initiate some discussion. How is it possible to multiply by 2 and have a product less than 1? Are these magic numbers? With youngsters who have trouble understanding, use the candy-bar approach—2 pieces, each the size of $\frac{1}{5}$ of a candy bar. That's how much of a candy bar?

You may want to add a bit of a challenge after checking answers. Ask in which problems the product is the same number as the numerator. Can anyone see a pattern? (Whole number the same as the denominator)

A

$\frac{3}{4}$ pie $\frac{3}{4}$ pie $\frac{3}{4}$ pie

How many pies in all? $\frac{9}{4}$ or $2\frac{1}{4}$

B.

$\frac{2}{3}$ page $\frac{2}{3}$ page $\frac{2}{3}$ page

How many pages in all? $\frac{6}{3}$ or 2

You have had enough work with addition for a while. Skip the drawings. You know you can multiply.

$$\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}$$
$$4 \times \frac{3}{5}$$

The easy part of multiplication is that you don't have to worry about a common denominator. Why not? You just multiply whole number and denominator's the

But you will still use your renaming skills. $4 \times \frac{3}{5} = \frac{4 \times 3}{5} = \frac{12}{5}$ or **?** $2\frac{2}{5}$

1. $4 \times \frac{3}{5}$ was an easy problem. Here are some more easy ones to do. Multiply and give the simplest name for the product.

a $2 \times \frac{1}{5}$ $\frac{2}{5}$ **b** $10 \times \frac{1}{6}$ $1\frac{2}{3}$ **c** $\frac{5}{6} \times 6$ 5 **d** $\frac{7}{8} \times 8$ 7 **e** $3 \times \frac{5}{6}$ $2\frac{1}{2}$ **f** $\frac{2}{3} \times 9$

g $8 \times \frac{3}{4}$ 6 **h** $6 \times \frac{1}{4}$ $1\frac{1}{2}$ **i** $\frac{4}{5} \times 4$ $3\frac{1}{5}$ **j** $3 \times \frac{3}{6}$ $1\frac{1}{2}$ **k** $5 \times \frac{5}{8}$ $3\frac{1}{8}$ **l** $10 \times \frac{7}{10}$

m Were all your products greater than 1? Can you find one example when you take a whole number times a fraction and the product is less than 1?
No; in the set above $2 \cdot \frac{1}{5}$; accept other good answers

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1. If you multiply two fractions that are less than 1, will the product be greater or less than 1? Let these problems help you answer.

   a $\frac{2}{3} \times \frac{1}{6} = \frac{2 \times 1}{3 \times 6} = \frac{2}{18}$ This can be renamed. Do it. The product of this problem is less than 1. $\frac{1}{9}$

   b $\frac{3}{4} \times \frac{1}{3} = \frac{3 \times 1}{4 \times 3} = \frac{3}{12}$ Rename the product. This product is also less than 1. $\frac{1}{4}$

   c $\frac{9}{10} \times \frac{9}{10} = \frac{9 \times 9}{10 \times 10} = \frac{81}{100}$ That product cannot be renamed. But it is still less than 1.

2. Keep on. Multiply. Write the simplest name for each product.

   a $\frac{1}{4} \times \frac{1}{2}$ $\frac{1}{8}$ b $\frac{1}{3} \times \frac{2}{5}$ $\frac{2}{15}$ c $\frac{2}{5} \times \frac{1}{6}$ $\frac{1}{15}$ d $\frac{5}{6} \times \frac{2}{5}$ $\frac{1}{3}$

   e $\frac{3}{4} \times \frac{2}{3}$ $\frac{1}{2}$ f $\frac{4}{5} \times \frac{4}{5}$ $\frac{16}{25}$ g $\frac{0}{2} \times \frac{1}{2}$ 0 h $\frac{5}{6} \times \frac{4}{5}$ $\frac{2}{3}$

   i Why will the product of two fractions that are less than 1 also be less than 1?
   This is the same as taking part of a part of a unit.

3. Do you think you have to learn something new to complete a problem such as $1\frac{1}{2} \times \frac{1}{4}$? No

   Try the old rules.
   Rename $1\frac{1}{2}$ as a fraction; then multiply. $\frac{3}{2} \times \frac{1}{4} = \frac{3 \times 1}{2 \times 4} = \underline{?}$ $\frac{3}{8}$

4. Review renaming mixed numbers.

   a $3\frac{1}{3}$ $\frac{10}{3}$ b $2\frac{3}{4}$ $\frac{11}{4}$ c $4\frac{2}{5}$ $\frac{22}{5}$ d $6\frac{1}{6}$ $\frac{37}{6}$ e $1\frac{7}{10}$ $\frac{17}{10}$ f $5\frac{1}{2}$ $\frac{11}{2}$

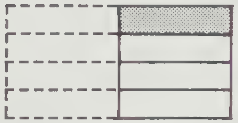
lesson Pages 231, 232, 233, 234

goal Multiplying fractions and mixed numbers

page 231 Multiplying two fractions, each less than one, is the first skill discussed and practiced. The size of the product is emphasized to help the learner look for signals that indicate an error in his computation. Knowing that both factors are less than 1, he should estimate a product that is less than 1. Encourage mental computation.

You may need to make a region diagram to help some youngsters understand question **2i.**

$\frac{1}{4} \times \frac{1}{2}$



We are looking for a part of a part, $\frac{1}{4}$ of $\frac{1}{2}$

Multiplying a mixed number and a fraction is the focus of the remainder of the page. Renaming mixed numbers is a prerequisite skill. Encourage each pupil to compute this renaming step mentally. Row 4 will help you identify who needs the extra help and practice that will be given on page 232.

goal Examining two methods for renaming mixed numbers as fractions

memo Pupils who are renaming mixed numbers with confidence and accuracy can skip this page.

page 232 The page is designed to provide help for the youngsters who are insecure in renaming mixed numbers. Many of them will be able to proceed with no additional help from you.

No fair explaining the shortcut. Let the pupil figure it out and explain it to you. Anyone who cannot figure it out should not be using it.

Encourage performing the computations that result in renaming mentally and recording only the fraction. This renaming step should always be done mentally when used for a problem.

Here's some help if you are having a hard time with renaming.

Renaming a mixed number is not hard.
You know that $\frac{3}{2} = 1\frac{1}{2}$, so you should also know that $1\frac{1}{2} = \frac{3}{2}$.

$\frac{3}{2} = \frac{2}{2} \quad + \quad \frac{1}{2} \longrightarrow 1 \quad + \quad \frac{1}{2}$, or $1\frac{1}{2}$

Look at another.

$3\frac{1}{2}$ **Think** How many $\frac{1}{2}$s in 1? 2
How many $\frac{1}{2}$s in 2? 4
How many $\frac{1}{2}$s in 3? 6
How many $\frac{1}{2}$s in $3\frac{1}{2}$? 7

$2\frac{2}{3}$ **Think** How many $\frac{1}{3}$s in 1? 3
How many $\frac{1}{3}$s in 2? 6
How many $\frac{1}{3}$s in $2\frac{2}{3}$? 8

Here's a shortcut. If you can figure it out, you can use it.

$1\frac{3}{4} = \frac{7}{4}$ (+, ×)

$2\frac{3}{8} = \frac{19}{8}$ (+, ×)

Stick with the top method if this doesn't make sense.

1. Practice these.

a $2\frac{1}{3}$ $\frac{7}{3}$ b $1\frac{1}{6}$ $\frac{7}{6}$ c $3\frac{7}{8}$ $\frac{31}{8}$ d $6\frac{7}{10}$ $\frac{67}{10}$ e $2\frac{1}{2}$ $\frac{5}{2}$ f $1\frac{3}{4}$ $\frac{7}{4}$

g $1\frac{5}{6}$ $\frac{11}{6}$ h $3\frac{1}{6}$ $\frac{19}{6}$ i $2\frac{3}{10}$ $\frac{23}{10}$ j $3\frac{3}{8}$ $\frac{27}{8}$ k $2\frac{2}{3}$ $\frac{8}{3}$ l $2\frac{3}{5}$ $\frac{13}{5}$

1. Finish these multiplication problems.

a $\frac{3}{4} \times 1\frac{1}{2}$

Rename $\frac{3}{2}$

Rewrite $\frac{3}{4} \times \frac{3}{2} = \frac{3 \times 3}{4 \times 2} = \frac{9}{8}$ Rename again. That's your job. $1\frac{1}{8}$

b $3\frac{1}{8} \times \frac{3}{5}$

Rename $\frac{25}{8}$

Rewrite $\frac{25}{8} \times \frac{3}{5} = \frac{25 \times 3}{8 \times 5} = \frac{75}{40}$

$\frac{40}{40} = 1$, $\frac{80}{40} = 2$ Nope! So $\frac{75}{40} = 1\frac{35}{40}$ or ? $1\frac{7}{8}$

c $1\frac{1}{3} \times \frac{5}{6}$

Rename $\frac{4}{3}$

Rewrite $\frac{4}{3} \times \frac{5}{6} = \frac{4 \times 5}{3 \times 6} =$? Your turn to finish. $\frac{20}{18}$ or $1\frac{1}{9}$

You are on your own.

d $\frac{1}{4} \times 5\frac{1}{2}$ $1\frac{3}{8}$ **e** $2\frac{3}{4} \times \frac{1}{6}$ $\frac{11}{24}$ **f** $\frac{1}{2} \times 3\frac{1}{2}$ $1\frac{3}{4}$ **g** $2\frac{2}{5} \times \frac{5}{6}$ 2

h $2\frac{2}{3} \times \frac{3}{8}$ 1 **i** $\frac{1}{7} \times 1\frac{2}{5}$ $\frac{1}{5}$ **j** $1\frac{1}{2} \times \frac{2}{3}$ 1 **k** $\frac{3}{4} \times 2\frac{1}{2}$ $1\frac{7}{8}$

2. Be brave. Try this one on your own. $2\frac{1}{2} \times 1\frac{2}{3}$ $4\frac{1}{6}$

goal Multiplying a fraction and a mixed number

page 233 Problem 1 sequences the learner through each step of the algorithm. The models are there for additional help with the remaining problems.

Types of errors to watch for:

- Renaming the mixed number
- Careless multiplication
- Careless division when renaming the product

Just in case someone is beginning to complain, problem 2 will let him know that things could get worse—two mixed numbers to rename rather than just one.

goal Multiplying with fractions, whole numbers, and mixed numbers as factors

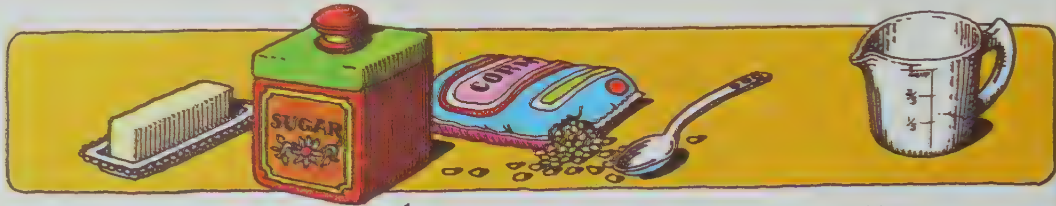
page 234 The focus is on multiplying with at least one mixed number as a factor. Long algorithms become tiresome. Encourage those who are able to rename mentally to do so and cut down on paper-and-pencil work. Those who are unsure should continue to write all the steps.

Pupils should be nearing independence with the skills practiced in this chapter. They therefore should not be required to write both the equation form and the computational form for each problem.

$$\frac{1}{3} \times \frac{7}{4} = \frac{1 \times 7}{3 \times 4}$$

equation form   computational form

Encourage each one to choose the form that he feels most **comfortable** with and has the most **success** with.



The recipe for caramel corn calls for $1\frac{1}{3}$ cups of sugar. How much sugar would be needed to make 2 times as much?

$2 \times 1\frac{1}{3}$

Rename $\frac{4}{3}$

Rewrite $2 \times \frac{4}{3} = \frac{2 \times 4}{3} = \frac{8}{3}$ or $2\frac{2}{3}$

You will be doing a lot of renaming.

Look at some more.

$\frac{1}{3} \times 1\frac{3}{4}$

Rename $\frac{7}{4}$

Rewrite $\frac{1}{3} \times \frac{7}{4}$

You might want to write it as $\frac{1 \times 7}{3 \times 4}$.
Anyway, you are ready to multiply. $\frac{7}{12}$

1. Finish these multiplication problems.

a $\frac{1}{4} \times 1\frac{3}{4}$

Rename $\frac{7}{4}$

Rewrite $\frac{1}{4} \times \frac{7}{4}$ or $\frac{1 \times 7}{4 \times 4} = \underline{?}$ $\frac{7}{16}$

b $5\frac{1}{2} \times \frac{1}{3}$

Rename $\frac{11}{2}$

Rewrite $\frac{11}{2} \times \frac{1}{3}$ or $\frac{11 \times 1}{2 \times 3} = \underline{?}$ $\frac{11}{6}$ or $1\frac{5}{6}$

You can make the rename step and the rewrite step just one step.

c $1\frac{5}{8} \times \frac{1}{4}$

Rename and rewrite $\frac{13}{8} \times \frac{1}{4}$ or $\frac{13 \times 1}{8 \times 4} = \underline{?}$ $\frac{13}{32}$

d $\frac{3}{5} \times 1\frac{1}{8}$

Rename and rewrite $\frac{3}{5} \times \frac{9}{8}$ or $\frac{3 \times 9}{5 \times 8} = \underline{?}$ $\frac{27}{40}$

e $1\frac{3}{4} \times \frac{9}{8}$ or $\frac{7 \times 9}{4 \times 8} = \underline{?}$ $\frac{63}{32}$ or $1\frac{31}{32}$

Don't write both of these.
Decide which form helps you most. Use that one.

The next problems are rigged. Think about the problems within a set. How are they alike? Multiplication
How are they different? Set 1 is fraction times fraction or whole number and set 2 is mixed number times whole number.

SET 1

	a	b	c	d	e
1.	$\frac{1}{2} \times \frac{1}{2}$ $\frac{1}{4}$	$\frac{1}{2} \times 1$ $\frac{1}{2}$	$\frac{1}{2} \times \frac{3}{2}$ $\frac{3}{4}$	$\frac{1}{2} \times 2$ 1	$\frac{1}{2} \times \frac{5}{2}$ $1\frac{1}{4}$
2.	$4 \times \frac{1}{5}$ $\frac{4}{5}$	$4 \times \frac{2}{5}$ $1\frac{3}{5}$	$4 \times \frac{3}{5}$ $2\frac{2}{5}$	$4 \times \frac{4}{5}$ $3\frac{1}{5}$	$4 \times \frac{5}{5}$ 4
3.	$3 \times \frac{1}{2}$ $1\frac{1}{2}$	$3 \times \frac{1}{3}$ 1	$3 \times \frac{1}{4}$ $\frac{3}{4}$	$3 \times \frac{1}{5}$ $\frac{3}{5}$	$3 \times \frac{1}{6}$ $\frac{1}{2}$

SET 2

	a	b	c	d	e
1.	$1\frac{1}{2} \times 2$ 3	$1\frac{1}{2} \times 3$ $4\frac{1}{2}$	$1\frac{1}{2} \times 4$ 6	$1\frac{1}{2} \times 5$ $7\frac{1}{2}$	$1\frac{1}{2} \times 6$ 9
2.	$1\frac{1}{7} \times 5$ $5\frac{5}{7}$	$1\frac{2}{7} \times 5$ $6\frac{3}{7}$	$1\frac{3}{7} \times 5$ $7\frac{1}{7}$	$1\frac{4}{7} \times 5$ $7\frac{6}{7}$	$1\frac{5}{7} \times 5$ $8\frac{4}{7}$

Only for the very brave.

	a	b	c	d	e
*3.	$1\frac{1}{10} \times 10$ 11	$1\frac{1}{10} \times 20$ 22	$1\frac{1}{10} \times 100$ 110	$1\frac{1}{10} \times 200$ 220	$1\frac{1}{10} \times 1000$ 1100

PROGRESS CHECK

Skill: Multiplying whole and mixed numbers

You're on your own. Make sure every product is in simplest form.

1. $2 \times 1\frac{2}{3}$ $3\frac{1}{3}$
2. $2\frac{1}{4} \times 5$ $11\frac{1}{4}$
3. $3\frac{3}{4} \times 3$ $11\frac{1}{4}$
4. $1\frac{1}{3} \times 3$ 4
5. $1\frac{1}{2} \times 10$ 15

STOP See activity 2, page 239a.

See activity 3, page 239a.

lesson Page 235

goal Practice in multiplying fractions, whole numbers, and mixed numbers; **Progress Check**—multiplying whole and mixed numbers

page 235 There's lots of practice provided on this page. Consider either using more than one day or being selective in making an assignment. Not much is accomplished once fatigue sets in.

You may want to hold the Progress Check for another day. Look for three common errors:
- Renaming the mixed number
- Multiplication computation
- Renaming the product

If multiplication facts are the problem, get out those flash cards and any drill kits you may have.

Have the pupil who makes renaming errors explain the steps to you orally. Try to determine where his thinking is faulty before providing more practice.

lesson Pages 236, 237

goal Development of a shortcut for renaming products

page 236 Just as there is "more than one way to skin a cat," there is more than one way to think about a problem in a real-world situation. The long algorithm presented will always work. But let's try simplifying it as much as possible. Is the numerator of the product greater than the denominator? Then divide the numerator by the denominator immediately.

Are all problems that multiply a whole number and a mixed number that bad? Look at some real-life situations.

He had 10 bricks. Each was $6\frac{1}{2}$ inches long. He wanted to put them around the edge of ground he dug up for plants. He wondered if he had enough. About how long would the bricks be if they were in 1 line?

$$10 \times 6\frac{1}{2} = 10 \times \frac{13}{2} = \frac{10 \times 13}{2} = \frac{130}{2}$$

How much is 130 halves?

THINK 2 halves = 1 You know this.
3 halves = $1\frac{1}{2}$ $2\overline{)3}$
10 halves = 5 $2\overline{)10}$
100 halves = 50 $2\overline{)100}$
110 halves = $2\overline{)110}$ or 55
120 halves = $2\overline{)120}$ or 60
130 halves = $2\overline{)130}$ or 65 inches

GOOD GRIEF! How can you take a shorter way?
Look at the product $\frac{130}{2}$

THINK $2\overline{)130}$ right away

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Try another problem.

She needed to put wood trim around the sides of a small square table. Each side of the table measured $14\frac{3}{4}$ inches. About how much trim would she need?

$$4 \times 14\frac{3}{4} = 4 \times \frac{59}{4} = \frac{4 \times 59}{4}$$

HOLD EVERYTHING!

This is a real-world problem. You wouldn't do this computation. You would estimate. $14\frac{3}{4}$ inches is about 15 inches. $4 \times 15 = 60$. She needs about 60 inches.

It is possible for you to find a problem like $34 \times 25\frac{1}{2}$. It is possible to do it in the same way you have done the other multiplication problems.
But look at another way:

34	
$\times\ 25\frac{1}{2}$	
17	$\frac{1}{2} \times 34$
170	5×34
680	20×34
867	

he way the problem is ritten makes sense hen you have such large umbers. Try just a ouple of them.

1.

	56	
	$\times\ 45\frac{1}{2}$	
28	▮▮	$\frac{1}{2} \times 56$
280	▮▮▮	5×56
2240	▮▮▮▮	40×56
2548	?	

2.

	48	
	$\times\ 32\frac{1}{2}$	
24	▮▮	$\frac{1}{2} \times 48$
96	▮▮	2×48
1440	▮▮▮▮	30×48
1560	?	

3.

	63	
	$\times\ 25\frac{1}{3}$	
21	▮▮	$\frac{1}{3} \times 63$
315	▮▮▮	5×63
1260	▮▮▮▮	20×63
1596	?	

goal Development of an algorithm for multiplying 2-digit whole and mixed numbers

page 237 When is an estimate a good enough answer? When is an exact answer necessary? This approach has been used before in estimation work. You may want to practice estimating with a few additional examples.

The example in the middle of the page uses the computational form of the distributive property of multiplication over addition. This vocabulary need not be pointed out to the pupils unless you feel it is necessary. The ability to use the property to simplify computation is far more important than technical language. This algorithm is used for the remaining problems.

lesson Page 238

goal Multiplying two mixed numbers

page 238 The ability to rename mixed numbers as fractions is an absolute prerequisite skill for this page.

A little competition might increase motivation. Consider turning the practice in sets 2 and 3 into a contest. Each pupil picks two problems from each row. When finished, he exchanges his paper with another pupil to check. For each error the checker finds and corrects, he gets 1 point to add to his own number correct. The papers are returned to their owners. If the owner disagrees with a correction and proves that he is right, he gets 2 points to add to his own number correct. He also gets 2 points if he finds an error on his own paper that the checker missed.

Here is a strategy that could be explained before the contest begins—

- There is an advantage to picking the most difficult problems in hopes that you can beat the checker.
- Direct sabotage can help increase your score. A clever, deliberate error that a checker might miss will yield 2 points. Remember—the owner gets to find his own errors too.

There is only one more type of problem to look at.
Grit your teeth and get ready.

1. It is $3\frac{2}{3}$ miles from Gary's house to the baseball field. He walked to the field and halfway back. (He got a ride the rest of the way home.) How far did he walk?

The math sentence for this problem is $1\frac{1}{2} \times 3\frac{2}{3} = \underline{?}$

You have been lucky so far. All you have had to do is rename and multiply. Do you suppose it will work again?

try it

$1\frac{1}{2} \times 3\frac{2}{3}$

Rename $\frac{3}{2} \times \frac{11}{3}$ or $\frac{3 \times 11}{2 \times 3} = \frac{33}{6} = \underline{?}$ $5\frac{1}{2}$

That wasn't so bad after all.

2. Practice on these.

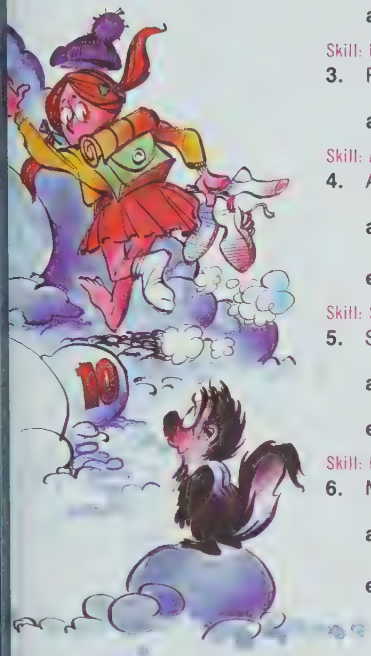
a $1\frac{1}{2} \times 1\frac{1}{2}$ $2\frac{1}{4}$ **b** $1\frac{1}{5} \times 1\frac{1}{2}$ $1\frac{4}{5}$ **c** $2\frac{1}{2} \times 1\frac{1}{3}$ $3\frac{1}{3}$ **d** $2\frac{2}{3} \times 2\frac{1}{4}$ 6 **e** $3\frac{3}{4} \times 1\frac{3}{8}$ $5\frac{5}{32}$

f $4\frac{1}{2} \times 3\frac{1}{3}$ 15 **g** $3\frac{1}{3} \times 1\frac{1}{3}$ $4\frac{4}{9}$ **h** $4\frac{1}{6} \times 3\frac{3}{5}$ 15 **i** $2\frac{1}{2} \times 1\frac{3}{5}$ 4 **j** $1\frac{1}{10} \times 1\frac{1}{2}$ $1\frac{13}{20}$

3. Multiply each number by $1\frac{1}{4}$.

a $\frac{2}{3}$ $\frac{5}{6}$ **b** $\frac{3}{4}$ $\frac{15}{16}$ **c** $\frac{4}{5}$ 1 **d** $\frac{5}{6}$ $1\frac{1}{24}$ **e** $1\frac{1}{4}$ $1\frac{9}{16}$ **f** $1\frac{1}{5}$ $1\frac{1}{2}$ **g** $3\frac{1}{2}$ $4\frac{3}{8}$ **h** $2\frac{2}{3}$

CHECKOUT



Skill: Renaming fractions with simplest names

1. Write simplest names for these fractions.

   a $\frac{12}{15}$ $\frac{4}{5}$ b $\frac{8}{14}$ $\frac{4}{7}$ c $\frac{6}{9}$ $\frac{2}{3}$ d $\frac{10}{12}$ $\frac{5}{6}$ e $\frac{10}{25}$ $\frac{2}{5}$ f $\frac{12}{16}$ $\frac{3}{4}$

Skill: Renaming fractions as mixed numbers

2. Rename each fraction as a mixed number. (Simplest form, please.)

   a $\frac{3}{2}$ $1\frac{1}{2}$ b $\frac{6}{4}$ $1\frac{1}{2}$ c $\frac{10}{4}$ $2\frac{1}{2}$ d $\frac{12}{9}$ $1\frac{1}{3}$ e $\frac{18}{8}$ $2\frac{1}{4}$ f $\frac{16}{6}$ $2\frac{2}{3}$

Skill: Renaming mixed numbers as fractions

3. Rename each mixed number as a fraction.

   a $2\frac{2}{3}$ $\frac{8}{3}$ b $1\frac{7}{8}$ $\frac{15}{8}$ c $3\frac{3}{4}$ $\frac{15}{4}$ d $2\frac{2}{5}$ $\frac{12}{5}$ e $4\frac{1}{2}$ $\frac{9}{2}$ f $1\frac{10}{11}$ $\frac{21}{11}$

Skill: Adding fractions (like and unlike denominators)

4. Add. Make sure each sum has its simplest name.

   a $\frac{2}{7}+\frac{8}{7}$ $1\frac{3}{7}$ b $\frac{7}{8}+\frac{5}{8}$ $1\frac{1}{2}$ c $\frac{4}{9}+\frac{1}{3}$ $\frac{7}{9}$ d $\frac{1}{2}+\frac{5}{8}$ $1\frac{1}{8}$

   e $\frac{3}{4}+\frac{1}{3}$ $1\frac{1}{12}$ f $\frac{1}{6}+\frac{1}{10}$ $\frac{4}{15}$ g $2\frac{1}{4}+1\frac{1}{5}$ $3\frac{9}{20}$ h $3\frac{1}{4}+1\frac{1}{3}$ $4\frac{7}{12}$

Skill: Subtracting fractions (like and unlike denominators)

5. Subtract.

   a $\frac{5}{6}-\frac{1}{6}$ $\frac{2}{3}$ b $\frac{5}{8}-\frac{3}{8}$ $\frac{1}{4}$ c $\frac{3}{4}-\frac{1}{8}$ $\frac{5}{8}$ d $\frac{7}{9}-\frac{2}{3}$ $\frac{1}{9}$

   e $\frac{4}{5}-\frac{1}{2}$ $\frac{3}{10}$ f $\frac{3}{4}-\frac{1}{3}$ $\frac{5}{12}$ g $4\frac{3}{5}-1\frac{1}{5}$ $3\frac{2}{5}$ h $3\frac{1}{4}-1\frac{3}{4}$ $1\frac{1}{2}$

Skill: Multiplying fractions

6. Multiply.

   a $3\times\frac{1}{2}$ $1\frac{1}{2}$ b $\frac{2}{5}\times 4$ $1\frac{3}{5}$ c $9\times\frac{1}{3}$ 3 d $\frac{2}{3}\times 6$ 4

   e $\frac{3}{4}\times\frac{2}{3}$ $\frac{1}{2}$ f $\frac{1}{2}\times\frac{4}{5}$ $\frac{2}{5}$ g $\frac{3}{8}\times 1\frac{1}{4}$ $\frac{15}{32}$ h $2\frac{5}{6}\times 1\frac{3}{4}$ $4\frac{23}{24}$

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STOP See activity 4, page 239b.

See activity 5, page 239b.

lesson Page 239

goal **Checkout** – renaming fractions and mixed numbers; adding and subtracting fractions with like and unlike fractions; multiplying fractions, whole numbers, and mixed numbers

page 239 Consider spreading this Checkout over two days. Fractions are not easy for many students. Numerous steps are involved. Careless errors can result from fatigue.

Be sure to check on the type of error made before providing any additional practice. If the error is not obvious, ask the youngster to tell you how to work the problem. Listen for faulty thinking or an error in learning. Then provide additional help. First the pupil must become aware of where he is making his mistake.

RESOURCES

another form of evaluation

for progress check–page 229

Add. Rename the sum if you can.

1. $\frac{7}{8}+\frac{3}{8}\left(\frac{10}{8}\right)\ 1\frac{1}{4}$ 2. $\frac{2}{3}+\frac{5}{6}\ \left(\frac{9}{6}\right)\ 1\frac{1}{2}$ 3. $1\frac{1}{4}+2\frac{3}{8}\ \ 3\frac{5}{8}$
4. $\frac{1}{3}+\frac{4}{5}\ \left(\frac{17}{15}\right)\ 1\frac{2}{15}$ 5. $\frac{1}{4}+\frac{2}{3}\ \ \frac{11}{12}$ 6. $3\frac{2}{5}+1\frac{1}{2}\ \ 4\frac{9}{10}$

Subtract.

7. $\frac{7}{9}-\frac{4}{9}\ \left(\frac{3}{9}\right)\ \frac{1}{3}$ 8. $\frac{11}{12}-\frac{1}{6}\ \left(\frac{9}{12}\right)\ \frac{3}{4}$ 9. $4\frac{9}{10}-2\frac{2}{5}\ \ 2\frac{1}{2}$
10. $\frac{8}{9}-\frac{5}{6}\ \ \frac{1}{18}$ 11. $\frac{5}{6}-\frac{1}{4}\ \ \frac{7}{12}$ 12. $3\frac{3}{4}-2\frac{1}{3}\ \ 1\frac{5}{12}$

for progress check–page 235

Multiply. Make sure every product is in simplest form.

1. $4 \times 2\frac{1}{3}\ \ 9\frac{1}{3}$ 2. $1\frac{5}{6} \times 2\ \ 3\frac{2}{3}$ 3. $1\frac{1}{2} \times 5\ \ 7\frac{1}{2}$
4. $3 \times 2\frac{2}{7}\ \ 6\frac{6}{7}$ 5. $1\frac{3}{8} \times 2\ \ 2\frac{3}{4}$ 6. $5 \times 1\frac{1}{5}\ \ 6$

for checkout–page 239

1. Write simplest names for these fractions.
a) $\frac{14}{16}\ \frac{7}{8}$ b) $\frac{8}{10}\ \frac{4}{5}$ c) $\frac{3}{9}\ \frac{1}{3}$
d) $\frac{25}{30}\ \frac{5}{6}$ e) $\frac{10}{18}\ \frac{5}{9}$ f) $\frac{8}{12}\ \frac{2}{3}$

2. Rename each fraction as a mixed number. (Simplest form, please.)
a) $\frac{5}{2}\ 2\frac{1}{2}$ b) $\frac{14}{8}\ 1\frac{3}{4}$ c) $\frac{12}{4}\ 3$
d) $\frac{4}{3}\ 1\frac{1}{3}$ e) $\frac{10}{6}\ 1\frac{2}{3}$ f) $\frac{15}{9}\ 1\frac{2}{3}$

3. Rename each mixed number as a fraction.
a) $3\frac{1}{2}\ \frac{7}{2}$ b) $1\frac{6}{7}\ \frac{13}{7}$ c) $2\frac{3}{4}\ \frac{11}{4}$
d) $1\frac{5}{6}\ \frac{11}{6}$ e) $2\frac{5}{8}\ \frac{21}{8}$ f) $3\frac{2}{9}\ \frac{29}{9}$

4. Add. Make sure each sum has its simplest name.
a) $\frac{4}{7}+\frac{5}{7}\ 1\frac{2}{7}$ b) $\frac{5}{9}+\frac{7}{9}\ 1\frac{1}{3}$ c) $\frac{7}{8}+\frac{2}{4}\ 1\frac{3}{8}$
d) $\frac{1}{3}+\frac{5}{12}\ \frac{3}{4}$ e) $\frac{1}{4}+\frac{3}{5}\ \frac{17}{20}$ f) $\frac{7}{9}+\frac{1}{2}\ 1\frac{5}{18}$
g) $1\frac{1}{3}+2\frac{1}{5}\ 3\frac{8}{15}$ h) $3\frac{3}{4}+1\frac{1}{6}\ 4\frac{11}{12}$

5. Subtract.
a) $\frac{5}{9}-\frac{2}{9}\ \frac{1}{3}$ b) $\frac{11}{12}-\frac{5}{12}\ \frac{1}{2}$ c) $\frac{5}{6}-\frac{1}{2}\ \frac{1}{3}$
d) $\frac{17}{20}-\frac{3}{5}\ \frac{1}{4}$ e) $\frac{7}{8}-\frac{2}{3}\ \frac{5}{24}$ f) $\frac{7}{10}-\frac{1}{4}\ \frac{9}{20}$
g) $6\frac{6}{7}-3\frac{3}{7}\ 3\frac{3}{7}$ h) $4\frac{3}{8}-2\frac{5}{8}\ 1\frac{3}{4}$

6. Multiply.
a) $2 \times \frac{2}{7}\ \frac{4}{7}$ b) $3 \times \frac{1}{4}\ \frac{3}{4}$ c) $6 \times \frac{2}{3}\ 4$
d) $\frac{4}{5} \times 3\ 2\frac{2}{5}$ e) $\frac{2}{5}\times\frac{3}{4}\ \frac{3}{10}$ f) $\frac{3}{8}\times\frac{1}{3}\ \frac{1}{8}$
g) $2\frac{1}{2}\times\frac{5}{6}\ 2\frac{1}{12}$ h) $1\frac{1}{4}\times 2\frac{1}{6}\ 2\frac{17}{24}$

activities

1. A modification in algorithm may help some youngsters. Insert a line so that fractions and whole numbers can be handled separately.

$$\begin{array}{r} 2\frac{1}{8} \\ +1\frac{1}{2} \\ \hline \end{array} \quad \begin{array}{r|l} 2 & \frac{1}{8}=\frac{1}{8} \\ +1 & \frac{1}{2}=\frac{4}{8} \\ \hline 3 & \quad\ \frac{5}{8} \end{array} \qquad\qquad \begin{array}{r} 3\frac{1}{2} \\ -1\frac{1}{6} \\ \hline \end{array} \quad \begin{array}{r|l} 3 & \frac{1}{2}=\frac{3}{6} \\ -1 & \frac{1}{6}=\frac{1}{6} \\ \hline 2 & \quad\ \frac{2}{6}=\frac{1}{3} \end{array}$$

This algorithm will help stress operating on fractions first.

2. things 24 index cards cut in half

Have the youngsters write one fraction in the upper left-hand corner of each card as shown.

$\frac{3}{4}$	$\frac{6}{8}$	$\frac{9}{12}$	$\frac{12}{16}$	$\frac{1}{4}$	$\frac{1}{8}$
$\frac{3}{12}$	$\frac{4}{16}$	$\frac{1}{2}$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{4}{8}$
$\frac{1}{3}$	$\frac{2}{6}$	$\frac{3}{9}$	$\frac{4}{12}$	$\frac{2}{3}$	$\frac{4}{6}$
$\frac{6}{9}$	$\frac{8}{12}$	$1\frac{1}{2}$	$1\frac{2}{4}$	$\frac{3}{2}$	$\frac{6}{4}$
$1\frac{1}{4}$	$1\frac{3}{12}$	$\frac{5}{4}$	$\frac{10}{8}$	$1\frac{2}{3}$	$1\frac{4}{6}$
$\frac{5}{3}$	$\frac{10}{6}$	$2\frac{1}{2}$	$2\frac{5}{10}$	$\frac{5}{2}$	$\frac{10}{4}$
$1\frac{1}{3}$	$1\frac{3}{7}$	$\frac{4}{3}$	$\frac{8}{6}$	$\frac{2}{1}$	$\frac{4}{2}$
$\frac{6}{3}$	$\frac{10}{2}$	$\frac{10}{4}$	$\frac{10}{6}$	$\frac{10}{8}$	$\frac{8}{6}$

Rules:
- Two to four players can play with one set of cards.
- Deal 7 cards to each player.
- The remaining cards are placed in a stack facedown.
- The first player draws a card from the pile and discards a card, placing it faceup to form a discard stack.
- Each following player draws one card, choosing from either stack.

The goal is to form sets of 2, 3, or 4 cards that name the same fraction.
Scoring:
- 1 point for each set of 2 cards laid down
- 5 points for each set of 3 cards laid down
- 8 points for each set of 4 cards laid down
- 2 bonus points for the simplest name
- 1 bonus point for being the first player to lay down all of his cards

3. things muffin pan; 6 small erasers; paper bag; felt pen

With the felt pen, write a whole number on each eraser and a mixed number in each cup of the muffin pan. The erasers are mixed in

the paper bag. The muffin pan is placed on a table about 3 feet from the edge. Or it may be placed on the floor and a line drawn about 3 feet from it.

The first player draws an eraser from the bag, stands at the table edge or back of the line, and tosses the eraser into the pan. To earn a point, the player must give the correct product for the number on the eraser and the number in the cup in which the eraser lands. Players predetermine how many points are needed to win.

Additional rules can be predetermined by the players. For example:

- For each correct product, the player can select another eraser from the bag and take another turn.
- Limit for any player at one time is three turns.
- Answer in simplest form earns a bonus.

4. things file folder; 1-inch square cards

On the file folder, make a 5-by-5 array of 1-inch squares. Write a fraction in the squares as indicated below. Four squares will be left blank. A fraction or mixed number is written on each square card. Make sure the denominators are compatible with those used throughout the chapter. These cards are mixed and placed in a stack.

$\frac{3}{4}$	$3\frac{1}{3}$	$\frac{1}{2}$	$\frac{3}{2}$	$1\frac{1}{6}$
$1\frac{5}{6}$		$3\frac{1}{4}$		$\frac{7}{6}$
$2\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{1}{3}$	$3\frac{3}{4}$
$\frac{1}{4}$		$\frac{1}{6}$		$\frac{4}{4}$
$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{2}$	$1\frac{1}{2}$	$\frac{7}{4}$

To begin the game, a player draws a card and places it faceup in any empty square on the game board. He then adds, subtracts, or multiplies the number on the card and the number in each square that the card touches. Points are earned by a predetermined scale. For example:

- Correct sum – 1 point
- Correct difference – 2 points
- Correct product – 3 points
- Answer in simplest form – 1 bonus point

Answers can be given on another part of the folder, making this an independent activity. The folder is used to store the cards needed for the game.

5. Divide the players into 2 teams. The teams huddle to make up 5 questions for each of the following categories: renaming fractions, addition of fractions, subtraction of fractions, multiplication of fractions. A quizmaster and a scorekeeper are chosen. Point values for each category of questions are decided. Each team chooses a captain.

To begin the game, the quizmaster asks the first player of one team to choose a category. The captain of the second team then asks a question in that category from his team's list. Answers should be computed mentally. The scorekeeper tallies the points (if any). The quizmaster asks the first player of the second team to choose a category and play continues. Teams predetermine how to decide the winning team.

additional learning aids

notation – chapter objectives 1, 2

SRA products

Mathematics Learning System, Activity Masters, level B, SRA (1974)
Spirit masters: F-11, 16

Computapes, SRA (1972)
Module 5: Lesson FR 10

Computational Skill Development Kit, SRA (1965)
Diagnostic test: 5
Use of Fraction cards: 1–16

Skill through Patterns, SRA *level 5*, (1974)
Spirit master: 60

other learning aids (described on page 288g) – Chip Trading, Experiments in Fractions, The Fat Fraction Game, Fraction Bars Student Activity Book (level II), Fraction Dominoes

operation – chapter objectives 3, 4

SRA products

Mathematics Learning System, Activity Masters, level B, SRA (1974)
Spirit masters: F-6, 8, 11, 12, 14, 15

Computapes, SRA (1972)
Module 5, Lessons: FR 15, 16, 19

Computational Skill Development Kit, SRA (1965)
Fraction addition cards: 5–16
Fraction subtraction cards: 3–9
Diagnostic Test: 6

Cross-Number Puzzles (Fractions), SRA (1967)
Addition cards: 6–18
Subtraction cards: 3–15
Multiplication cards: 1–12

diagnosis: an instructional aid – Mathematics Level B, SRA (1972)
Probes: M-5, 6, 7

Skill through Patterns, level 5, SRA (1974)
Spirit masters: 62, 66, 67, 70, 71

other learning aids – Fraction Multifax, Mathimagination (book D-Fractions)

11 FINDING, ORGANIZING, AND REPORTING INFORMATION

before this chapter the learner has—

1. Mastered preparing a tally chart to organize data
2. Experienced reading bar graphs

in chapter 11 the learner is—

1. Examining the reliability of claims and statements
2. Preparing graphs to report data
3. Examining a variety of graphs for completeness and to extrapolate information
4. Mastering computing the mean for data
5. Examining charts as a method for reporting data
6. Reading maps, schedules, and timetables
7. Examining the likelihood of an outcome of a probability experiment
8. Making predictions

in later chapters the learner will—

1. Master reporting data in a chart, table, and bar graph
2. Express the probability of the outcomes of a probability experiment

Notes & Things

This chapter's title, "Finding, Organizing, and Reporting Information," is truly descriptive. This is a good chapter for experimenting with large and small groups of pupils. A child at any ability level will have something to offer during this study.

This chapter can be made even more meaningful and full of enriching experiences if you can collect a few things in advance.

Keep your eyes open for colorful graphs in magazines and newspapers. Have just one or two on hand. This will get the youngsters in the collecting mood.

And gather up your vacation file and share it with the children. Find all the different kinds of maps you can.

Any time schedules around? They don't have to be up to date. Maybe the teacher next door can help you.

Lay the groundwork to borrow the telephone book from the office for one day. Does the principal have a school district boundary map you can borrow? Does the Chamber of Commerce have any free maps of your area? This chapter presents all sorts of charts, graphs, and maps. You can make it even more real world with resources from your own community.

By means of activities, the learner is led to question the reliability of information. How many trials or how great a sample is necessary to make a reliable prediction? He is led to observe that the greater the number of trials, the more accurate his prediction.

Statistical techniques for tallying data, constructing a graph, and finding the mean and the range are developed—all with little emphasis on vocabulary and independent mastery.

The learner will use actual charts, graphs, and maps to extrapolate information. The focus is learning by doing; the computation involved seems secondary. The activities build readiness for more technical work in later levels. A primary objective is to develop the ability to think critically.

things

newspapers and magazines
graph paper
rulers or tape measures
35 cubes
jelly beans (or substitute objects) of 2 colors
paper bags
coins - 1¢, 5¢, 10¢
six-sided pencils
checkers (or disks) of 2 colors
plastic spoons

For the extra activities you will want to have these things available:

city map
financial pages of a newspaper
overhead projector transparency film
spirit masters
World Almanac
province map
circle compass

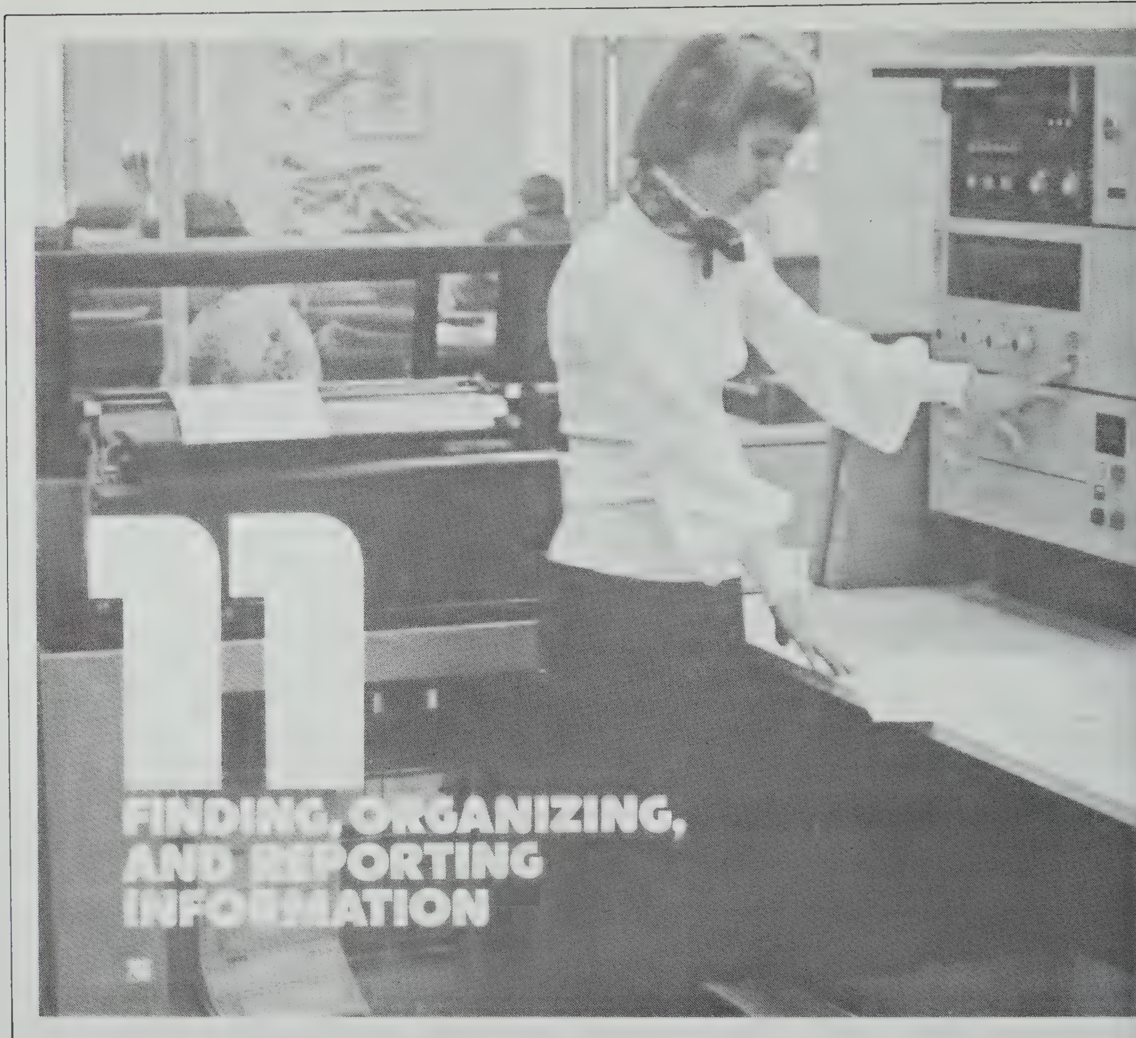
goal Think about and explore ideas through a picture clue

page 240 The computer is the great modern storehouse and organizer of information. If a visit to a computer is possible, this will certainly enrich the work of this chapter. Pupils should appreciate that someone has to collect the information that is fed to the computer. People have to know how to organize it so that it tells what they need to know. Much of the work for this chapter is collecting data, organizing it into reports, and learning to read such forms of presenting information as charts and graphs.

Testing the reliability of data is also important. With the flood of advertisements to which we are all exposed, a good way to start is by examining their claims with objectivity. Each student should be able to bring at least one advertisement from a magazine or newspaper to class. Try to find some that make some sort of claim. Then start analyzing them. What do they say when you read them quickly? What do the words **really** say?

What claims do the pupils remember being stated on radio or TV? Write down the claims as they are remembered. Listen for them again to hear the exact words said. Are there any differences in what seemed to be claimed and the exact words?

The youngsters should really enjoy keeping this an ongoing assignment.



A RECENT POLL SHOWS MUSIC IS A POPULAR STUDY.

YOU GET TWICE THE SHINE WITH WAX-O-WOW.

COMPUTERS ARE THE GREATEST INVENTION OF ALL TIME.

'CANADIAN CAKE' IS THE BEST RECORD BY THE BALLPOINT BANANAS.

Every day you hear claims like those above. Some are true. Some are completely false. Some are partly true — they begin with facts, but the claims go beyond the facts.

All claims should be based on facts. Today we know more facts about our world than ever before. We have so much more information that people talk about the "knowledge explosion." Much of the information is stored in computers, machines that can find, organize, and report facts.

You don't want to depend on reports from people or machines. *You* want to find the facts, make the decisions, and THINK for yourself.

YOUR
GOAL
is to learn more about
— finding and collecting information
— organizing the information you find
— reporting what your information shows

1. Suppose someone says, "Music is the most popular subject at our school." He should have some **data** (facts) to show that his claim is true.

a Suppose there are 300 pupils at your school. Suppose you ask ten of them and each says, "I like Music best." Would you agree that Music is the most popular subject at your school?
Probably not — 10 people out of 300 may not show what most people think

b What if six say they like Music best and four say they dislike it? Still the same problem — too few people

c Would you have to ask all three hundred to find out whether Music was most popular? No

d What would be a reasonable number to ask?
Answers will vary An appropriate range: 30 to 100 persons

241

lesson Pages 241, 242

goal Critical examination of various claims and statements, and of the importance of data

memo Work pages 241 and 242 together.

page 241 There's much to discuss and perhaps argue about. Points to consider while discussing the claims and statements given are—
- Can the claim or statement be tested?
- How many tries are necessary to give an adequate picture of the outcomes? The results of a test or survey are known as DATA. This may be a new word for some youngsters.

Your students may want to try a survey of their own. Perhaps they will choose to find what are the most popular subjects at school. Or they may choose some quite different topic. Such a project requires planning and organization.

- Who will survey? Who will record?
- What is a reasonable number of students to survey?
- Should everyone in the class be asked?
- Should students from other classes be asked?
- Should an equal number of boys and girls be asked?
- Might results vary, depending on how many girls and how many boys are asked?
- Might results vary, depending on the ages of the students asked?

Decisions all over the place! The results are some meaningful data for your specific school. Would these data be valid for another school?

goal Examining the reliability of claims and statements

things newspapers and magazines

page 242 As the various advertising claims are discussed, wonder aloud whether what they say actually exists. One purpose of this chapter is to develop critical thinking. Discuss and question the reliability of each claim.

If you have a supply of newspapers and magazines in your classroom, distribute them and have the youngsters go on an ad hunt. Have them look for statements similar to those on the page. Talk about their reliability.

Answers to these discussion questions will vary. Insights are given.

Think of ads that claim products are

Were any data used to show the claims were true? You don't know.
What kind of data? **Do you believe it?** You don't know unless its givi… you can't tell whether to belie…

1. Do you know what is meant by "you get twice the shine with Wax-O-Wow"? No
Twice what? Compared with what? You don't know You don't know

2. Many radio stations play "the top ten." How do they decide which records are top ten? Is record No. 1 the same for every station? Not necessarily R… listeners… disc jock…

3. Some advertisers say, "We're Number 1." What does this mean? You don't know; it seems to say that they sell the most, but it could mean anything

4. For sports fans. Who do you think is th… best player in the sport you like best? Why? Did you use data in deciding (matches won, goals scored, or points gained)? It depends on the resource use…

5. "There are 100 more students at Central School than at West School. So Central's school band will be better." Must this be true? Why might West's band be better? No — they might have better players

If you count the cars passing your home from 5:00 P.M. to 5:15 P.M., you have some *data* about the traffic. The number of people in your family is *data* about your family.

There are lots of ways to collect data. Tallying is a simple way to keep track of things being counted.

VOTE FOR CLASS PRESIDENT

Bill 𝍸 𝍸 𝍸 =15

CINDY 𝍸 𝍸 𝍸 | =16

One mark for each vote.

Every fifth mark a diagonal.

1. Jim Larsen was helping count the supplies in the school storeroom. Here's his tally of the packages of paper.

   a White 𝍸 𝍸 𝍸 𝍸 |||| 24

   b Blue ||| 3

   c Green 𝍸 𝍸 || 12

   d Pink 𝍸 𝍸 𝍸 15

   Write these tallies as standard numerals.

2. A week later a new paper shipment came. Tally the number of each kind. Then write a standard numeral for each.

												Pink	Green
Green	Green	Blue	Pink	Blue	White	Pink	Blue	White	White	Blue	White	Pink	Blue
Blue	Blue	Green	White	Green	Blue	Green	Blue	White	Blue	Blue	Pink	White	Blue

White 𝍸 || 7
Blue 𝍸 𝍸 || 12
Green 𝍸 | 6
Pink 𝍸 5

lesson Page 243

goal Examining and preparing tally charts to organize data

page 243 Tally charts serve to organize the gathering of data so that it is readable. Every possibility that could be counted should be listed before beginning a tally chart. This involves critical thinking, and also speeds up the actual tallying process.

lesson Pages 244, 245

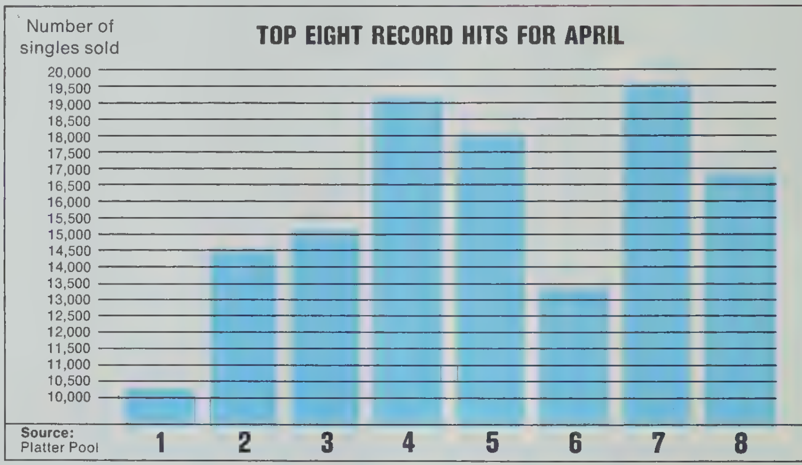
goal Examining a graph as a means of reporting data

page 244 The type of graph shown is called a BAR GRAPH. Why is this an appropriate name?

On their search for examples of graphs and charts, have the youngsters watch for other types of graphs as well—picture, circle, line. The youngsters' examples could be displayed.

Collecting data is not enough. You need to organize and report it. Charts and graphs are used to organize information. One way of reporting information is through pictures. Few words are needed if the right picture is picked.

What does this graph tell? The sales of the top 8 records for April; What doesn't this graph tell? The year and the titles of the records.



Graphs and charts are used to—

1. present information,
2. show relationships,
3. catch the reader's attention, and
4. make information easily available.

Start a search. Look in newspapers and magazines. Get permission to cut them out. Bring them to class.

244

Every graph needs to have certain labels.

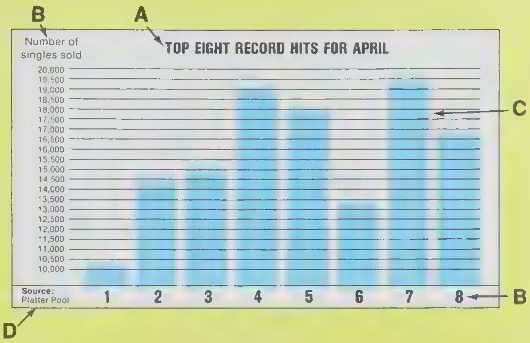
A It must have a title.

B You must know what the numbers refer to on the vertical scale and the horizontal scale.

C It needs some sort of grid so that you can read it easily.

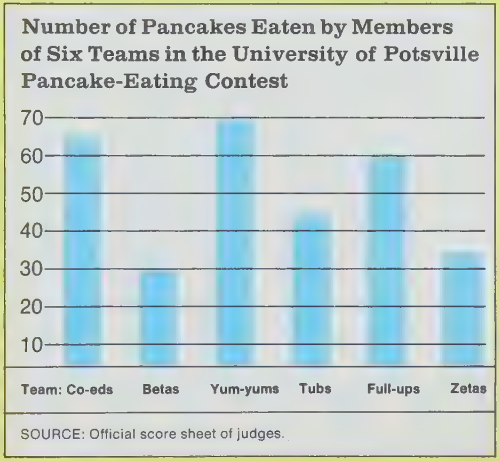
D It should have the source of this information stated.

E It should have complete information. This graph doesn't. You don't know the year or title of the records in the graph.



Does this graph have all the necessary information?

Yes



goal Examining the parts of a complete graph

page 245 You'll probably want to discuss the parts of a graph in a group. But checking out the graph at the bottom of the page should be an independent activity.

You might want the youngsters to look at some of the samples of graphs they brought in. Consider whether the graphs are complete or, if not, what parts are missing.

lesson Pages 246, 247, 248

goal Tallying data and making a bar graph to report the data

things graph paper (preferably large-square)

page 246 The purpose of the page is clearly stated. Discuss why graphs are made. (To picture data, to communicate information) Why are labels so important? (To communicate correct information) Ask the youngsters to cover the labels on the first example. What does the graph tell? Have them uncover one label at a time. Repeat the question each time.

Pupils may work in groups or as an entire class for problem 2. What needs to be done before a graph can be made? (The data needs to be tallied.) Certainly they won't forget the necessary labels!

When you count,

decide first what it is you want to count.

1. Mr. Kaplan asked his class to graph the number of people in their families.

   What a mess! Julia said there were 18 in her family: mother, father, 6 children, 2 cats, 1 dog, 7 gerbils.

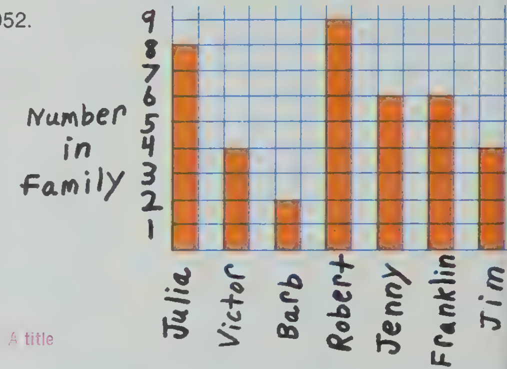
   Victor didn't know whether to count his grandmother. She had lived at his house since 1952.

   There was lots of confusion. Barb lived alone with her mother. But her aunt was visiting them. Two families were staying together because of a fire. One boy didn't live with any of his relatives.

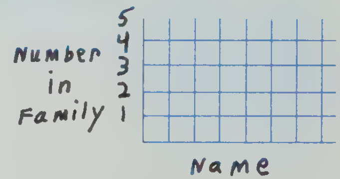
   Finally the class agreed on what they meant by "family." They made a bar graph. Part of their graph is shown at the right.

   a What's the smallest family shown? 2 (Barb's)
   What's the largest? 9 (Robert's)

   b What information is missing from the graph? A title

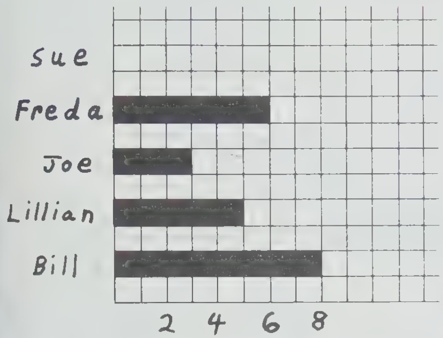


2. The graph from Mr. Kaplan's class is called a bar graph. Make a bar graph for family size in your class (or a smaller group within your class). You will need to make a grid like the one on the right. Put names along the bottom. Above each name, shade in one square for each family member. Make the graph as large as necessary.
   Answers will vary

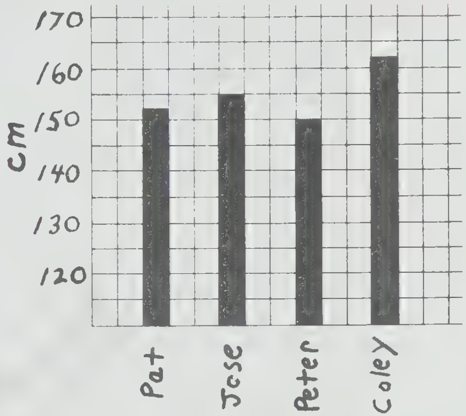


246

. Bar graphs can go across. This bar graph shows the number of years each member of the Reid family has spent in school. Make a graph like this for your family, relatives, or friends. Each rectangle across should stand for the same number of years. Answers will vary.



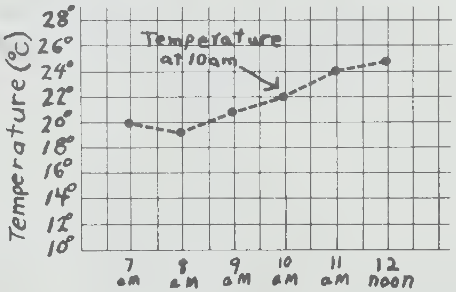
2. Bar graphs can go up. Make a bar graph showing heights of your classmates. (On a big sheet of paper, the bars can be the same heights as the people.) Answers will vary.



lere is a different kind of graph. The dots show ne temperature at different times. At 7 a.m. it was 0°. What was it at 10 a.m.? at noon? About 25°

22°

he dots are connected by a broken line. The line elps show how the temperature changed. But the emperature readings were made only on the hour.



247

See activity 1, page 268a.

goal Tallying data and making bar graphs to show the data; examining a line graph

things graph paper
rulers or tape measures

page 247 Groups of five or six pupils would be the best organization for handling problems 1 and 2. Again, tallying the data for the group is the first step. Perhaps they had better check their finished graphs with the list at the top of page 245 to make sure the information really is complete.

You might be able to reallocate for problem 2 some of that kindergarten or first-grade paper that comes in a large roll. (You can return the favor at some later time.)

Make sure to emphasize the difference between bar graphs and line graphs.

goal Tallying data and making a line graph to show the data; examining how graphs can be made to distort data

things graph paper

page 248 Problems 1 and 2 would make excellent outside-of-class projects—unless you just happen to have a thermometer outside your window.

The collections for problem 2 could be pasted into a simple scrapbook. Mark each example as being complete or indicate places where information is missing.

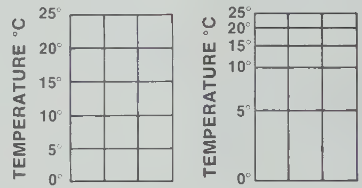
Just as claims are sometimes used to convince, graphs are sometimes used to present a more or less favorable picture that could help win or lose an argument. The graph should present an **accurate** picture. Examine the examples in problem 3. What false picture could someone get in each case? Spacing is not the problem in the graph for **3b**. What is? (The scale selected)

Sometimes you can use either kind of graph to show something.

1. Make a line graph showing the outside temperature at the same time each day for a week. (Use a thermometer at home or on a building, or use a radio or television report.) Or make a line graph showing the outside temperature each hour for at least five hours. Answers will vary

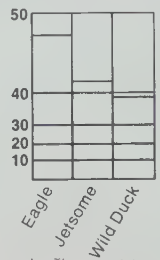
2. Collect at least six examples of line or bar graphs. Look in newspapers, textbooks, or reference books.
Collections will vary

3. People sometimes change the spacing on graphs. Sometimes they leave off parts of graphs. There might be a good reason for doing so. Or maybe the reason is to fool someone!

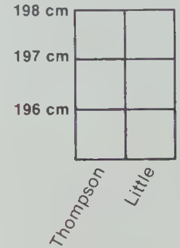


Equally spaced Not equally spaced Why not? *

a 50 flights on 3 different airlines were checked. The graph shows how many were on time. Is this a good graph? Why? No Numbers aren't equally spaced and you don't know how many flights a day each airline has



b The graph makes Tiny Thompson look a lot taller than Midge Little. Is he really that much taller? No—only

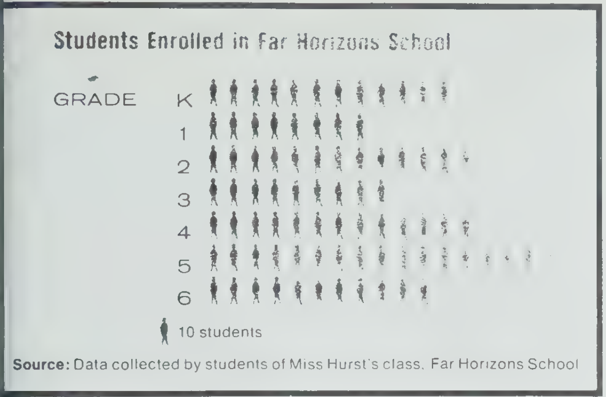


*Answers will vary. Example: They want to emphasize the lower (cold) temperatures

248

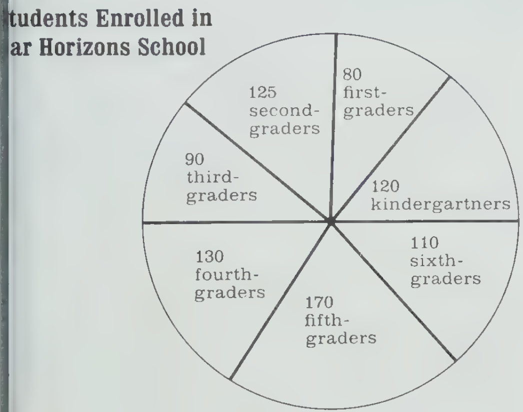
Have the youngsters hunt for examples of graphs that distort information. Look particularly for those showing different-sized figures—where the area of the figure distorts the scale.

Talk about the various examples brought to class. What favorable or unfavorable impression is being made?



Graphs can have pictures to replace bars.

What information does this picture graph have?
The number of students enrolled in each grade of Far Horizons School



The same information could have been presented this way. This is a

CIRCLE GRAPH

1. Use either graph to answer these questions:

   a Which grade had the most students? the fewest? First
   Fifth

   b How many students in all were enrolled in the school?
   825

249

lesson Pages 249, 250, 251

goal Examining a picture and a circle graph

page 249 With the pupils check whether these graphs contain the information outlined at the top of page 245. When might one of these types of graphs be used rather than a bar graph? Could a line graph be used in place of one of these graphs? (No, the information reported is not constantly changing.)

Problem 1 is an independent activity. Which type of graph is easier to use when looking for information? Why do pupils think so?

lesson Pages 250, 251

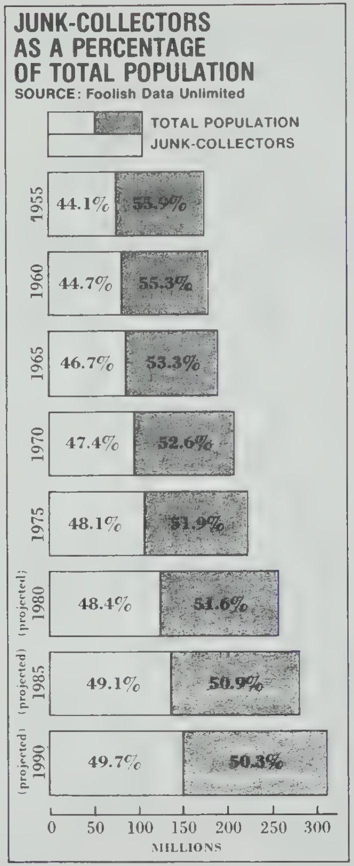
goal Examining graphs to determine completeness of information

memo Pages 250 and 251 go together. Directions are given once on page 250.

page 250 Everyone on his own. There's sure to be some confusion because all answers do not stand out neatly and clearly.

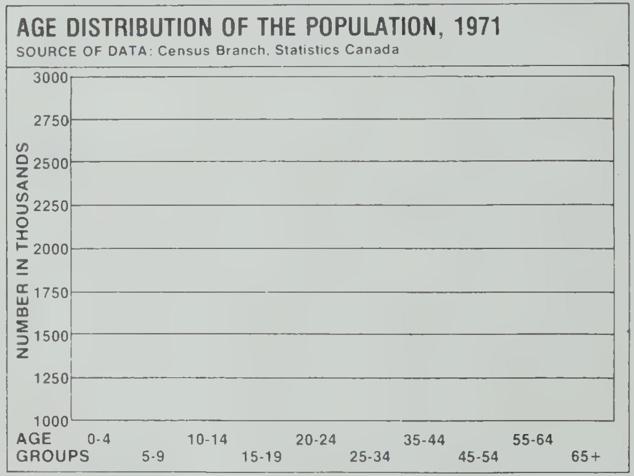
Hold off any discussion until page 251 has also been completed. Then examine these graphs for possible conclusions that could be made. Consider these or similar questions:

- What is happening to the population in general?
- What seems to be happening to the junk-collectors? Is there reason to believe this graph is not to be taken seriously?
- What age group has the largest population? (Careful—there is a catch in this one.)
- Why does the 25-34 year group appear so much larger on the graph than the 20-24 group?
- What age group has the smallest number of people?
- Does the graph tell how the age groups in the population have changed over the past ten years?

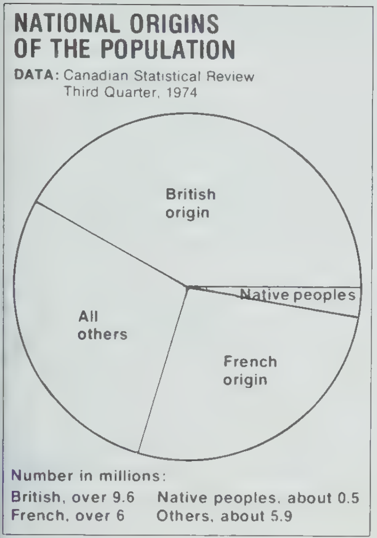


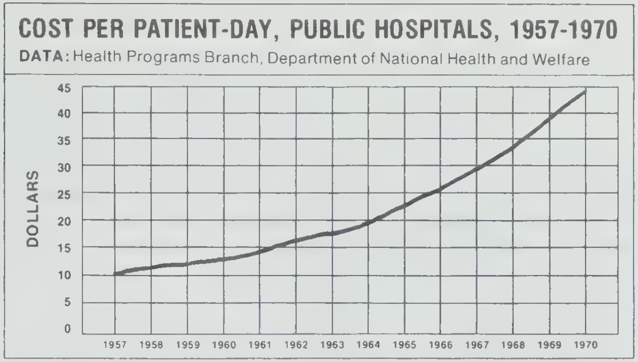
Study the graphs on this page and the next. Does each of these graphs have—

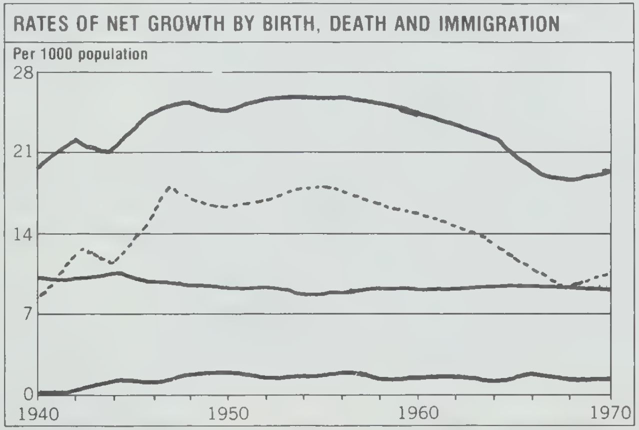
1. A title? What is it? Yes
2. Numbers on the horizontal and vertical scale? All but the ci
   What do they refer to? Years, number of dollars, or people
3. A grid? Can you read it easily? Yes, all but the circle graph
4. The source of information stated? Rates of Net Growth does no the others do.
   Is it a source you trust? Yes, except for Foolish Data Unlimited



250







251

goal Examining graphs to determine completeness of information

page 251 Discuss the completed answers. Those who had trouble can gain by such a discussion. Any misinterpretations will also come out in the open.

Examine the graphs for information that can be extrapolated.

- Did the cost of staying in hospital rise more quickly in one period than in another?
- In what years did it rise fastest?
- Does the graph show the total population of Canada?
- About how much is the total population?
- Does it show how many people were born in Canada and how many immigrated from other lands?
- Does the population grow more from births or from immigration? Does the graph tell this?
- Is the birth rate going up? What about the death rate? the immigration rate?
- Is this graph (lower right) a satisfactory report? What is missing?

lesson Pages 252, 253, 254

goal Introduction to finding an arithmetical mean (average)

things 35 cubes

page 252 The word **average** is used to denote several different things—and sometimes it's hard to know exactly what is meant. For example, what does **average** mean in the rhyme at the top of the page?

Consider the question: How many children in an average-size family? Tally the number of children in each family represented in your class.

No. of children in family	1	2	3	4	5	6	7
No. of families							

Which number has the most tallies? This number is sometimes called an **average**.

The ARITHMETICAL MEAN is considered to be an **average** because it evens off or balances all the numbers. The example in the text can be illustrated with cubes. Stack five sets—one for each number given.

9 **8** **11** **5** **2**

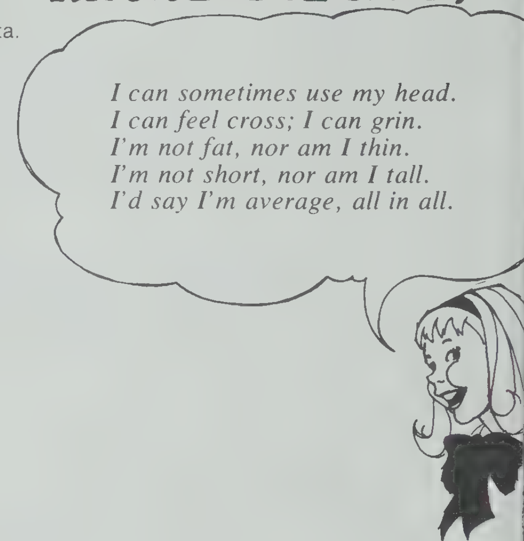
Ask a pupil to rearrange the cubes so that each stack contains the same number of cubes. Amazingly, the solution turns out to be the computed MEAN.

Data are numbers collected by observing events or counting things. But data alone usually aren't very interesting. We use graphs to make pictures of data. Now we'll look at some other ways of presenting data.

Statistics is the study of data: how to collect data, what data mean, and how to use data. The idea of "average" is important in statistics.

You've probably heard the word *average* before: average size, average speed, average height, and so on. We'll look at a type of average called the *mean*. (That's short for *arithmetic mean*. People usually think of the mean when they say "average.")

A STUDENT WITH SOME KNOWLEDGE SAID,



Here's a set of numbers:

9, 8, 11, 5, 2
Find their sum.

$$\left.\begin{array}{r} 9 \\ 8 \\ 11 \\ 5 \\ 2 \end{array}\right\} \text{5 numbers}$$
$$\underline{}$$
$$35$$

Divide the sum by the number of numbers added. The quotient is the *mean* of the numbers in the set.

$$\underset{\text{sum}}{35} \div 5 = \underset{\text{mean}}{7}$$

The mean of a set of numbers is one way to describe the size of the numbers. It's somewhere between the smallest number and the biggest number. Does the mean of a set of numbers have to be one of the numbers in the set? No

252

things *Wall Street Journal;* transparency film or spirit master

Obtain a copy of the *Wall Street Journal* and make an overhead transparency or a spirit master of the Dow Jones Industrial Averages graph on the next to the last page.

Talk about the data shown on the graph, such as the rise and fall of prices, the range of the average during a particular day (high, low, close pattern). What does the word **average** mean here? Local newspapers often have other financial graphs.

Find the mean of each set of numbers.

a 5, 7, 18 10 **b** 12, 3, 0, 10, 15 8 **c** 25, 17, 46, 4 23

d 100, 300, 600, 100 275 **e** 0, 6, 25, 5 9 **f** 4, 5, 9, 8, 11, 25, 15 11

What is the mean height of these five children? 147 cm
Pat, 145 cm; John, 155 cm; Linda, 145 cm; Sheila, 140 cm; Mike, 150 cm .

a Is anyone's height equal to the mean height? No

b How many are taller than the mean height? 2

c How many are shorter? 3

Pam, Pat, and Paul worked in a shop.
They made these sales.

Pam: 5¢, 15¢, 20¢, 5¢, 5¢ 10¢

Pat: 5¢, 5¢, 10¢, 10¢, $3.25 71¢

Paul: 25¢, 15¢, 5¢, 10¢, 10¢ 13¢

Find the average sale for each of the three.

a Why is the one average so different from the other two? Pat's sale of $3.25

b What is the average of all the sales? $31\frac{1}{3}$¢ (real world – 32¢)

Look back at problems you did early in this chapter. Calculate the mean of at least two sets of data. (For example, what's the mean family size?) Answers will vary

Find the word *average* in ads, newspaper articles, and other places. Is *average* used to mean the mean? Can you tell? Probably; can't tell for sure

For sports fans. How is each of these found?

a A batting average
Number of hits divided by official number of times at bat

b A hockey-player's goal average
Number of goals divided by number of games

c A fullback's rushing average
Number of yards he gained divided by times he carried the ball

253

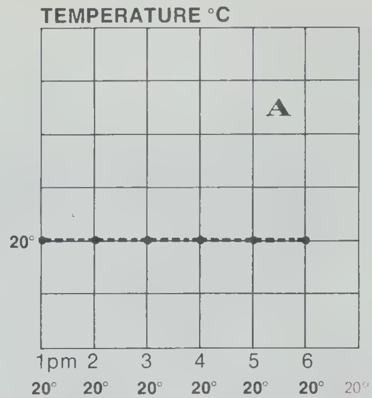
goal Practice in computing an arithmetical mean

page 253 These are independent activities, involving much computation or research. Consider dividing up the activities among groups and then sharing the information in a discussion. Make sure each group has at least one activity that requires computation of the mean.

goal Examining average temperature

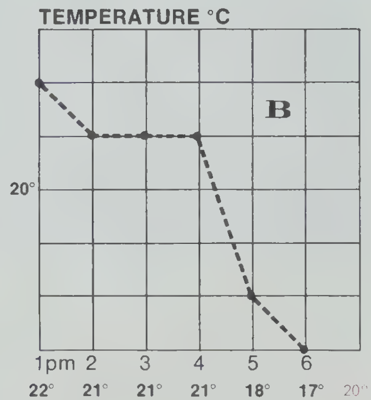
page 254 You'll want to talk about problem 1. Take a look at how average temperature is reported in the local paper(s) and on TV.

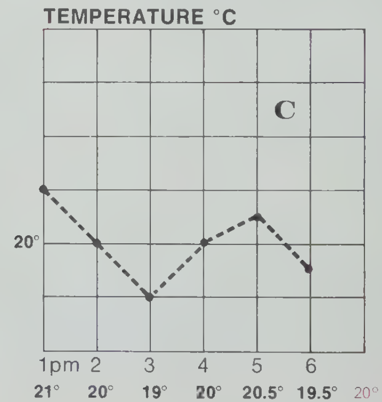
Computing the mean for each group in problem 2 only to find that it is the same for each example should stimulate some discussion. How can this be? Remember the cubes used for page 252? Finding the mean is a balance problem.



1. "The average temperature yesterday was 20°." Do you know exactly what that means? Or do you need more facts?
   - a What does *yesterday* mean? (Daytime? 24 hours?) You don't know
   - b How often was the temperature recorded? (Hourly? Three times?) You don't know.
   - c Where was it recorded? (City? Country? Shore?) You don't know.
   - d When was it recorded? You don't know.

2. Graphs A, B, and C show temperatures taken at s different times. Find the mean temperature for each set of data. If two sets of data have the same mean, do their graphs look alike? Not necessarily





things *World Almanac*

Look in the *World Almanac* or contact the local weather bureau to find out how the **mean temperature** for a day is computed. Is the mean temperature a mean as defined on the page? Collect data for hourly temperatures from a newspaper and compute the **average temperature.** Compare this average temperature with the mean temperature.

re are the heights of five children:
t, 145 cm; John, 155 cm; Linda, 145 cm;
eila, 140 cm; Mike, 150 cm . Who's the
ortest? Who's the tallest? Subtract Sheila; John
e height of the shortest from the height
the tallest. Your result is the *range*
these data.

0 cm, 145 cm, 145 cm, 150 cm, 155 cm

$$\begin{array}{r} 155 \text{ cm} \\ -\,140 \text{ cm} \\ \hline 15 \text{ cm} \end{array} \longleftarrow \text{range}$$

e range of a set of numbers shows the
fference between the smallest number
d the largest number in the set.

Find the range of each set of numbers.

a 5, 7, 18 13 | b 14, 3, 0, 10, 15 15
c 25, 17, 46, 5 41 | d 100, 300, 600, 100 500
e 0, 7, 25, 15 25 | f 4, 7, 9, 8, 11, 25, 15 21

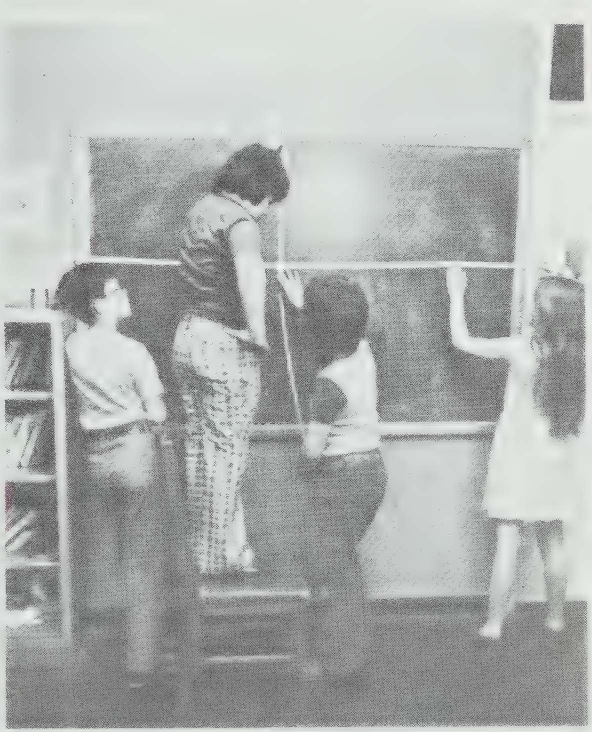
How wide do you think the board in your classroom is? Everyone in the class should write down a guess (to the nearest centimetre).

a Tally the guesses.

b Make a bar graph showing the tally.

c Find the range of the guesses.

d Find the mean of the guesses.



Actually measure the board.
How close is the mean to the measurement?
Answers will vary

255

lesson Pages 255, 256

goal Introduction to computing the range for a set of data

page 255 Computing the range for a set of numbers is not difficult, nor is it very exciting unless related to a real-world situation. Consider paying for a party. As few as 13 or as many as 25 persons may attend. What is the range? Suppose you order everything for 13 people. For how many could you have too little? If you order for 25, what could happen? Many things will enter into your final decision.

Everyone completes problem 1. Problem 2 requires some whole-group work (**2a**) as well as independent work (**2b, c,** and **d**). The youngsters might want to try guessing the length of the chalkboard in metric units. No fair actually measuring until after the assignment is completed.

goal Applications of mean, range, and graphing

things graph paper

page 256 You decide how best to handle this page with your pupils. You'll want to make sure everyone knows how to read the tally chart of passengers and what information is given on the graphs.

1. A traffic engineer wanted to know how many people caught buses at the train station. She studied four buses. Each leaves the station ten times a day, on the hour. Here are the figures.

PASSENGERS EACH TIME											
Bus	7 am	8 am	9 am	10 am	11 am	12 noon	1 pm	2 pm	3 pm	4 pm	Total
A	21	22	25	25	24	20	21	19	18	18	213
B	25	39	60	60	60	57	55	49	52	47	504
C	5	9	13	11	12	17	60	60	42	12	241
D	4	6	53	60	60	60	35	10	7	6	301

Bus capacity is 60.

a Tally the number of times each bus left the station with 60 passengers.

A 0
B III
C II
D III

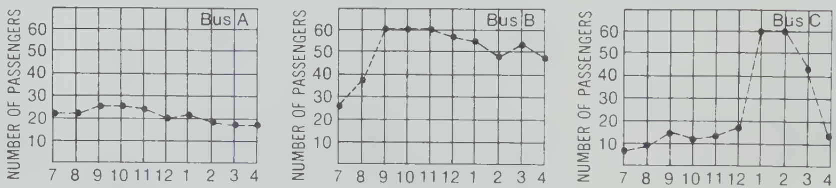
b What is the passenger range for each bus?

A, 7; B, 35; C, 55; D, 56

c What is the mean number of passengers for each bus?

A, $(21\frac{3}{10})$ 22; B, $(50\frac{2}{5})$ 51; C, $(24\frac{1}{10})$ 25; D, $(30\frac{1}{10})$ 31*
* You cannot have a fractional part of a person.

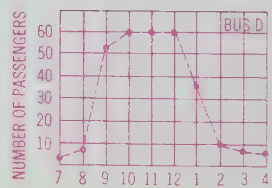
2. Here are some graphs the traffic engineer prepared.



a Compare the graphs. Why is bus A's line almost flat? Why is there a big hump in bus C's graph? What might cause such a hump?

It had the least range; lots more passengers; rush hour for lunch

b Make a graph showing the data for bus D.



256

Not every kind of information can be put into a graph.

1. Charts can give a lot of information. They save time, too. What does this chart tell? Scores for a test having 20 questions

ANSWERS CORRECT	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
THEN YOUR SCORE IS	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100

This chart was made to use when a test had 20 questions. What would your score be if you got all 20 answers correct? 100 Here are the number of correct answers that ten people got. You write the score each will get.

a Amy – 17 85 b Bob – 19 95
c Carol – 18 90 d Dennis – 20 100
e Ellen – 20 100 f Frank – 12 60
g Glen – 16 80 h Helen – 15 75
i Ida – 2 10 j Jack – 0 0

(Ida and Jack had a bad day. They got 20 correct on the next test, though.)

2. What information does this chart give? *
   a What good is the information? Useful for meal planning
   b Who might use this information? Dietitians, cooks, homemakers
   c If a store were out of No. 4 tins, what could you buy instead? Two No. 2 tins
   d If a store were out of No. 5 tins, would three No. 1 tins be enough? No

APPROXIMATE TIN SIZES

TIN SIZE	MASS	HEIGHT
No. 1	170 g	8 cm
No. 2	225 g	8 cm
No. 3	300 g	10 cm
No. 4	450 g	12 cm
No. 5	575 g	16 cm
No. 6	800 g	21 cm

For research

Look at some tins of food at home. Try to find the size. Tell what is in the tins. Tell how much the label says is inside.

*Approximate mass and capacity for various tin sizes

257

lesson Pages 257, 258, 259

goal Examining charts as a method for reporting information

page 257 Charts are generally used when more than two kinds of information are being reported. Sometimes a chart is used instead of a graph, even with only two kinds of information, because a chart is easier to read. A chart also can take less space than a graph, if space must be considered.

Everyone should be able to handle the problems independently unless reading is a problem.

See activity 2, page 268a.

goal Reading a train schedule

page 258 Help! How do you read this thing? It is a feat for many adults—let alone youngsters. You would think someone could figure out an easy-to-read timetable. For many of your pupils this may be the first exposure to such a mystifier. In what order do the pupils think the cities are listed? (Order of train stops) Each column headed by a numeral (533, 21, 535, and so on) indicates a different train. Where does each train begin? (Chilton) What is the last stop for train 533? (Hilton) for train 33? (East Hilton)

Once your pupils feel secure about reading the schedule, the remainder of the page can be completed independently.

Does anyone have a train schedule he could bring to class? Seeing more than one type of schedule is very helpful.

From CHILTON
SATURDAYS ONLY

Read Down		533	21	535	29	31	33	35
		AM	AM	AM	AM	AM	PM	PM
CHILTON (Albert St. Station)	Lv	5.10	6.50	7.55	9.40	10.40	11.40	12.40
Kedzie	Ar	5.19	6.59	8.04	9.49	10.49	11.49	12.49
Oak Park (Marion St.)		5.27	7.06	8.11	9.56	10.56	11.56	12.56
River Forest								
Maywood		5.30	7.10	8.15	10.00	11.00	12.00	1.00
Melrose Park		5.32	7.12	8.17	10.02	11.02	12.02	1.02
Bellwood		5.35	7.15	8.20	10.05	11.05	12.05	1.05
Berkeley		5.38	7.18	8.23	10.08	11.08	12.08	1.08
Elmhurst		5.41	7.21	8.26	10.11	11.11	12.11	1.11
Villa Park		5.45	7.24	8.29	10.14	11.14	12.14	1.14
Lombard		5.49	7.28	8.33	10.18	11.18	12.18	1.18
Glenn Ellyn		5.54	7.32	8.37	10.22	11.22	12.22	1.22
College Avenue			7.35	8.40	10.25	11.25	12.25	1.25
Wheaton		5.59	7.38	8.43	10.28	11.28	12.28	1.28
Winfield			7.42	8.47	10.32	11.32	12.32	1.32
East Hilton	Ar	6.10	7.47	8.52	10.37	11.37	12.33	1.37
HILTON	Ar	6.18	7.55	9.00	10.45	11.45		1.45
		AM	AM	AM	AM	AM	PM	PM

Does anyone in your family ride a train to work? If so, you have seen one of these charts. You have to have good eyes to read charts like this. This type of chart is called a schedule.

1. If you leave Chilton at 10:40 a.m., when would you get to Wheaton? 11:28
2. Could you use this schedule if you wanted to go from Maywood to Elmhurst? Yes
3. Could you use this if you were going on Sunday? No
4. Pretend you are in the Chilton train station. Pick a place to go. Tell when you will go and when you will get there. Stay there at least one hour. Continue by train to Hilton. Tell when you will go and when you will get there.

258

things *World Almanac*

Have the youngster find the perpetual calendar in the *World Almanac*. Challenge him to find the following information:

- The next year his birthday will be on a Sunday
- The day of the week Christmas was on 100 years ago
- The day of the week Christmas will be on next year
- The day of the week the year 2001 will begin

Some charts look hard to read. Small print is not the only problem. You may need special information to decode the chart.

The chart below lists some flights from Edmonton to other cities. The letters after each city show that the times are Standard: EST (Eastern), CST (Central), MST (Mountain), and PST (Pacific).

D	A	🍴	#	✈	†	①②③④⑤⑥⑦
Edmonton MST						
Toronto/Hamilton EST						
01.50	07 10	B	164	JET	-	①②③④⑤⑥⑦
09 25	14 45	L	106	L10	-	①②③④⑤⑥⑦
15 10	20.30	D	154	L10	-	①②③④⑤⑥⑦
Via Winnipeg						
15 55	23.05	D	232-264	JET-L10	2	①②③④⑤⑥⑦
Vancouver PST						
08.50	09 15		283	JET	-	①②③④⑤⑥⑦
10.00	10.25		275	JET	-	①②③④⑤⑥⑦
15 30	15.55		273	JET	-	①②③④⑤⑥⑦
17 35	18.00	S	281	JET	-	①②③④⑤⑥⑦
20.00	20.25		287	JET	-	①②③④⑤⑥⑦
20.10	20 35		237	JET	-	①②③④⑤⑥⑦
20.50	21 15		293	JET	-	①②③④⑤⑥⑦
Windsor/Detroit EST						
Via Toronto						
01.50	10.15	B	164-307	JET	1	①②③④⑤⑥⑦
09.25	18.15	L	106-325	L10-JET	1	①②③④⑤⑥⑦
Via Winnipeg						
13.55	20 30	D	284-206	JET	1	①②③④⑤⑥⑦
Winnipeg CST						
07.05	10.25	B	212	JET	1	①②③④⑤⑥⑦
10.10	13.30	L	272	JET	1	①②③④⑤⑥⑦
13.55	16.35		284	JET	-	①②③④⑤⑥⑦
15 55	19 15	D	232	JET	1	①②③④⑤⑥⑦
22 40	01 20	S	296	JET	-	①②③④⑤⑥⑦

Courtesy of Air Canada

Each symbol that heads a column has a meaning.

1. The letters D and A head columns that give times of departure and arrival. Both times are given on the 24-hour clock, beginning at midnight.

   You reach Toronto at 7:10 a.m. (07 10 on the 24-hour clock). When did you leave Edmonton? 1:50 a.m.

2. A knife and a fork are the symbol for meals on the flight. Letters tell which meals: B, breakfast; S, snack; L, lunch; D, dinner.

   Which meal are you served if you leave for Winnipeg at 7:05 a.m.? at 10:10 a.m.? Breakfast; lunch

3. The symbol # heads a column of flight numbers. Two numbers show that passengers change flights.

   At which airport do passengers to Windsor/Detroit change to flight 307 or 325? Toronto

4. A tiny plane heads a column describing the aircraft. L10 means a Lockheed 1011.

   How many flights to Toronto use *only* L10s? 2

5. The dagger symbol is over numbers telling how many stops. The mark - means a non-stop flight.

   Are all flights to Vancouver non-stop? Yes

6. Circled numbers ending each row tell days of departure, from 1 for Monday to 7 for Sunday.

   Do all flights listed here depart daily? Yes

goal Reading an airline timetable

page 259 Probably your help will be needed in reading this timetable also. Be sure the youngsters understand that where two cities are listed with a slash between (Toronto/Hamilton), this means that both are served by the same airport.

The main difficulty with reading the timetable may well be the smallness of the print and the amount of type crowded into a small space. Why are airline (and other) timetables printed in this way? The pupils may find it useful to use a ruler or piece of cardboard to keep their eye on the horizontal line as they read the timetable.

Each airline uses its own timetable format. The tables change every few months—for example, changes to and from daylight saving time always necessitate a new timetable. The youngsters might try to obtain actual schedules for comparison. Does each one give the same basic information?

ˈoal Finding streets and locations on a ity map or plan

ıage 260 This is a relatively clear map, ɜrge enough that names are easy to find nd read. Let the children have lots f practice with finding streets and vorking out routes. They will enjoy etting each other destinations, and vorking out the streets they must travel o get there. This can be fun when the ɔcation of a bridge interferes with the nost direct as-the-bird-flies route.

After practising on this map, the pupils nay want to try the same thing with a eal map of their own city or area. This nay be much more difficult, but will have he advantage of being a familiar area, ınd the problems can be real ones for hem. What would be your shortest route o the museum? the library? the wimming pool?

A street map like this one — Downtown Ottawa — can give you useful information.

1. If you wish to visit the Parliament Buildings, which street must you get to? Wellington
2. If you go right across the Rideau Canal from Wellington, which street are you on? Rideau
3. What park is situated near the canal? near the curve of Wellington Street? Major's Hill Park; Garden of the Provinces
4. The map names two bridges across the canal. What are the names? Mackenzie King, Laurier
5. Which streets must you cross to go from Rideau to St. Patrick along Dalhousie? from Sussex Drive to King Edward along St. Patrick? George, York, Clarence, Murray; Parent, Dalhousie, Cumberland
6. Which streets must you cross to go from Wellington to Laurier along Kent? from Kent to Elgin along Laurier? Sparks, Queen, Albert, Slater; Bank, O'Connor, Metcalfe

260

Collect some maps.
Make sure one of them is a road map.
Look on your maps to see if there is a

LEGEND.

The legend below contains information you need to understand an airline schedule. The legend at the right — for a highway map — is quite different.

Which legend, airline or highway, helps to find out about:

a	distances? highway	b	equipment? airline	c	boundaries? highway
d	times? airline	e	stops? airline	f	ferries? highway

LEGEND/LÉGENDE

D	— Departure/Départ	①	— Monday/Lundi
A	— Arrival/Arrivée	②	— Tuesday/Mardi
X	— Meals/Repas	③	— Wednesday/Mercredi
#	— Flight/Vol	④	— Thursday/Jeudi
✈	— Equipment/Équipement	⑤	— Friday/Vendredi
†	— Stops/Escales	⑥	— Saturday/Samedi
◗	— Begins/Commence	⑦	— Sunday/Dimanche
◖	— Ends/Se termine	15.03.75	Day ● Month ● Year Jour ● Mois ● Année

Origin City/Ville d'origine, e.g. Toronto
Destination City/Ville de destination, e.g. Montréal

747	— Boeing 747	JET	— DC8/DC9
L10	— Lockheed 1011	PRP	— Propellor/Avion à hélices
727	— Boeing 727		
B	— Breakfast/Petit Déjeuner	D	— Dinner/Diner
L	— Lunch/Déjeuner	S	— Snack/Collation

All Flights are shown in the 24-hour clock.
Tous vols indiqués sur cadran de 24 heures.

0700	— 7 AM
1200	— 12 Noon
1900	— 7 PM
2015	— 815 PM

Courtesy of Air Canada

Highways of
CANADA
and
NORTHERN UNITED STATES
· EASTERN SHEET ·

SCALE 1 cm equals approximately 25 km

25 0 25 50 75

REFERENCE

Toll .... Divided Highway
..... Principal Through Highway (Paved)
..... Principal Through Highway (Gravel)
.... Other Main Highways (Paved)
.... Other Main Highways (Gravel)
.... Highway Under Construction
.... Other Roads
.... Trans-Canada Highway Marker
11 . 67 .... Provincial, State, Highway Markers
3 . 9 .... Federal Highway Marker (U.S.A.)
.... Interstate Highway Marker (U.S.A.)
.... International Boundary
.... Provincial, State, Boundary
.... Steamer Route and Ferries

POPULATED PLACES

OTTAWA	Nat. Capital	TORONTO	Prov. State Capital
Montreal	over 200 000	Sarnia	50 000 to 200 000
Alma	10 000 to 50 000	Renfrew	2000 to 10 000
Margaretsville	0 to 2000		

Selected from material supplied by the Department of Energy, Mines and Resources
SI usage adopted for scale and populations

goal Reading the legend on a road map, and on an airline schedule

page 261 Oral questioning will be very helpful to the children in learning to read map legends. Why are the symbols completely different on the two legends? Will the symbols be the same on two different road maps? (Someone can check this.) Will the markers for the Trans-Canada Highway be the same on all maps? Will divided roads and roads under construction necessarily be shown in the same way on all maps?

A group of pupils may want to carry out a research project on the method for assigning numbers to highways. Letters to departments of highways may bring some interesting information.

Now is the time to start working with maps you have in your classroom. A game of 20 questions would be perfect. One person finds a town. The questions asked by fellow students will get better and better each time the game is played, and reading skills will be developed in the process.

goal Reading the distance scale on a map

page 262 Nice and easy. Reading a scale on a map is not all that simple. Many adults are mystified by the markings.

Examine the legend on several maps. Do you always have a ruler when traveling? Are all roads straight? Discuss how it is possible to estimate distances on a map. Has anyone ever heard of a map wheel? Do a little research. Find out what a map wheel is and how it is used.

A map scale in the legend tells what distance is represented by a given length on the map.

SCALE OF KILOMETRES

1. The bar on this scale is 4 cm long. The scale says that every 4 cm on the map represents 400 km . What does 1 cm on the map represent? 100 km

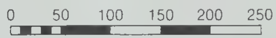
APPROXIMATE SCALE IN KILOMETRES

2. This bar is 7 cm long. The scale says that 7 cm represent how many kilometres? 7 km

One centimetre equals approximately 10 km

3. One centimetre would represent 10 km if the scale were on a map. How many kilometres would be represented by 2 cm? by 5 cm? 50 k by 10 cm? 20 km 100 km

SCALE OF KILOMETRES



4. If this scale were marked on a map, how many kilometres would be represented by 2 cm? 10 by 5 cm? by 10 cm? 250 km 500 km

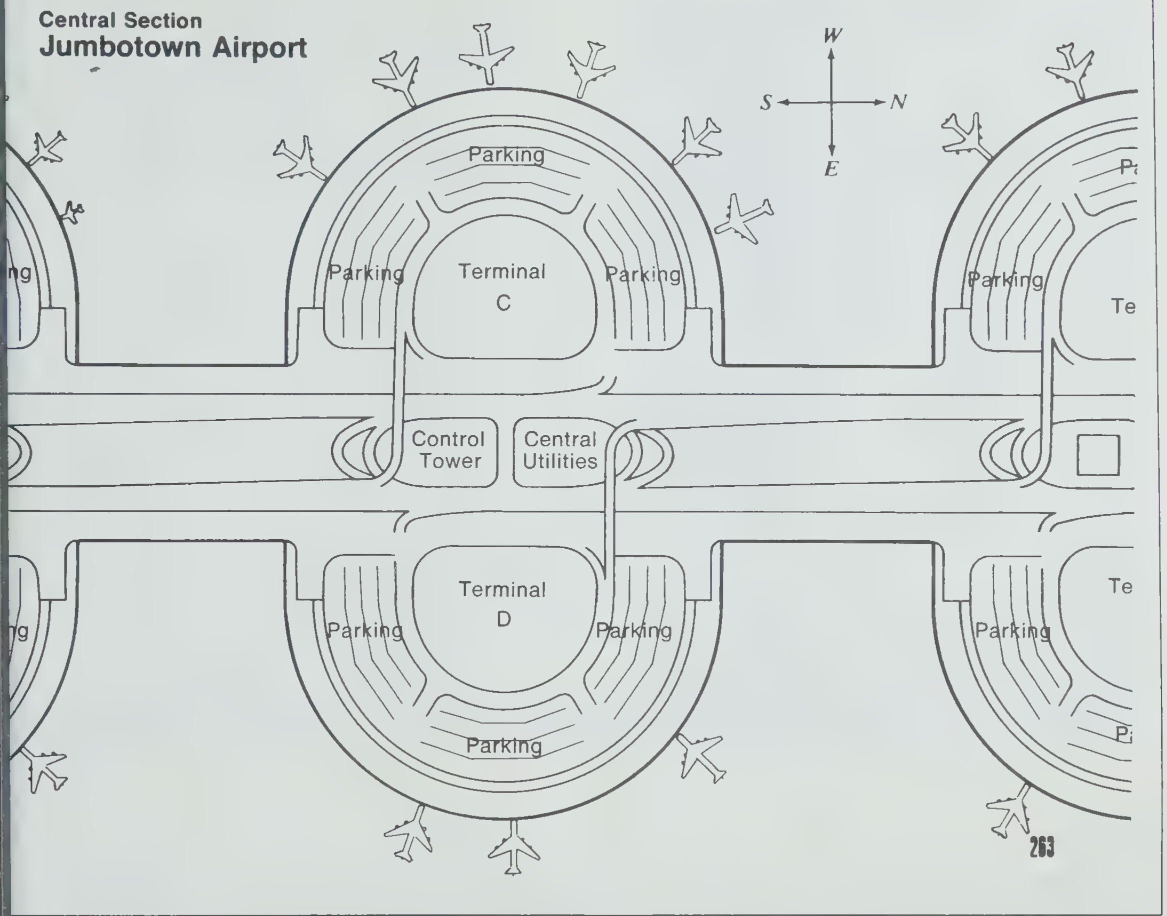
5. If you use your centimetre rule, you will find that 1 cm represents 12.5 km . How many kilometres are represented by 2 cm? by 3 cm? by 5 cm? 25 km 37.5 k 62.5 km

262

things map of province; circle compass

Have the pupil obtain a map for his province. Have him set the compass for a distance of 100 km, using the scale given on the map, and draw a circle indicating a radius of 100 km from his home city. Challenge him to plan a round trip of interest that stays within the circle. What is the total distance for the trip? You may need to point out the small numbers on the map.

Not all maps are scaled maps. Their purpose is to tell you location, not size or distance.



goal Examining a map not drawn to scale

page 263 A drawing as simple as the one on this page does not do justice to the grand scale of many modern large-city airports. Put your researchers to work to assemble some facts on the actual size of the local airport, or the largest one in your area. How many flights leave and depart daily? What is the total number of passengers passing through the airport in a day? a year? Have provisions been made for adding more terminals if this becomes necessary? What are the major causes of difficulty or delay at the airport? What suggestions can be made for removing or lessening these problems?

Take time to find out who has been to the airport in your own area. Is your local airport like the one shown? How is it different?

Tracing paper over the page will let you see if the youngsters can find their way around. Give them problems similar to this one: You are coming from the north and you want to take a plane from Terminal D. Trace a path to show your route.

goal Using telephone area codes and postal codes for specific locations

page 264 Postal codes for mail are not to be confused with area codes for telephone calls. The numbering systems are quite different. A research team will find it interesting to write the post office department to find out how the numbers are assigned for postal codes, and how the system works to speed up the delivery of mail.

Canada and the United States are divided into more than 120 telephone areas, each with its own area code. Some major cities or metropolitan regions have an area code of their own, because there are so many phones concentrated there. The curious may want to look up some of these. What is the area code for your city?

Be sure the children know how to use the telephone area codes to make long distance calls, dialling 1 and the code number for another area before the local number.

This is part of an area code map in a telephone directory for Windsor, Ontario.

AREA CODE LIST FOR SOME PLACES YOU CAN DIAL

Please Do Not Dial Area Code for Places in Area Code 519.

Courtesy of Bell Canada

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1. What is an area code? A number assigned to an area to help in calling long
2. When would you use an area code? When calling outside your own area
3. If you lived in Windsor, what area code would you dial to call someone in Sault Ste. Marie? in Toronto? in Detroit? 705 416 313
4. When would you use a postal code? On all letters or parcels you send by mail.
5. Postal codes include three letters as well as three numbers. Write AC for area code or PC for postal code for each code below.
   a 807 AC b M5P 1E8 PC c T5N 1L2
   d E3B 5H1 PC e 403 AC f 506 AC
6. What is your area code? postal code?
7. You will need a telephone directory for these answers. Find the area code for each city in the list.
   a St. John's, Newfoundland 709
   b Sherbrooke, Quebec 819
   c Moose Jaw, Saskatchewan 306
   d Grande Prairie, Alberta 403
   e Nanaimo, British Columbia 604

See activity 3, page 268a.

You've been using statistics, with data organized in graphs and charts. Statistics helps in understanding what's happened. Another kind of mathematics, called *probability*, helps in understanding what's going to happen in the future.

Suppose you flip a coin. It might land with heads up. What other outcome is possible? You have Tails up 1 chance in 2 to get tails.

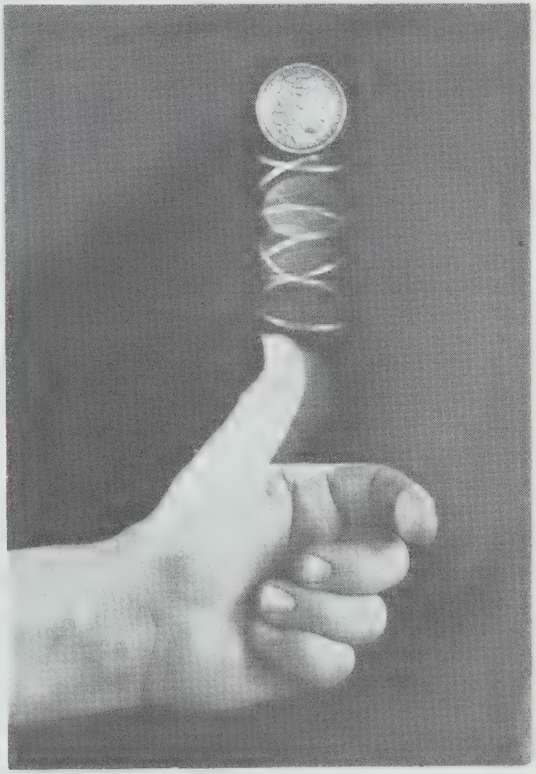
Suppose there are 15 orange jelly beans in a bag. If you reach in without looking, what kind will you get? Orange

Suppose instead that there are 14 orange jelly beans and 1 green jelly bean in the bag. If you reach in without looking, which kind do you think you'll get? Orange *Must* you get that kind? No You have only 1 chance in 15 to get green. The outcome is more likely to be orange.

When you have time, try flipping a coin 10 times. Does it land heads up 5 out of the 10 times? Should it? Yes Probably not—too few trials

Try flipping it 20 times. Does it land heads up 10 out of the 20 times? Should it? Yes It's doubtful.

The more times you flip it, the more likely it will land heads up half of the time. Why? More chances



265

lesson Pages 265, 266, 267

goal Examining the likelihood of an outcome

things jellybeans (or substitute objects) of 2 colors
paper bags
pennies, nickels, dimes

page 265 The ideas presented are sophisticated. Activities, however, create interest and build understanding. They also provide the nonmathematician an opportunity to experience success. Organize into small groups. Have each group try the experiments discussed on the page.

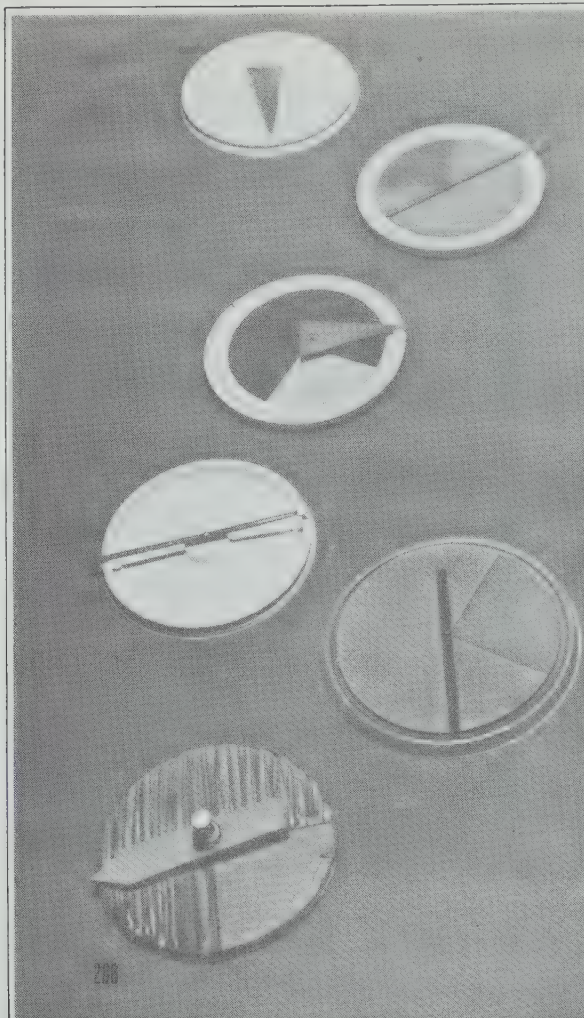
Jellybeans may not be available. Substitute any type of candy, beads, buttons, or marbles. **Important**: the shape and size of the objects must be the same; only the color should differ.

Have the youngsters increase the coin-flipping experiment to 50 or 100 trials. Someone had better be a careful counter. As the number of trials increases, what happens to the OUTCOMES? Does the type of coin (penny, nickel, dime) make any difference? (Shouldn't)

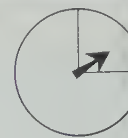
goal Performing several probability experiments

things six-sided pencils
checkers (or disks) of 2 colors
paper bags
coins

page 266 Continue the group organization for problems 1, 2, and 3. Have each group do at least two of the three experiments. Share results. Are they identical for each group?



Spinners are used in many games. This one has a shaded part and a plain part. Do you know where it will stop? Do you think it's more likely to stop on plain or on shaded?



1. Mark an X on one side of a six-sided pencil. Roll the pencil 12 times. Tally the number of times the X ends up on the top. Is this about wha you expected? Repeat 12 more times. Is the resu the same? Results will vary.

   Plain
   X

2. Put 8 red and 2 black checkers in a bag. (Use disks or other things if you wish. But these should be two different colors.) Draw a checker without looking. Look at it and tally the color. Then put it back in the bag. Repeat for a total of 30 draws. How often did you draw red? black? Is this what you expected? Results will vary.

   Black
   Red

3. Flip two coins at the same time. Write "1" if you get two heads. Write "0" if you didn't. Keep writing 0 or 1 until you've flipped both coins 36 times.

   0 0 0 1 0 0 1 0 0 0 0 0 1 1 0

   What's the longest string of 0s before you get a 1? What's the shortest string of 0s? Results will vary.

In the preceding section you could tell the chances of something happening just by looking at the possibilities. For example, if there's an X on one side of a six-sided pencil, there's 1 chance in 6 of rolling the X.

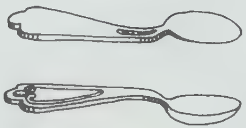
Sometimes it's impossible to tell the chances ahead of time. Then we might use statistics to see what happened in the past. This could show what's likely to happen in the future.

If you flip a spoon, it can land with the bowl up or down. Which do you think is more likely? Actually flip a spoon 12 times. Tally the results. Repeat 12 more times. Are the results about the same? If you flipped the spoon 12 more times, what result will you get?
Depends on the spoon – results will vary

Predictions about the future are usually based on what happened in the past. Sometimes statistics is used in predicting. Often predictions about what a lot of people will do are based on what a few people *have* done (or say they *will* do).

Weather predictions are based on information about past weather.

Before an election a small number of people are asked who they'll vote for. Predictions are then made about what will happen when everyone votes.



Temperature and Precipitation Data

	Av. Dates of Freezing Temperatures (0°C or Lower)		Av. Number of Days of Precipitation (All Forms)
	Last in Spring	First in Autumn	
Prairie Provinces —			
Manitoba —			
Churchill	June 22	Sept. 12	141
The Pas	May 28	Sept. 20	128
Winnipeg	May 25	Sept. 21	121
Saskatchewan —			
Regina	May 27	Sept. 27	114
Saskatoon	May 27	Sept. 15	103
Swift Current	May 28	Sept. 19	112
Alberta —			
Beaverlodge	May 22	Sept. 7	129
Calgary	May 28	Sept. 12	113
Edmonton	May 14	Sept. 19	121
Medicine Hat	May 17	Sept. 20	89

Canada Year Book 1973

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goal Examining making predictions

things plastic spoons

page 267 Use plastic spoons for the spoon activity. No injuries, please. Continue to increase the number of tosses. As the number increases, can a more certain prediction be made?

Put your researchers to work.
- How are weather predictions made?
- What election polls are traditionally taken? How are they taken? by whom? Must people vote for the person they say they will vote for?

goal **Checkout** – reading a map; predicting outcomes; finding the mean and the range of data

page 268 Do not let reading stand in anyone's way. Observe as pupils work independently. The ease with which each youngster works will indicate those who need additional experience.

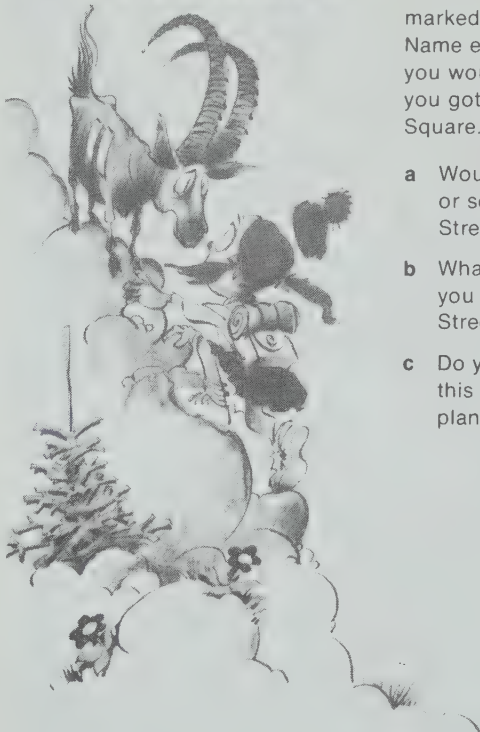
Youngsters who have trouble with the map-reading problem will need continued experience. Try a local map. Give the youngster one or two specific questions as well.

Here's an experiment for those who have trouble with problem 2. Draw 2 beads at a time from a bag or box containing 10 yellow and 10 green beads. What are the possible outcomes? (2 yellow, 2 green, or 1 yellow and 1 green) Which outcome do you actually predict is most likely to happen? Perform 20 trials, replacing the beads each time.

You may want to try enacting problem 2 to help the pupil see the reasoning. Have the pupil decide how many blank caps and how many at each price to include. **He** is the cola company. This is **his** money.

Have those who had trouble with problem 3 repeat the activity, using actual test scores from the class.

CHECKOUT



Skill: Reading information on map

1. Pretend you are going to walk the path that is marked. Start at the star. Name every street that you would walk before you got to Portsmouth Square. Foxhill, Maiden Lane, Church, Queen, River, Montgomery, Wellington
   - a Would you walk north or south on Foxhill Street?
   - b What direction would you walk on River Street? Northeast
   - c Do you know from this map how far you plan to walk? No

1. Foxhill Bus Terminal
2. Commonwealth Square
3. Memorial Fountain
4. Portsmouth Square

Skill: Predicting outcomes

2. A company offers to give you 10¢ if you find "10¢" printed under its bottle cap. You get 25¢ if it says "25¢," and so on, up to $25. But most bottle caps are blank. If you managed the company, would you print "$25" on many caps? No Would 10-cent caps be more common than 25-cent caps? Yes

Skill: Finding the mean and range of data

3. Here are ten test scores. There were 25 questions on the test. The scores are the number of questions each person got right. Find the range of the scores. Find the mean (average) of the scores.

   20, 18, 24, 8, 15, 12, 17, 23, 24, 21

   Range = 16; mean = 18.2

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See activity 4, page 268a.

See activity 5, page 268a.

RESOURCES

another form of evaluation

for checkout–page 268

1. Draw a map that shows the route you take to get from your home to school. Label the streets. Draw a compass on your map to show where north is. Answers will vary.
2. You own a grocery store and have to decide how much of certain brands of soap to order. You read the results of a survey that said 350 people out of 500 used Brand A, 125 out of 500 used Brand B, and 75 out of 500 used Brand C. Will you order more of Brand A, B, or C?
3. Five children collected newspapers for recycling. Here are the totals for each person's collection. Find the range of the masses. Find the mean (average) of the masses.

29 kg 21 kg 32 kg 18 kg 30 kg

Range = 14 kg; mean = 26 kg

activities

1. Question: In what month of the year were you born? Everyone should have an opportunity to answer. Someone will need to make a tally chart. Tally the data in three different ways:
- Boys only
- Girls only
- Everyone together

Have the youngsters graph the data on three bar graphs. Compare the graphs. In which month do the most boys have a birthday? the most girls? the most children in the class? Is there any month in which no one has a birthday? Can you tell from the tally charts or the graphs in which month a particular person has a birthday? on which day of the month this person's birthday falls?

2. things large, wide rubber band; strip of soft wood; stapler

Extend problem 1 by having the youngsters make a "rubber-band chart." Cut the rubber band and lay it flat. Mark off 10 same-length segments. Label the marks made 0 through 100. Add marks midway between the multiples of 10. Label these 5, 15, 25, . . . , 95.

On the piece of wood, mark off 30 same-length segments and label the marks made 0 through 30. Fasten the 0 end of the rubber band to the wood so that the 0 marks on both scales line up.

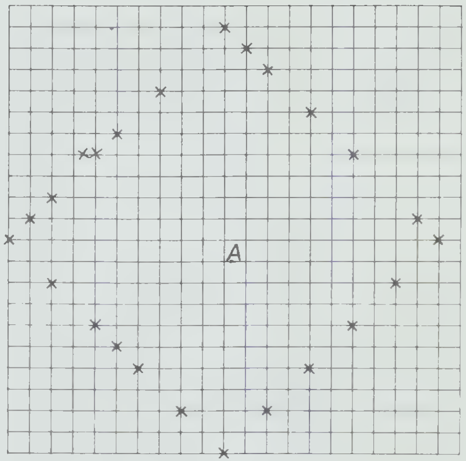
The chart may now be used for any test having 30 questions or fewer simply by stretching the rubber band so that the 100 lines up with the number of questions on the test. The mark on the rubber band nearest to the number correct on the test gives the score based on 100.

Challenge your sharpies to make a chart similar to that in problem 1 for a 10-item test–or perhaps for a 25-item test.

3. things city map; graph paper

Locate 2 places on the map. Compare the "block distance" and the "straight-line distance" between these 2 places. Think about walking the sidewalk distance as opposed to cutting through lots.

Try to find a section of the city laid out in regular square blocks, or use graph paper as if it were a map. Pick a location. Mark it with a big dot. Find 10 to 15 places that are 10 blocks away from the dot. Mark these places with an **X**. Do these **X's** form a circle?



4. things newspaper sports section

Have the pupil find the scoring record of his favorite athletic team–basketball, baseball, football, hockey, or soccer. From the information given, have him find either of the following:
- The average (mean) number of points scored by a player thus far in the season
- The average (mean) number of points scored by a player for the latest game played

How many players scored above the average? How many scored below? Can any conclusions be made?

5. What do you think is the most common color of car? Find a place with lots of traffic. Pupils may want to work in teams–one observing and the other recording. A tally chart will help organize the data. Agreements will need to be reached, since each automobile manufacturer varies shades and tones of basic colors. How many cars should be counted before deciding which color is most common? Try this experiment again at a different hour of the day and at a different location. Are the results the same?

additional learning aids

measurement of events – chapter objectives 1, 2, 3, 4, 5, 6, 7, 8

SRA products

Mathematics Learning System, Activity Masters, level B, SRA (1974)
Spirit masters: P-5, 12, 13; S-1, 2, 3, 4, 5
diagnosis: an instructional aid – Mathematics Level B, SRA (1972)
Probe: M-23
Math Applications Kit, SRA (1971)
Occupations card: 49
Skill through Patterns, level 5, SRA (1974)
Spirit masters: 13, 15, 16, 21, 22, 68, 69

other learning aids (described on page 288g) – Block Graph, Good Time Mathematics, Histogram Board, Making and Using Graphs and Nomographs, Probability Maze, Probability Set, Toward Probability

12 GEOMETRY SYMMETRY

before this chapter the learner has—

1. Experienced paper-folding activities
2. Identified line segments, parallel lines, and perpendicular lines
3. Explored the concept of symmetry
4. Explored the concept of congruence
5. Identified various polygons by the number of sides

in chapter 12 the learner is—

1. Making symmetrical figures
2. Testing whether a figure has symmetry by tracing and folding
3. Mastering the identification of lines of symmetry by folding
4. Finding the congruent parts of a figure by tracing, folding, or measuring
5. Exploring some concepts of motion geometry

in later levels the learner will—

1. Determine whether two figures are congruent
2. Make a congruent copy of a figure

Notes & Things

This chapter explores the sophisticated idea of symmetry in such a way that a big idea is reduced to something very simple—and interesting too. Pupils are to explore, discuss, and verify ideas. Tracing, cutting, folding, and moving models are all part of the study.

Little emphasis is placed on vocabulary. Vocabulary is developed through experience, not by memorization. Pupils are expected to use the following terms correctly: *congruent, symmetrical, line of symmetry.*

The door is opened to any number of individual pupil activities—making scrapbooks, designs, mobiles, and so on. A group approach is used to facilitate the sharing of ideas and discoveries. For these activities, pupils need to feel the concepts and relationships with their hands rather than simply to state the rules. Formality will come later. Now is the time to develop readiness. On completion of the chapter, the pupil is expected to be able to determine whether a figure is symmetrical, employing tracing and folding. Enjoy and have fun!

things

paper for tracing
scissors
mirror
leaves, shells, objects that appear symmetrical
crayons
patterned shelf paper or gift-wrapping paper (optional)
rulers
graph paper (optional)

For the extra activities you will want to have these things available:
- spirit masters
- symmetrical and nonsymmetrical shapes (see page 288f)

goal Think about and explore ideas through a picture clue

page 269 This classic stained-glass window is a very striking and beautiful example of symmetry. Take time to look at the details of the window design. Do the youngsters recognize that parts of the design are the same?

Are there things in your community that have parts that repeat in form or design? Are there things in the classroom? The chapter is full of applications of symmetry. The youngsters will have many opportunities to understand this geometric concept so if you don't get a good discussion right now, you will soon. Go right on into the chapter and wait for ideas to develop.

lesson Pages 270, 271

goal **Survey**—concepts of congruency and symmetry

memo Now is an excellent time to ask the art specialist in your school to share some knowledge (as well as slides or pictures) of the world's great architecture. Symmetry can be found in practically all buildings built before the 20th century. Check your own picture folder for examples. Coming to appreciate various cultures and their achievements is especially appropriate when studying geometry.

page 270 The cathedral on the right is classic Gothic in form. Can the youngsters see that half the building looks much like the other half. The Far East temple in the photograph below is a building of lesser height with much of the same majesty of height found in the cathedral. How is this done?

This style of building has been popular for cathedrals over centuries. Why do you think so many people find this style beautiful? What is pleasing about its looks? Answers will vary. Example: Lofty spires, ornateness give people a special feeling.

Some very modern buildings have one part of their style in common with this very old style of building. What is it? Symmetry.
There is an idea of geometry that can help you answer the last question. It's called symmetry. The idea is used a lot.

goal **Survey** – concepts of congruency and symmetry

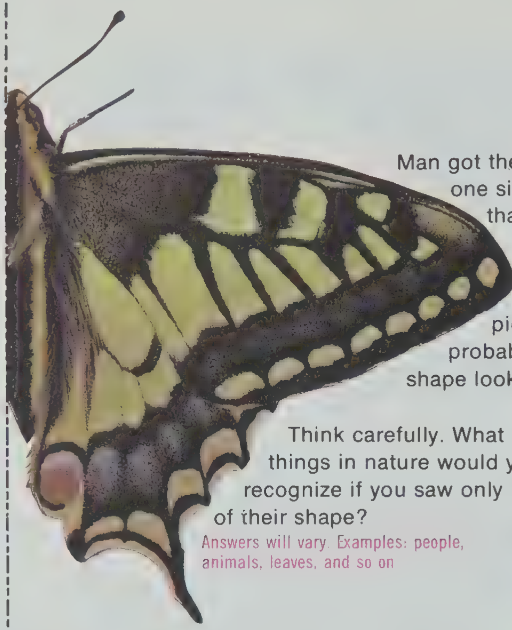
page 271 As the questions are discussed, expect a wide range of responses relating to size, shape, and number. Your pupils may not use the technical terms **congruent** and **symmetry**, yet they may express these ideas with such statements as "Everything on this side is the same as on that side," or "The doors match." Listen for concepts.

lesson Pages 272, 273, 274

goal Examining symmetry in nature

memo Pages 272, 273, and 274 are exploratory in approach. This is a time to investigate and experiment.

page 272 Have pupils jot down their answers to the questions before discussing the ideas presented.



Man got the idea of balancing one side of an object so that it matches the other side from nature.

Only half of a shape is pictured on the left. But you probably know what the full shape looks like.

Think carefully. What other things in nature would you recognize if you saw only half of their shape?

Answers will vary. Examples: people, animals, leaves, and so on

Cover half of this creature with a sheet of paper. If you saw only half, would you know what it was? Yes

Shape is a big clue that helps you know what an object is.

Think again. What man-made objects would you recognize if you saw only half of their shape? Remember—you have no clues other than shape. Answers will vary. Examples: Tables, chairs, glasses, wheels, and so on

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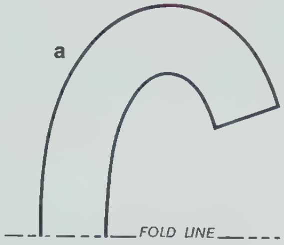
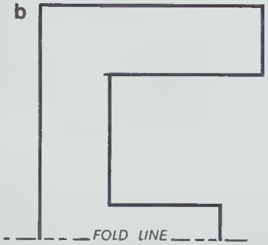
1. Half of each of these figures is folded under the figure itself. What would each figure look like it it were unfolded and you could see the whole shape? C E Y

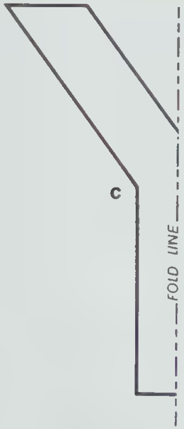
a



b



c



2. Name other letters of the alphabet you think can be cut from folded paper. Name some that cannot. Prove it. F, G, J, L, N, P, Q, R, S, Z
A, B, C, D, E, H, I, M, O, T, U, V, W, X, Y (Consider K)

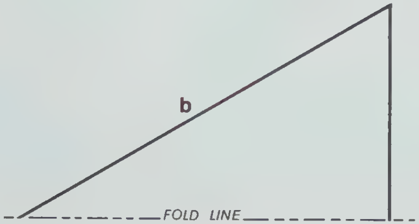
3. Half of each of these figures is folded under itself. What would each figure be if it were unfolded?

Could figure **a** and figure **c** be the same size and shape? No

a

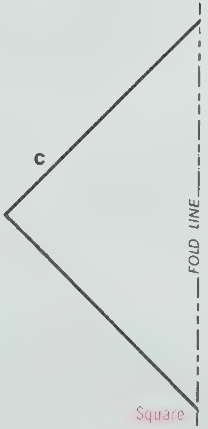
Rectangle

b



Triangle

c



Square

273

goal Exploration of symmetry in alphabetic letters and geometric shapes

things paper for tracing
scissors

page 273 Not sure what the whole shape looks like? Then find out. Trace the half shown, fold along the line, cut out the figure, and unfold.

Problem 2 lends itself well to small-group organization. Various ideas can be challenged and tested, generating enthusiasm and discovery. The results of the exploration can be recorded on charts. Block letters should be used for this lesson.

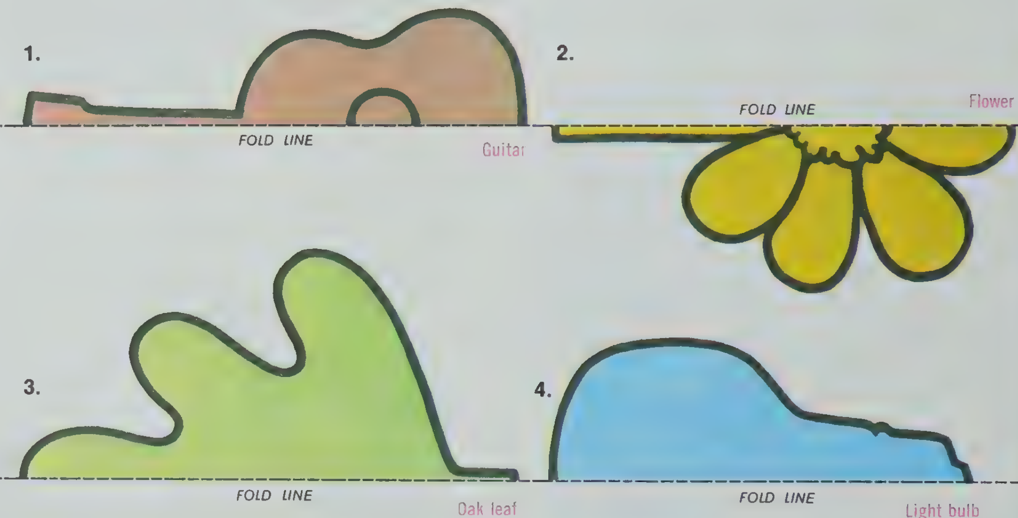
Do discuss **how** the pupils decided whether figure **3a** and figure **3c** are the same size and shape.

goal Making symmetrical figures

things paper for tracing
scissors

page 274 Purely an independent-activity page. Don't worry about technical vocabulary here. The words will be repeated many times. For now the activity is still directed toward building the concept.

Here are some designs to cut from folded paper. Trace each design, and then fold your paper along the dotted line. Before you cut, try to guess the shape you will have when you open the cutout. Did you guess right?



lesson Pages 275, 276

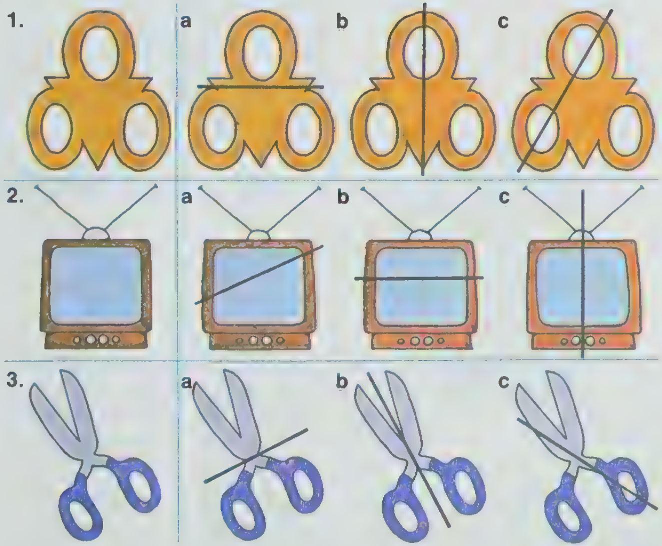
goal Identifying lines of symmetry

page 275 The concept of LINE OF SYMMETRY has been developed by the trace, fold, and cut activities on the preceding page. Some shapes have more than one line of symmetry. Do any of these?

The shapes you have been cutting out look the same when they are flipped over because they have two halves that match each other exactly.
Such shapes are called **symmetrical** shapes.

The lines where they can be folded to make matching parts are called
lines of symmetry.

Which figure on the right shows the line of symmetry for the figure on the left?



things 2 posters or spirit masters; set of symmetrical and nonsymmetrical shapes

Prepare 2 posters or spirit masters – one consisting of symmetrical shapes, the other of nonsymmetrical shapes. Describe the difference(s) between the two sets.

Have the pupils examine several shapes or pictures of shapes. To which set – symmetrical or nonsymmetrical – does each shape belong? How did the pupil decide?

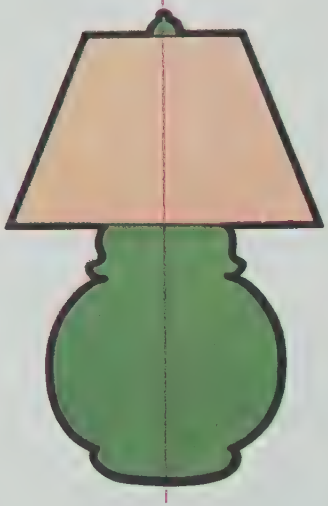
goal Developing a test to find a line of symmetry

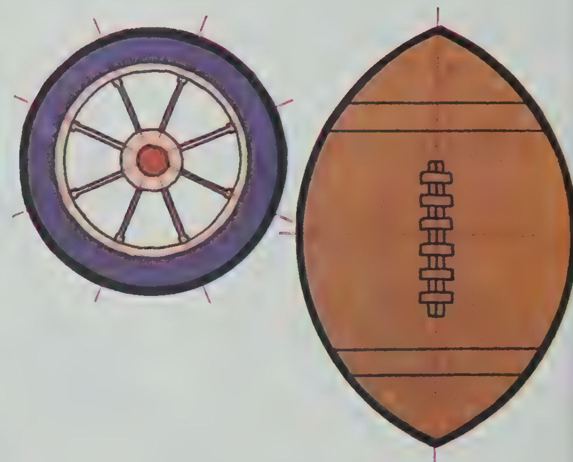
things paper for tracing

page 276 Students who make sloppy tracings may have difficulty testing the figures. Cutting out the figure isn't necessary – making a tracing with a clear outline is.

You might have the youngsters share their findings for problem 2 and then summarize them in a chart.

1. Here are some other shapes that have lines of symmetry. To find a line of symmetry, imagine where the shape could be folded. You can test your ideas by tracing the shape on another sheet of paper and then folding it.





2. Find some other shapes that are symmetrical. If possible, bring pictures or drawings to class. Try to find at least five things in your classroom, five things in your home, and five things outdoors that look symmetrical.

lesson Pages 277, 278, 279

goal Examining symmetry in the real world

page 277 The questions are designed to stimulate thinking and an awareness of symmetry in nature and in manufactured objects. It's a time to share ideas and perhaps even argue a little.

Things to think about.

1. Is the front of your school symmetrical? Is the front of your home symmetrical? Find some buildings that are symmetrical and some that are not.

2. Are cars symmetrical? Look at one from several different sides and ask the question each time. From the front and back they usually are, but not from the sides. Inside, the dashboard certainly isn't. What else?

3. Study some fruits and vegetables from different sides. Find some that appear symmetrical and some that do not. Challenge students to think about bananas cut the "long way," a peanut, berries, and so on.

4. Some things seem to be symmetrical in some ways but not in others. Look at your classroom door. Would you consider it symmetrical? In every way? What about the cover of this book? traffic signs? telephones? All these things are symmetrical in some ways, but not in all ways.

5. Look at dogs, cats, birds, and other animals. Are they symmetrical? Not really

6. There are many things in real life that appear symmetrical. This fact is very important to scientists. Can you think of any animal that is not symmetrical in at least some way? Do you see any ways in which a symmetrical structure is helpful to an animal?

Some microscopic animals—  ; balance, agility, ability to move in more than a single direction

Encourage everyone to collect models or pictures illustrating symmetrical objects found at home or outdoors. This collection should stimulate thinking and discussion. Encourage the pupils to devise their own intuitive tests for symmetry, in addition to folding. Bulletin boards, charts, and a display are possible outgrowths of this activity. You may want the pupils to indicate the line(s) of symmetry for each model.

goal Examining the human body for symmetry

things mirror

page 278 The picture a child sees of himself in his mind's eye may be far different from what other people see. A mirror will be a handy thing to have ready for this discussion. Some youngsters may think of their internal body as well as their external body.

Pair the youngsters. Have them trace each other's hands. Then each one can lay the tracing of his right hand over the tracing of his left hand to test for symmetry.

1. Are you symmetrical? Do you have matching parts on the two sides of your body? Are your two hands exactly the same? What about No your feet or your eyes? No Take a good look at yourself and other people.

Not really symmetrical; matching parts on the outside but not inside (two lungs, two kidneys, but your heart is in just one side of your body)

278

1. Which of these appear to be symmetrical? All
2. Which are symmetrical? (Test by tracing.)
3. Which are man-made? Which are things found in nature? a, d
4. Most natural objects would not pass the strict rules of *exact* matching that we use with our paper designs. Try tracing some real leaves, shells, or other natural objects that appear symmetrical. Do the halves always match exactly when you fold the tracing? Probably not

279

goal Finding symmetry in manufactured objects and in nature

things paper for tracing
leaves, shells, objects that appear symmetrical

page 279 **Symmetry** in nature is only an approximation. People probably observed what they believed to be symmetry and then perfected it in the things they made. You'll want to emphasize this idea.

The youngsters will find out whether it's possible to make a symmetrical model by tracing a leaf in problem 4. Then challenge them to trace only half of the same leaf and to complete the model by folding and tracing. Is there a difference between the two models?

A walk in the great outdoors can be a meaningful part of this lesson.

lesson Pages 280, 281

goal Discovering that figures cut from a folded pattern will fit that pattern in at least two ways

memo Pages 280 and 281 work together. Patterned shelf paper or gift wrappings eliminate the need for crayons.

things paper for making a figure
crayons
scissors

page 280 Here is an opportunity for creativity. Papers can be folded in half from any direction. Encourage your pupils not to match the figure illustrated. Caution them not to cut any of the outer edges of the piece of paper. Holes carefully made with a pencil point will be large enough for these activities.

These are exploratory activities. Reading should not impair discovery. If necessary, dictate directions to less capable readers.

Here is an alternate method for making a tracing of the cutout if the paper is too thin to trace around easily.
- Lay the cutout on a sheet of paper.
- Use a flat piece of crayon. Rub over the outer edge of the cutout—from the cutout onto the sheet of paper.
- Go completely around the cutout.

Lift the cutout. The sheet of paper will show a clear uncolored copy of the cutout.

1. Color a piece of paper on one side. Fold it in half. Cut out a shape. Now open out the cutout and the piece of paper from which it was cut. Smooth them out. Fit the cutout back into the hole in the paper. How many ways can you fit the shape into the hole? 2 If the shape is cut carefully, there will always be at least two ways to put the cutout back. Why do you think this is true?
Because the shape is symmetrical
2. Get a sheet of clean paper. Trace the shape of the cutout on the paper. Punch a small hole near the edge of your open cutout. Then fit it back into the outline you have just traced. Mark a point on the tracing through the hole.
3. Flip your cutout. Fit it back into the traced outline. Mark a point through the hole again. Is the second point you marked in the same place as the first one?
No, it's on the opposite side of the outline.

4. Get a ruler. Connect the two points with a *straight* line segment. What is the distance from the point on the left to the fold line? What is the distance from the fold line to the point on the right? How many square corners are formed at the point where your segment passes through the fold line? Answers will vary, but the distance from the points to the fold line should be the same and there should be four square corners where the segment passes through the fold line.

5. Punch another hole on one side of your cutout. Locate the point on your tracing. Flip the cutout. Locate the second point. Connect the two points with a straight line segment. How far is it from the left point to the fold mark? How far is it from the fold mark to the right point? Is this line segment longer or shorter than the first one? Answers will vary (same criteria as in problem 4).

6. Now try to punch a hole in the cutout that will locate *only one* point when the cutout is flipped. It can be done only if the hole is punched through the fold line.

7. Have you got a place on your cutout to punch one more time? This time fold the cutout. Punch the hole so that is goes through both sides of the cutout. Open the cutout. Locate these two new points on your tracing. Flip the cutout. Do you have to locate two new points? Why? No; because it is symmetrical.

Repeat this activity with other cutouts if you have time.

281

goal Exploring whether symmetrical figures have matching points on opposite sides of a line of symmetry

things rulers

page 281 Use the same procedure as on page 280. The children will want to talk about what they discovered after they have completed the activities.

lesson Pages 282, 283

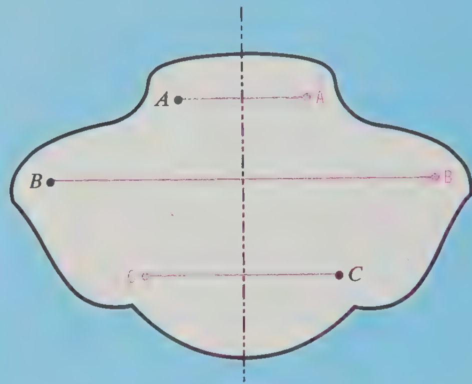
goal Investigation of line segments drawn between matching points of a symmetrical figure

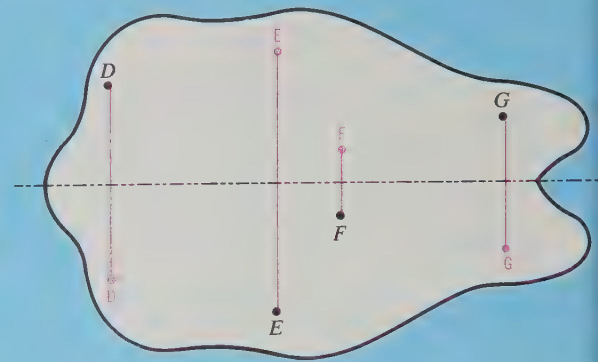
memo Pages 282 and 283 work together.

things paper for tracing
rulers

page 282 Again, please don't let reading impair discovery. Folding will help students who are having difficulty matching points. Making the line of symmetry solid will emphasize the square corners formed by perpendicular lines and contribute to the discovery that every line segment drawn between matching points of a symmetrical figure is perpendicular to the line of symmetry.

Some students may have forgotten what they learned about perpendicular lines in chapter 3.





1. Trace the two shapes above on a sheet of paper. Don't forget the points that are marked. Then find the matching point for each marked point.

2. Use your tracings. Pick two matching points and draw a *straight* line segment connecting them. Does your line cross the line of symmetry? Yes Connect all the matching points in the same way. See drawings above.

3. Study all the lines drawn between matching points. Does each cross the line of symmetry? Yes Is each perpendicular to the line of symmetry? Yes

4. If you have drawn them carefully, each pair of lines will be the same distance apart along their entire length. Do you remember what lines like this are called? Parallel lines

282

Name of point	Distance of point from line of symmetry	Distance of matching point from line of symmetry	Distance of point from matching point
A / D	12 - 13 mm / 2 cm	12 - 13 mm / 2 cm	25 mm / 4 cm
B / E	38 mm / 25 mm	38 mm / 25 mm	76 mm / 5 cm
C / F	19 mm / 6 mm	19 mm / 6 mm	38 mm / 12 mm
/ G	/ 8 mm	/ 8 mm	/ 16 mm

5. Can you find any pair of matching points joined by a line that is not parallel to the connecting lines of other pairs? No

6. Measure the distance between point *A* and the line of symmetry. Record it on a chart like the one above.

Measure the distance between the matching point and the line of symmetry. Finally, measure the distance between point *A* and its matching point. Record all your findings.

7. Repeat this for other points until you find some patterns. Then make up some rules to describe the patterns you have found. Test your rules to see if they work for all points on any symmetrical figure.

Distance of point from line of symmetry = distance of matching point from line of symmetry
Distance between two matching points twice the distance from point to line of symmetry
Lines connecting matching points will be parallel to each other AND perpendicular to the line of symmetry

283

goal Further investigation of line segments drawn between matching points of a symmetrical figure

things rulers
graph paper (optional)

page 283 Carry on, using the same procedure as for the preceding page. Parallel lines also may have been forgotten since chapter 3. Again, the focus is on exploration, not on mastery. But it is hoped that the learner will discover that line segments drawn between matching points of a symmetrical figure are parallel, and that the distance between a point and the line of symmetry is one-half the distance between that point and its matching point.

You are the best judge of how accurately your students are able to measure for problem 6. Here is an opportunity to practice rounding units. Squares on graph paper make good countable units. Consider measuring with graph paper rather than with rulers.

Be sure to discuss the rules made up for problem 7. Technical language is not as important as the ideas. This is the time to catch faulty conclusions.

See activity 1, page 288a.

lesson Pages 284, 285

goal Finding congruent parts of symmetrical figures

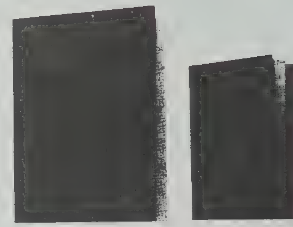
memo Page 285 will continue the activities started on page 284.

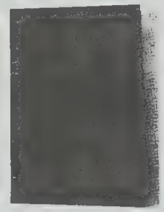
things for each pupil:
several small sheets of paper
(Waxed paper is great!)
scissors
rulers

page 284 Directions are short and clear, but reading can be a problem for some. You might pair a less capable reader with one who has no problems and have them work as a team.

The youngster who cuts an equilateral triangle for problem 1 – by luck or by design – will come up with the answers shown in parentheses. You may want to discuss the two possible sets of answers after the activities have been completed.

YOU WILL NEED SEVERAL SMALL SHEETS OF PAPER AND SCISSORS.





1. Fold a piece of paper in half.
   Make sure straight sides match.
   Cut off one corner. →
   Take the corner.
   What shape do you think it will be? A triangle

   Open it.

   a Measure. How many sides are the same length? 2 (Maybe 3)

   b Match. How many angles are the same size? 2 (Maybe 3)

   c Is the fold mark a line of symmetry? Yes

   d Are there other lines of symmetry? No (Unless 3 sides are same length)

   Experiment. How can you cut a folded sheet of paper and be sure each side will be the same length? Use $\frac{1}{2}$ the length of one side to measure the base.

2. Use another piece of paper.
   Fold in half.
   Fold again so that the folded side comes together.
   Measure 3 cm up from the folded corner.
   Mark a point.
   Measure 3 cm out on the other folded side.
   Mark another point.
   Connect the points with a straight line segment.
   Cut along the lines.
   Use the corner.
   What shape do you think you will get? Answers will vary.
   Unfold.

   a Measure. How many sides are the same length? 4

   b Match. How many angles are the same size? 4

   c Are the fold marks lines of symmetry?

   d Are there others? How many? Yes; 2

284

See activity 2, page 288a.

DO THE SAME SORT OF THING AS YOU DID ON THE LAST PAGE. THIS TIME FOLD SO THAT YOU WILL HAVE EIGHT PARTS.

Make sure one folded edge always folds down to match another fold.
OR YOU WILL HAVE A MESS.

1. Mark the same length on each of the folded sides. Connect those points with a straight line segment. Cut along the line. Use the corner piece. Open it.
   - **a** How many sides? 8
   - **b** What is the length of each side? Answers will vary.
   - **c** How many angles? 8
   - **d** Does a fold mark cut through each angle? Yes
   - **e** Is each fold mark a line of symmetry? Yes
   - **f** Are there other lines of symmetry? Yes
     How many? 4

2. Fold another piece of paper in eight parts. This time make some different designs. Your first try will probably look like a paper doily. Keep on. You get some great designs. And each will be symmetrical.

goal Finding congruent parts of an octagon and the lines of symmetry of an octagon

things for each pupil:
several small sheets of paper
scissors
ruler

page 285 Nice and easy with the folding directions for problem 1 – thin paper and sharp creases will help. Everyone knows how to organize for these activities by now.

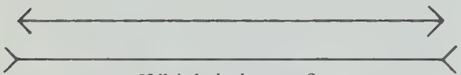
The results from problem 2 make excellent room decorations.

lesson Pages 286, 287

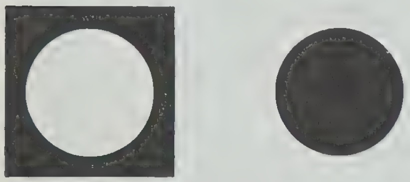
goal Examining how symmetry can cause an optical illusion

page 286 This page begins simply and then the world of optical illusions unfolds. Surely you will have more than one youngster anxious to find still other optical illusions. Maybe the art resource person and the librarian will help with research.

Whet your pupils' research appetites. Try showing some of the old standbys on the chalkboard.



Which is longer?

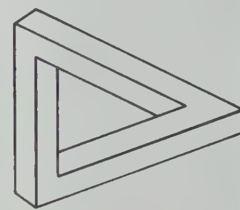


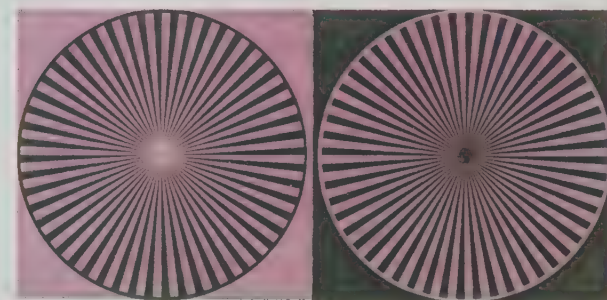
Which is larger?

Many optical illusions are tricks played on your eyes because of symmetry.

Do you see a vase? Yes
Do you see two faces? Yes

Look for two different images in each of these pictures.





286

MEASURE

TRACE

FOLD

ANSWER THESE QUESTIONS ABOUT EACH SHAPE

1. How many sides? A, 3; B, 4; C, 5; D, 6
2. Are they the same length? Yes
3. About how long is each side? A. 38 mm; B. 38 mm; C. 25 mm; D. 22 mm
4. About how far is it around all the sides? A. 114 mm; B. 152 mm; C. 125 mm; D. 132 mm
5. How many angles? A. 3; B, 4; C, 5; D, 6
6. Are the angles the same size? Yes
7. Is the figure symmetrical? Yes
8. How many lines of symmetry? A, 3; B, 4; C, 5; D, 6

DO YOU REMEMBER—

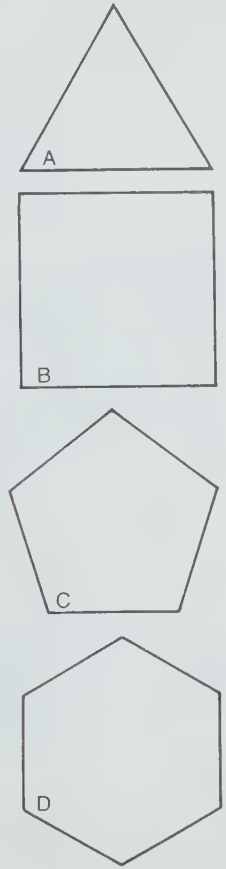
that these are plane figures?

that they are called regular figures?

the name given to each shape?

What are the names?

Triangle
Square
Pentagon
Hexagon



goal Examining optical illusions related to plane geometric figures

things paper for tracing
rulers

page 287 Plane geometric figures can fool you too! The length of the sides of several figures appears to be almost the same. The test to verify this illusion is given at the top of the page.

lesson Page 288

goal **Checkout**—identifying symmetrical figures and lines of symmetry

things paper for tracing

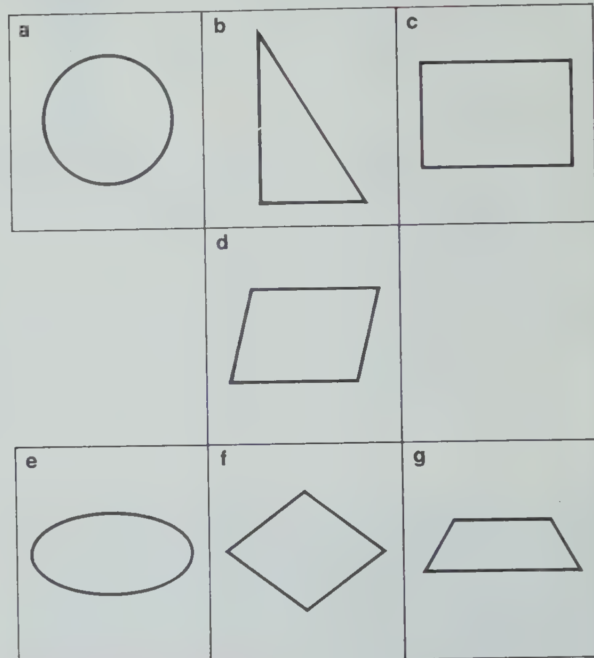
page 288 Everyone knows how to check hunches by now—trace and fold. The Checkout should be completed independently.

Make use of those pictures that the youngsters have collected. Anyone who had trouble with the Checkout needs more tracing, cutting, and folding experience to help him identify symmetrical figures and locate lines of symmetry.

CHECKOUT

Skill: Identifying symmetrical figures

1. Name the shapes that are symmetrical. Trace and fold if you want to make sure. a, c, e, f, g



Skill: Finding lines of symmetry

2. Which of the shapes above have only one line of symmetry? [illegible]

3. Which of the shapes above have more than two lines of symmetry? a

288

things for each pupil: set of symmetrical and nonsymmetrical figures; mirror

Challenge the pupil to use the mirror to help decide whether each shape is symmetrical. Then challenge him to use the mirror to find the line(s) of symmetry.

See activity 3, page 288a.

RESOURCES

another form of evaluation

for checkout–page 288

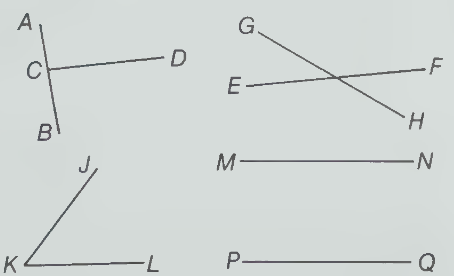
On your paper draw a square, a rectangle, a circle, a triangle, and a trapezoid.

1. Mark the lines of symmetry in your figures.
2. Which figure(s) have only one line of symmetry? Isosceles triangle, some trapezoids
3. Which figures have more than two lines of symmetry? Square, circle, equilateral triangle

activities

1. things spirit master

Prepare a spirit master consisting of the figures shown and the following questions.



1. Is line AB a line of symmetry for line CD? Is line CD a line of symmetry for line AB? Why?
2. Is line EF a line of symmetry for line GH? Is line GH a line of symmetry for line EF? Why?
3. Does a pair of parallel lines have a line of symmetry? If so, where is it?
4. Does an angle have a line of symmetry? If so, what does it do to the angle?

2. things squares of paper; scissors

Individual activity (Provide the pupil with the following directions and questions.)

1. Fold a square of paper in half. Cut off 1 corner. Without opening the paper, draw what you think the figure will look like when the paper is opened. Unfold the paper. Were you right? Would the figure look different if you had cut off a different corner? if you had folded along a different line of symmetry? Try it to make sure.
2. Fold a square of paper into fourths. Do not fold along the diagonals. The result should be a small square. Cut off one corner. Without opening the paper, draw what you think the figure will look like when the paper is opened. Unfold the paper. Were you right? Would the figure look different if you had cut off a different corner?
3. Fold another square of paper in half 3 times. Cut off 1 corner. Without opening the paper, draw what you think the figure will look like when the paper is opened. Unfold the paper. Were you right? Would the figure look different if you had cut off a different corner?

3. things pieces of paper; scissors

Individual activity (Provide the pupil with the following directions and questions.)

1. Fold a rectangular piece of paper in half and cut out an irregular shape from the single folded edge. Open your paper. Is the result symmetrical? How many lines of symmetry does the result have?

2. Fold a rectangular piece of paper in half and in half again. Cut out an irregular shape from the single folded edge. Open your paper. How many lines of symmetry does the result have? How many cut out pieces are there?

3. Fold a piece of rectangular paper 3 times. Cut out an irregular shape from the single folded edge. Open your paper. How many lines of symmetry does the result have? How many cut out pieces are there? What do you notice about these pieces?
4. Can you predict the results for 4 folds? Complete the following chart.

Number of folds	Number of lines of symmetry in the result	Number of cut out pieces
1		
2		
3		
4		

additional learning aids

notation — chapter objectives 1, 2, 3, 4

SRA products

Mathematical Learning System, Activity Masters, level B, SRA (1974)
  Spirit masters: P-2; G-4, 5
diagnosis: an instructional aid — Mathematics Level B, SRA (1972)
  Probe: M-29
Math Applications Kit, SRA (1971)
  Occupations card: 49
Mathematics Involvement Program, SRA (1971)
  Cards: 65, 75, 196, 206, 306
Skill through Patterns, level 5, SRA (1974)
  Spirit masters: 33, 34, 35, 60, 64, 65

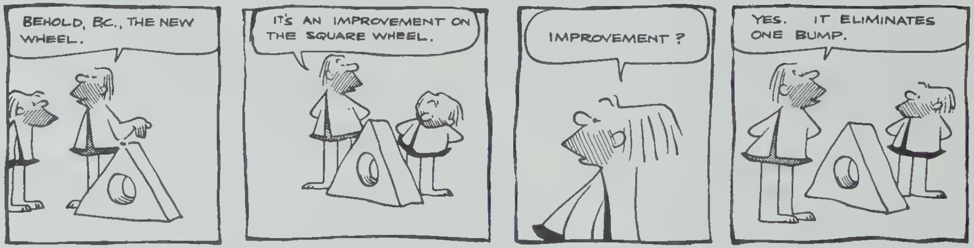
other learning aids (described on page 288g) — Geoboard Activity Cards (intermediate set), Geoboard Kit, Good Time Mathematics, Great Shapes, Mira, Mira Math for Elementary School, Mirror Magic

concept — chapter objective 5

SRA products

Mathematics Involvement Program, SRA (1971)
  Cards: 196, 206
Skill through Patterns, level 5, SRA (1974)
  Spirit masters: 57, 58, 59

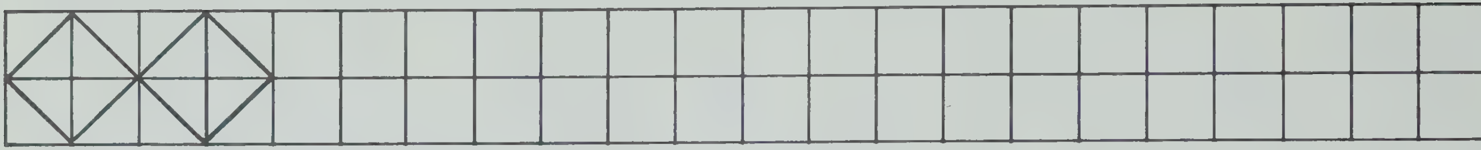
other learning aids — Geoboards and Motion Geometry, Rotation, Translation and Reflection Kit



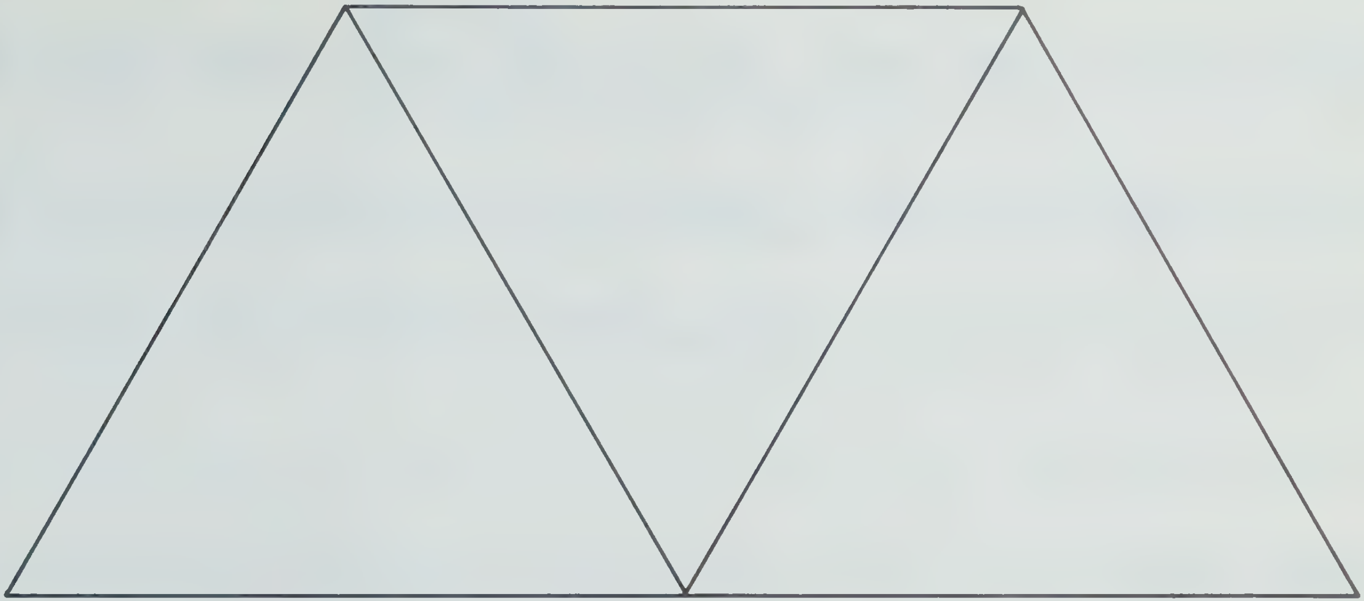
multiplication mini-cards

$\begin{array}{r} 9 \\ \times 9 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ \times 8 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ \times 3 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ \times 2 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ \times 1 \\ \hline \end{array}$
$\begin{array}{r} 8 \\ \times 8 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 3 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 2 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 1 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 0 \\ \hline \end{array}$
$\begin{array}{r} 7 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 3 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 2 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 1 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 5 \\ \hline \end{array}$
$\begin{array}{r} 6 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 3 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 2 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ \times 3 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ \times 2 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ \times 1 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ \times 0 \\ \hline \end{array}$

Use your pencil and ruler. Make a repeating border pattern for each row of squares. Add color to make the design more interesting.

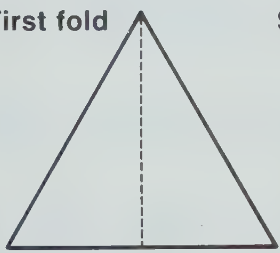


Cut apart these 3 triangles. (Use them as a pattern to make more if you need to.)

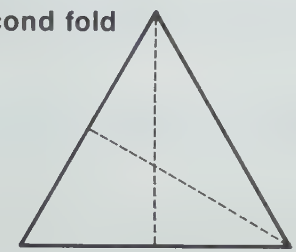


Use one triangle. Fold it.

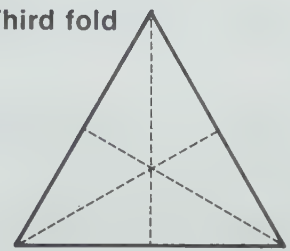
First fold



Second fold



Third fold



Experiment

Cut along any of the fold lines.
Get triangles that are different sizes.

Use another triangle. Fold again.
Cut on any of the fold lines.
Can you find still more different sized triangles?

Use the third. Cut apart.
Get paper and paste.
Now make a symmetrical design.

Look carefully. Mark each symmetrical letter.
Which type has the most symmetrical letters?

ABCDEFGHIJKLMNOPQRSTUVWXYZ

ABCDEFGHIJKLMNOPQRSTUVWXYZ

ABCDEFGHIJKLMNOPQRSTUVWXYZ

ABCDEFGHIJKLMNOPQRSTUVWXYZ

ABCDEFGHIJKLMNOPQRSTUVWXYZ

ABCDEFGHIJKLMNOPQRSTUVWXYZ

ABCDEFGHIJKLMNOPQRSTUVWXYZ

ABCDEFGHIJKLMNOPQRSTUVWXYZ

ABCDEFGHIJKLMNOPQRSTUVWXYZ

13 COMPUTATION

before this chapter the learner has—

Developed the concepts and practised the skills to be mastered in this chapter

in chapter 13 the learner is—

1. Reviewing rounding and estimation skills
2. Reviewing and practising estimating and finding the sum of 3- and 4-digit addends
3. Reviewing and practising estimating and finding the difference of 3- and 4-digit numbers
4. Mastering estimating and finding the product of 2- and 3-digit factors
5. Mastering estimating and finding the quotient for any 3- or 4-digit number and any 2-digit number
6. Working with fractions of whole numbers
7. Reviewing and practising the addition and subtraction of two fractions with like or unlike denominators
8. Reviewing and practising multiplication with fractions and mixed numbers
9. Mastering the addition and subtraction of decimals expressed in tenths or hundredths
10. Reviewing metric units and changing to related units
11. Finding prime and composite numbers
12. Reviewing the indentification of plane geometric figures and solid geometric shapes
13. Reviewing and extending the concept of symmetry
14. Exploring a potpourri of mathematical ideas

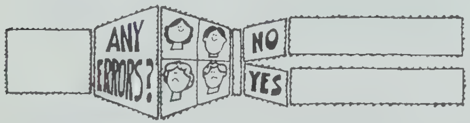
in later chapters the learner will—

1. Maintain the skills mastered
2. Apply and extend the concepts and skills mastered

Notes & Things

This is a chapter that really covers the whole works. It reviews all the learning in this level. It's mainly a self-study chapter and it contains a wild variety of things to do. The most important feature is the series of diagnostic exercise sets. Each problem in these sets is different from the next. Each problem evaluates a child's thinking pattern or a subskill of the major skill that is to be mastered by the end of the level.

You will see a chart like this following each diagnostic set.



On the basis of his errors, the pupil will be directed right on the text page to special help and practice that should help strengthen any weak skill. (You will want to do some snooping to make sure the child doesn't also need your personal help or encouragement.) If a pupil doesn't make any mistakes, there is time for some special activities that will put his general knowledge of math and problem-solving skills to work.

Because of this chapter's organization, there is no need for the usual Progress Checks. The pupil has already been through that routine in the earlier chapters. There is a rich source of additional suggestions for you, however, just in case you think a child needs still more practice.

Please notice that the answers to the diagnostic exercises are printed on page 336 in the text. You may want to take time out and talk about how each individual needs to use these answers in order to get maximum benefits.

things

references containing tables of measure (optional)
markers
mirrors
coffee filters

goal Think about and explore ideas through a picture clue

page 289 The blur of taillights in this time-exposure photograph hints of speed and progress. Let this be the mood for this last chapter in the book. It should be a joy for everyone to finish the level and be ready to go on to something new.

lesson Pages 290, 291

goal Establishing the organization of the chapter

page 290

Look & enjoy!

Your goal is to review your computational skills and have some fun, too!

There were no supermarkets in the early 1900s. There were very few department stores. Everything a family needed was found at the general store. Many foods were sold from big barrels. (They didn't have fancy packages in those days.) People used to gather at the general store and talk. These talks were called cracker-barrel sessions. They talked of things in the past and things in the future.

You can think of this chapter as a sort of math general store. It presents number ideas from the past. It also has some number ideas for the future. It has other things, too. There are three types of pages in this chapter.

pages. These pages will give everyone a chance to share ideas and discuss what's coming next.

Very Important Problems pages. You will work through these pages on your own. If you show that you know how to do them, you won't have to do many. If you don't know how to do them, you will get some help and have a chance to practise.

Mixed Bag pages. These are scattered throughout the chapter. They are some interesting extra activities you can do when you have time.

goal Establishing the organization of the chapter *(continued)*

page 291 The three types of pages around which the chapter is organized are explained on the page. You'll want to talk about these together.

Cracker-Barrel Session pages have ideas that can be shared.

Very Important Problems pages contain some problems that will determine whether or not the pupil has mastered a specific skill. Each person will correct his own work—the answers are at the end of the chapter on page 336. A chart will direct the pupil to his next assignment. The first of the charts is found on page 294. The skill tested is identified in each chart. You'll want to make sure everyone knows how to read the charts. The additional practice problems are designed so that most pupils directed there can continue independently. You will be free to provide individual or small-group help.

Mixed Bag pages are well named. They present enrichment activities that will help prove math can be something more than computing problems. Everyone should have the experience of working with some of these activities. Those pupils who find themselves practicing a lot deserve a little relief from time to time. Others, whose skills check out, will do nearly all these pages.

All in all, the chapter is a potpourri—it provides a little of everything—as did the old general store. Do encourage the children to ask for help when they do not understand or are not succeeding.

lesson Pages 292, 293

goal **Survey**—ability with numbers and numeration

memo Pages 292 and 293 are used together in answering the questions.

page 292 These pages show real-world uses of numbers. The questions will help you identify each pupil's ability in reading and using numerals.

1. What is the largest number you can find on these two pages? 20 000 000

2. What is the smallest number you can find? 0.04

3. Find the roman numerals. Write them down. Then rewrite them as standard numerals.
MCMLX-1960; CMXCIX-999; XI-11 or IX-9

4. Find all the numbers that are in measurement situations. Write them. Tell whether the unit of measure is length, mass, capacity, time, temperature, or money. $100-money; 8:00 P.M.-time
24°C-temperature; 35 kg-mass; 67¢-money; 2:31-time, and so on.

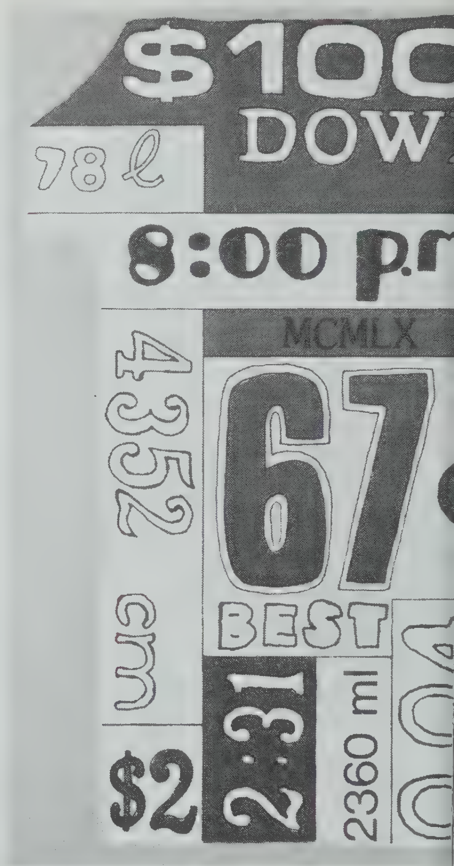
5. Write the fractions you can find. $\frac{7}{8}$, $23\frac{1}{2}$; also consider 0.04, 339.05

And here is a real challenge!

As soon as mathematics class is over, try not to say any number of any kind until tomorrow at this same time. Keep track of the times you had to say a number. Jot down the reason you had to say it.

Remember — no numbers! No talk of time, money or temperature. No street numbers, or bus numbers, or TV channels, or phone numbers either.
Get the idea?

Good luck!



292

goal Helping the pupil become aware of his dependence on number and numerals

page 293 A fun way to make the pupil aware of why numbers are necessary and how often he uses numbers without thinking about using them.

lesson Pages 294, 295, 296

goal Diagnosis of ability in rounding numbers

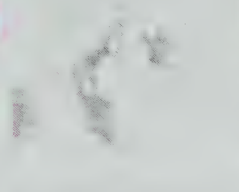
page 294 The diagnostic check will automatically initiate the grouping process:
- Pupils going directly to page 296
- Those directed to the practice problems

These groupings will change with each diagnostic check.

Very Important Problems

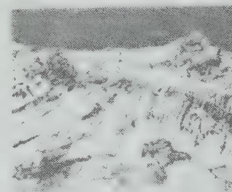
Diagnostic check on rounding to the nearest ten, hundred, thousand, ten-thousand

Pictured below are several of the tallest mountains in the world. Their exact heights are given. Usually the heights of mountains are given as rounded numbers. That job is left to you.



Mount Everest in Asia

8948 m



Mount Aconcagua in South America

6960 m

Mount Kilimanjaro in East Africa

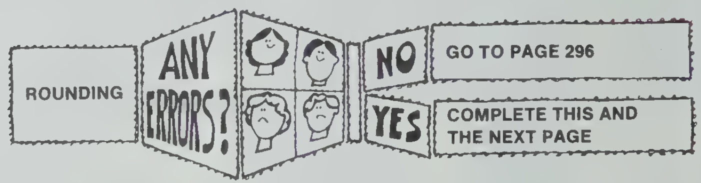
5895 m

a Round each height to the nearest ten. 8950; 6960; 5900

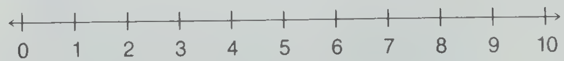
b Round each height to the nearest hundred. 8900; 7000; 5900

c Round each height to the nearest thousand. 9000; 7000; 6000

Check your answers with the key on page 336.



Look at the number line below.



Which numbers on the line are closer to 0 than to 10? 1, 2, 3, 4

Which are closer to 10 than to 0? 6, 7, 8,

Is 5 closer to 0 or to 10? Neither

When rounding, always round 5 up to 10.

294

rounding

Skill: Rounding to nearest ten

When rounding to the nearest ten, look at the ones. If there are 5 or more ones, round up. If there are less than 5, round down.

For example: 27 rounds to 30. 495 rounds to 500. 3574 rounds to 3570.

Now it's your turn. Round these numbers to the nearest ten.

1. 9 10 **2.** 43 40 **3.** 89 90 **4.** 104 100 **5.** 644 640 **6.** 898 900

7. 2350 2350 **8.** 4788 4790 **9.** 73 70 **10.** 10,515 10,520 **11.** 17 20 **12.** 666 670

Skill: Rounding to nearest hundred

Is $153 closer to $100 or to $200? When rounding to the nearest hundred, look at the number to the right of the hundreds. If there are 5 or more tens, round up. If there are less than 5 tens, round down.

For example: 237 rounds to 200. 71 rounds to 100. 4672 rounds to 4700.

Your turn again. Round to the nearest hundred.

13. 508 500 **14.** 381 400 **15.** 35 0 **16.** 299 300 **17.** 650 700 **18.** 1783 1800

19. 1870 1900 **20.** 992 1000 **21.** 6430 6400 **22.** 12,745 12,700 **23.** 57 100 **24.** 4673 4700

Skill: Rounding to nearest thousand

When rounding to the nearest thousand, look at the numbers to the right of the thousands.

For example: 23,532 rounds to 24,000. 4499 rounds to 4000.

Round these numbers to the nearest thousand.

25. 1570 2000 **26.** 3845 4000 **27.** 2220 2000 **28.** 6399 6000

29. 7904 8000 **30.** 15,038 15,000 **31.** 34,762 35,000 **32.** 84,360 84,000

Skill: Rounding to nearest ten-thousand

Can you round to the nearest ten-thousand? Try it with these numbers. (You might want to think of the numbers as automobile mileages.)

33. 18,724 20,000 **34.** 33,786 30,000 **35.** 25,000 30,000 **36.** 76,369 80,000

295

goal Practice in rounding numbers

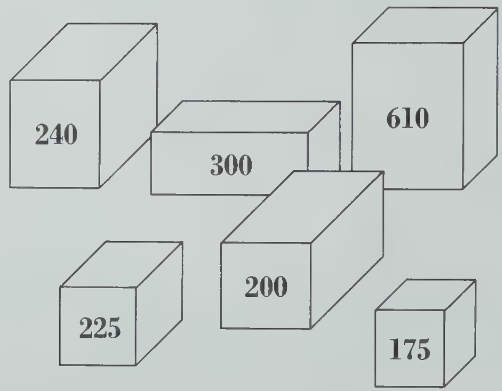
page 295 Check each youngster after he has completed the first two or three problems in each group. The skills are identified on the reduced pupil page to help you quickly identify the pupil's need. Make sure that he is succeeding before he continues to practice. Provide help and adjust additional assignments to meet individual needs.

goal Application of rounding and estimation skills

page 296 Youngsters who have been involved with the practice on the previous page may want to work this page in their spare time or as homework. Everyone need not complete this page—but it does provide challenging practice.

Mixed Bag

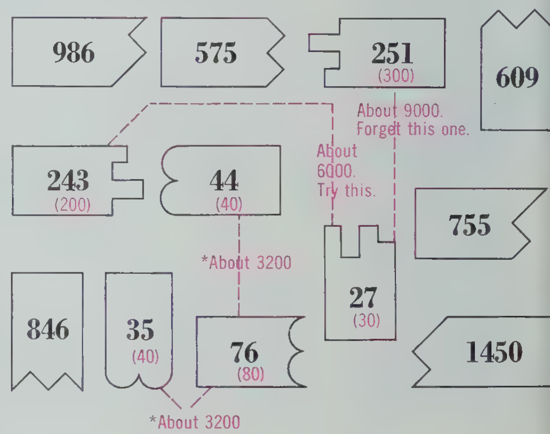
1. You have a boat that can carry 900 pounds of cargo. There are 6 boxes that need to be taken across the lake. What boxes can you take at one time so that you have to make only two trips? ① 610, 240 ② 300, 225, 200, 175



2. Here's another easy puzzle. Use estimates to help you. Find two pieces that fit together, and name each one of these products.

Each product a
should be roun
examples given

*(3000) 2660 35 × 76	(500,000) 515,214 846 × 609	6561	744,430



296

Cracker Barrel Session

You use rounding to estimate answers to addition problems.

Jay wanted to buy these things:

Book \$2.98
Shirt \$3.59

He had \$5.00. Did he have enough money?

\$2.98 rounds to \$3.00
\$3.59 rounds to \$3.60

No, he didn't have enough money.
You don't need pencil and paper for that.

Using rounded numbers, estimate and answer these questions.

1 He needed about 500 trading stamps.
He had 185 in one book.
And 335 in another book.
Did he have about 500? Yes; 200 + 300 = 500

2 They hoped about 700 people would vote.
372 people voted in one place.
229 people voted in another place.
Did about 700 people vote? No; 400 + 200 = 600

3 There were about 100 chairs in the room.
36 parents planned to come.
24 students would attend.
And 9 teachers would be there.
Were there enough chairs? Yes; 40 + 20 + 10 = 70

4 Are these reasonable answers?

a 395 + 475 = 760
No; 400 + 500 = 900

b 525 + 675 = 1110
No; 500 + 700 = 1200

297

lesson Page 297

goal Applications requiring estimated answers

page 297 From time to time the learner is reminded that often in the real world an estimate is a sufficient answer. This page illustrates the idea.

You will probably want to discuss this page with those youngsters who were directed from page 294 to page 296 so that they can continue independently.

Youngsters who are working with the practice activities on pages 294 and 295 can jump to this page when they show improved skill in rounding.

lesson Pages 298, 299

goal Diagnosis of ability and practice in estimating and finding the sum of 3- and 4-digit addends

page 298 The diagnostic check identifies the pupil's ability to estimate as well as to compute. The estimate should determine the reasonableness of the computed answer.

You'll want to check the progress of youngsters directed to the practice problems after they have completed two of those problems. Additional help may be needed with—

- Rounding and estimating
- Addition facts

Adjust additional assignments to meet the needs of the individual.

Diagnostic check on estimating sums with 3- and 4-digit addends

Very Important Problems

Estimate the sums. Then compute exact sums.

a	375	400	**b**	5267	5000	**c**	525
	+ 195	+ 200		+ 1956	+ 2000		296
	570	600		7223	7000		+ 713 +
							1534

Check your answers with the key on page 336.

An estimate will not tell you if your answer is right. It will tell you if your answer is reasonable.

695	rounds to	700
+ 225	rounds to	200
		900 is a good estimate.
		The answer is 920.
		It is reasonable.

Skill: Estimating sums with 2-, 3-, and 4-digit addends

Here are some completed problems. Some are right. Some are unreasonable. Round the numbers computed. Then estimate to find which answers are not reasonable. Do not compute.

1. 476 + 341 = 817 (500 + 300 = 800 O.K.)

2. 222 + 993 + 316 = 5131 (200 + 1000 + 300 = 1500)

3. 11 + 198 + 224 = 533 (10 + 200 + 200 = 410)

4. 8825 + 2914 = 11,739 (9000 + 3000 = 12,000 O.K.)

5. 326 + 555 + 189 = 970 (300 + 600 + 200 = 1100)

6. 75 + 284 + 407 = 666 (100 + 300 + 400 = 800)

7. 4008 + 8986 = 12,994 (4000 + 9000 = 13,000 O.K.)

8. 51,294 + 3,414 = 55,706 (51,000 + 3,000 = 54,000)

298

Mixed Bag

You can add the first three counting numbers in your head: $1 + 2 + 3 = 6$

Now add the first ten counting numbers in your head.
No pencil and paper, please.
That's not quite so easy, right?

1	Look at them in a column.		1
2	Can you add them now? Yes		2
3			3
4	The column on the right shows		4
5	an easier way of doing it.		5
6		$1 + 10 = 11$	6
7		$2 + 9 = 11$	7
8		$3 + 8 = 11$	8
9		$4 + 7 = 11$	9
10		$5 + 6 = 11$	10

5

How many 11s? What's the product? 55

What's the sum of the first ten counting numbers? 55

Try to find shortcuts for adding these numbers.

1. $2 + 3 + 4 + 5 + 6 + 7$ = 27

2. $5 + 6 + 7 + 8 + 9 + 10$ = 45

3. $4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12$ = 72

4. $2 + 4 + 6 + 8$ = 20

5. $2 + 4 + 6 + 8 + 10 + 12$ = 42

6. Experts only! Find the sum of the first 28 counting numbers without writing all the numbers down. Now try the first 39. **GOOD LUCK!**
406 780

goal Finding a pattern for adding consecutive numbers

page 299 This is meant to be a challenge for young mathematicians. No help, please—except to clarify directions for anyone who does not understand.

lesson Pages 300, 301, 302

goal Diagnosis of ability and practice in estimating and finding the difference of 3- and 4-digit numbers

page 300 Again, the emphasis is on using the estimate to verify the reasonableness of the computed answer. Ability in both computation and estimation is diagnosed.

Do check the success of youngsters sent to the additional practice after they have done the first two problems. More help may be needed with—

- Rounding and estimating
- Subtraction facts

Use your discretion in making additional assignments. Once a youngster shows evidence of having mastered a skill, he does not need to continue practicing. The skill will be maintained throughout the chapter.

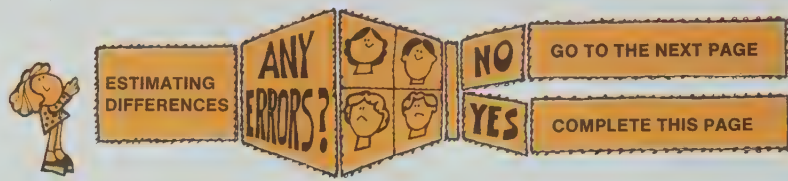
Diagnostic check on estimating differences with 3- and 4-digit numbers

Very Important Problems

Estimate the differences. Then compute exact differences.

a	705	700	**b**	6020	6000	**c**	4250	4000
	− 496	− 500		− 2856	− 3000		− 987	− 1000
	209	200		3164	3000		3263	3000

Check your answers with the key on page 336.



An estimate will tell you if your answer is reasonable.

963	rounds to	1000
− 225	rounds to	200
		800 is a good estimate.

The exact answer is 738. Is it reasonable?

Skill: Estimating differences with 3- and 4-digit numbers

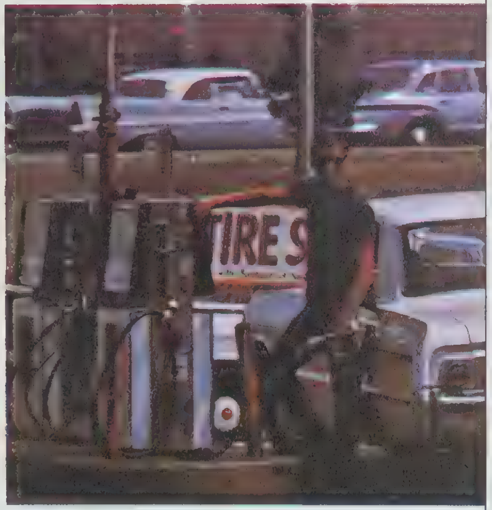
Here are some completed problems. Some are right. Some are unreasonable. Round; then estimate to find which answers are not reasonable. Do not compute.

1.	824	800	**2.**	601	600	**3.**	8523	9000	**4.**	7030	7000
	− 387	− 400		− 225	− 200		− 3918	− 4000		− 942	− 1000
	537	400		376	400 O.K.		4605	5000 O.K.		6988	6000
5.	2725	3000	**6.**	5104	5000	**7.**	426	400	**8.**	1403	1400
	− 1826	− 2000		− 3313	− 3000		− 149	− 100		− 649	− 600
	1899	1000		1791	2000 O.K.		377	300		854	800 Maybe O.K.

300

You use rounding to estimate answers to problems.
Here is an example:

Mr. White filled his car's gasoline tank. According to the gasoline pump's gauge, it pumped 17 gallons for $8.50. The price shown was 31 cents per gallon. Was the meter working properly?



You can do this problem in your head. **Watch!**

17 rounds to 20.
31 rounds to 30.
How much is 20×30? 600
Is 850 close to 20×30? No

No
Was the pump working properly? Check to make sure.
Multiply 17×31 with pencil and paper. $17 \times 31 = 527$

An estimate will not tell you if your answer is right.
It will tell you if your answer is reasonable.

Are these reasonable?

1. Belle earns $.75 an hour. She worked 12 hours one week. She said she earned about $75.00 that week.
Is that reasonable?
No; $\$.80 \times 10 = \$8.00$

2. Bob had 7 boxes. Each box had 750 clips in it. He said he had less than 5000 clips.
Is that reasonable?
No — $7 \times 800 = 5600$; should be more than 5000

3. Mr. Jones traveled 37 miles every day to get back and forth from work. He said that he drove over 200 miles each week just traveling to and from work.
Is that reasonable?
Maybe; $40 \times 5 = 200$. Need to know how many days of work — 5 or 6.

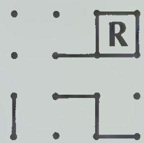
goal Application of rounding skills to real-world situations

page 301 Estimation, as applied to real-world situations requiring multiplication, is explored before this skill is checked on page 303. Using the estimate to verify **reasonableness** is emphasized again. You will need to examine this page as a group if there are pupils who have a reading problem.

goal Connecting points to form a regular hexagon

page 302 An old game with a different twist—a game to enjoy and play whenever time permits. Challenge your gamesmen to design additional patterns of their own. Can a figure other than a square, a regular hexagon, or a triangle be used?

Mixed Bag

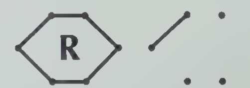


You may already know how to play the game of Dots. Each player connects two dots with a vertical or horizontal line segment. Eventually a player will complete a square. When he does, he puts his initials inside it and gets an extra turn. The player who makes the most squares wins.

You can play another version of Dots with regular hexagons instead of squares.

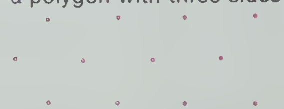
Copy or trace the following pattern of points. Connect the dots with horizontal or diagonal line segments only.

When a player forms a hexagon, he puts his initial inside and gets an extra turn.
Example:



Why is this pattern necessary for a game of Dots with regular hexagons? A hexagon has 6 sides.

Can you design a pattern for a game of Dots with a polygon with three sides?



302

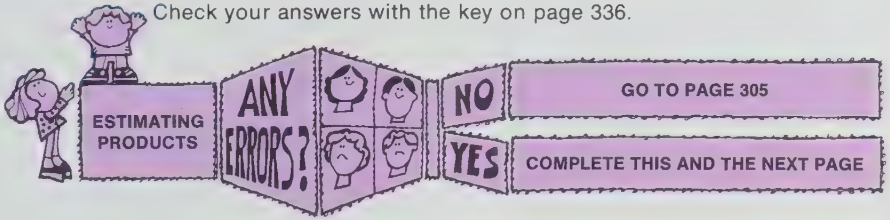
Diagnostic check on estimating and finding exact products with
2- and 3-digit divisors

Very Important Problems

Estimate the products. Then compute the exact products.

Item	Number of boxes	Number in each box	Estimated product	Exact product
Paper hats	12	12	100	144
Music makers	18 (20)	36 (40)	a ? 800	b ? 648
Balloons	237 (200)	72 (70)	c ? 14,000	d ? 17,064
Licorice sticks	578 (600)	144 (100)	e ? 60,000	f ? 83,232

Check your answers with the key on page 336.



When estimating in multiplication, you round both factors and then multiply. But do you always know which place to round to?

Your goal is to estimate the answer in your head.
Use numbers you can multiply without using pencil and paper.

538	rounds to	500
× 74	rounds to	70

You can do this in your head.

Then when you compute the exact answer, you'll know if it's reasonable.

Would an answer close to 350 be reasonable? No

Would an answer close to 3500 be reasonable? No

What number should the exact answer be close to? 35,000

lesson Pages 303, 304, 305

goal Diagnosis of ability and practice in estimating and finding the product of 2- and 3-digit factors

page 303 Both computational skill and ability to estimate are diagnosed. Additional help is provided at the bottom of the page. Practice sets are on page 304.

Youngsters who have difficulty with computation should begin with the estimation work at the bottom of this page. This will help you pinpoint those who are really in trouble.

goal Practice in estimating and finding the product of 2- and 3-digit factors

page 304 In each set of problems, check the youngster's success after he has computed the first row of problems. No sense in continuing to practice without success.

Examine the following pattern with learners who have difficulty with problems 1 through 10.

tens × tens = hundreds
10 × 30 = 300 hundreds – 2 zeros
tens × hundreds = thousands
80 × 300 = 24,000 thousands – 3 zeros
hundreds × hundreds = ten-thousands
400 × 700 = 280,000 ten-thousands – 4 zeros

Anyone shaky on place value may require the assistance of a place-value chart.

Note that for the last set of problems, the learner is directed to estimate **and** to compute. Checking the reasonableness of the computed answer should help eliminate silly mistakes.

Do these in your head. Write the answers only. Skill: Multiplying multiples of 10 and 100

1. 30 × 10 — 300
2. 50 × 20 — 1000
3. 70 × 40 — 2800
4. 200 × 50 — 10,000
5. 300 × 80 — 24,000
6. 500 × 70 — 35,000
7. 900 × 60 — 54,000
8. 300 × 200 — 60,000
9. 700 × 400 — 280,000
10. 800 × 700 — 560,000

First round the numbers. Then write the product of the two rounded numbers. Do not find the exact product. Skill: Estimating products with 2- and 3-digit factors

11. 24 × 13 — 20 × 10 = 200
12. 57 × 49 — 60 × 50 = 3000
13. 84 × 75 — 80 × 80 = 6400
14. 115 × 38 — 100 × 40 = 4000
15. 250 × 95 — 300 × 100 = 30,000
16. 473 × 46 — 500 × 50 = 25,000
17. 851 × 99 — 900 × 100 = 90,000
18. 777 × 129 — 800 × 100 = 80,000
19. 501 × 291 — 500 × 300 = 150,000
20. 699 × 549 — 700 × 500 = 350,000

Use rounding to estimate the product. Then compute the exact answer. Skill: Estimating and finding exact products with 2- and 3-digit factors

21. 34 × 12 — (300) 408
22. 97 × 45 — (5000) 4365
23. 54 × 48 — (2500) 2592
24. 125 × 43 — (4000) 5375
25. 383 × 71 — (28,000) 27,193
26. 899 × 99 — (90,000) 89,001
27. 715 × 14 — (7000) 10,010
28. 481 × 149 — (50,000) 71,669
29. 809 × 305 — (240,000) 246,745
30. 555 × 499 — (300,000) 276,945

These look like ***awful*** problems, but you'll be surprised. Multiply and see what happens.

Mixed Bag

1. $\begin{array}{r} 12345679 \\ \times \quad 9 \\ \hline 111{,}111{,}111 \end{array}$

2. $\begin{array}{r} 12345679 \\ \times \quad 18 \\ \hline 222{,}222{,}222 \end{array}$

3. $\begin{array}{r} 12345679 \\ \times \quad 27 \\ \hline 333{,}333{,}333 \end{array}$

4. What do you think the product will be if you multiply by 36? You'd better check to make sure. 444,444,444

5. If the pattern holds, what will the product be if you multiply by 45? 54? 63? 72? 81? 555,555,555; 666,666,666; 777,777,777; 888,888,888; 999,999,999

The poor digit 8 was almost ignored in the problems above. That's not fair. Complete this computation and see what happens.

6. $(9 \times 9) + 7 = \underline{?}$ 88
7. $(98 \times 9) + 6 = \underline{?}$ 888
8. $(987 \times 9) + 5 = \underline{?}$ 8888

9. If the pattern holds, what would be the next complete sentence in order? $(9876 \times 9) + 4 = 88{,}888$

10. Only one other digit has not been considered. That's zero. You can take care of that in a hurry. Zero times any number equals what number? 0 Now all digits are present and accounted for.

goal Finding patterns in multiplication problems

page 305 Don't hesitate to use a page like this as a homework assignment for those youngsters who have been working with the practice exercises. They need to see the fun side of math from time to time.

lesson Page 306

goal Estimating quotients in division

page 306 Time out for a how-to page—how to round and estimate quotients in division. This concept needs to be discussed. Estimation is more important in division than in any other operation, because it is used at each step in computing the exact quotient.

Cracker Barrel Session

You can use estimation for a quick quotient that does not need to be exact. For example:

The winner of the Grand Sports Car Race averaged 88 miles per hour. The course was 579 miles long. How many hours did the race last?

Find an estimated answer without using pencil and paper.

Is this what you did?

$88\overline{)579}$ Round 88 to 90.

Think about how many 90s in 579?

$6 \times 90 = 540$ and $7 \times 90 = 630$.

So the race lasted between 6 and 7 hours.

Now compute to find the exact answer.
What is it? Is the estimated answer close? 6 R51; yes

Estimate these answers. Do not compute.

1. About how many 20s in 120? 6 (Think ? × 2
2. About how many 25s in 120? 4 (Think ? × 3
3. About how many 20s in 1200? 60 (Think ?
4. About how many 25s in 1200? 40 (Think ?
5. About how many 50s in 500? 10 (? × 50 =
6. About how many 52s in 500? 10 (? × 50 =
7. About how many 57s in 500? 8 (? × 60 =
8. About how many 70s in 4800? 70 (? × 70
9. About how many 70s in 4900? 70 (? × 70
10. About how many 70s in 500? 7 (? × 70 = 5
11. About how many 47s in 250? 5 (? × 50 = 2
12. About how many 53s in 250? 5 (? × 50 = 2
13. About how many 58s in 250? 4 (? × 60 = 2
14. About how many 62s in 250? 4 (? × 60 = 2
15. About how many 85s in 6300? 70 (? × 90 =
16. About how many 89s in 6300? 70 (? × 90 =
17. About how many 93s in 6300? 70 (? × 90 =
18. About how many 95s in 6300? 63 (? × 100

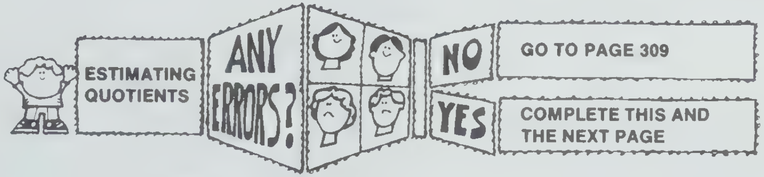
Diagnostic check on estimating and finding exact quotients with 2-digit divisors

Very Important Problems

Here are some other plans for cycling trips.
First estimate the days needed to complete the trip.
Then compute the exact number.
(Days for stopovers are not included.)

Kilometres per day	Length of trip	Number of days (estimated)	Number of days (exact)
45	135	2	3
79	237	a ? 3	b ? 3
63	756	c ? 12	d ? 12
99	1089	e ? 10 or 11	f ? 11

Check your answers with the key on page 336.



$30\overline{)720}$ About how many 30s in 720?
If you say 20, you're right. $20 \times 30 = 600$
But there may be more than 20.
If you say 30, you're also right. $30 \times 30 = 900$
But there are fewer than 30.
The answer is between 20 and 30.

Skill: Estimating quotients with a multiple of ten as the divisor

Estimate the quotients. Do not compute.

1. $40\overline{)320}$ 8
2. $50\overline{)450}$ 9
3. $20\overline{)1800}$ 90
4. $60\overline{)3000}$ 50
5. $30\overline{)900}$ 30
6. $90\overline{)550}$ between 6 and 7
7. $70\overline{)500}$ between 7 and 8
8. $50\overline{)4100}$ between 80 and 90
9. $80\overline{)5000}$ between 60 and 70
10. $60\overline{)7000}$ between 100 and 200

lesson Pages 307, 308, 309

goal Diagnosis of ability and practice in estimating and finding quotients with 2-digit divisors

page 307 Again, skill in both estimation and computation are diagnosed. Learners who have difficulty only in estimating should concentrate on this single skill in the practice work.

Assure your pupils that there is no one right estimate for a problem.
About how many 40s in 340?
The estimate 8 answers the question "About how many?"
The estimate 9 also answers the question "About how many?"
Sure, there are less than 9, but 9 is not a wrong estimate. It is a high estimate—too high for the purpose of computing.
This idea is presented in the middle of the page—for discussion with those pupils who need such a discussion.

Pupils who have difficulty computing should practice estimating as well as computing exact answers.

Note that for problems 1 through 5 the dividend is a multiple that is clearly seen, whereas for problems 6 through 10 the learner may respond that the quotient is between two multiples. For example, in problem 6 the pupil may respond that the quotient is between 6 and 7 or he might select either of these two numbers.

goal Practice in estimating and finding quotients with 2-digit divisors

page 308 Your help may be needed here. For many of your pupils the review and development presented will be sufficient; others may need to be led step-by-step through several problems. Estimation is used repeatedly when dividing. This skill must be mastered before speed and accuracy can be attained in division. Verify mastery of multiplication facts for youngsters who experience difficulty.

Check progress after one or two examples. Have the pupil identify where estimation is used—then multiplication, subtraction, and addition. Adjust the assignment for individual pupils.

NOW FOR ESTIMATION AND COMPUTATION

$27\overline{)6264}$ Estimate. About how many 30s in 6264?

$$\begin{array}{rl} 200 & \text{Was this your estimate?} \\ 27\overline{)6264} & \\ \underline{5400} & 200 \times 27 \\ 864 & \text{remain} \end{array}$$

Estimate again.
How many 30s in 864? → 20

$$\begin{array}{rl} 20 & \\ 200 & \\ 27\overline{)6264} & \\ \underline{5400} & \\ 864 & \\ \underline{540} & (20 \times 27) \\ 324 & \\ \underline{270} & (10 \times 27) \text{ Where does the 10 go?} \\ 54 & \\ \underline{54} & (2 \times 27) \quad \text{Where does the 2 go?} \\ 0 & \end{array}$$

Estimate again. How many 30s in 324?

You don't have to estimate this.

What is the final quotient? 232

Division is a lot of work. You have to know how to estimate. You have to know how to multiply. You have to know how to subtract. And you have to add, too.

Pick any six of these division problems to complete. Skill: Finding exact quotients with 2-digit divisors

1. $17\overline{)541}$ 31 R14
2. $21\overline{)684}$ 32 R12
3. $62\overline{)471}$ 7 R37
4. $78\overline{)8325}$ 106 R57
5. $56\overline{)1573}$ 28 R5
6. $20\overline{)8071}$ 403 R11
7. $91\overline{)3763}$ 41 R32
8. $88\overline{)7541}$ 85 R61
9. $69\overline{)7892}$ 114 R26
10. $33\overline{)3994}$ 121 R1

308

Mixed Bag

3	?	4	?	6	=	2
?		?		?		?
2	?	3	?	4	=	10
?		?		?		?
7	?	6	?	2	=	11
=		=		=		=
13	?	6	?	4	=	23

Copy this puzzle on paper. All you have to do to complete the puzzle is put operation symbols in the empty boxes. You can't put them any old place. You must complete a true sentence in every row or column that has numbers.

?	×	?	÷	?	=	4
×		−		×		+
?	×	?	÷	?	=	16
÷		+		−		−
?	+	?	+	?	=	11
=		=		=		=
3	×	1	+	6	=	9

This one is not so easy. Copy this puzzle too. All you have to do in this one is find the correct numbers to place in each empty box. You must complete a true sentence in each row and column that has numbers.

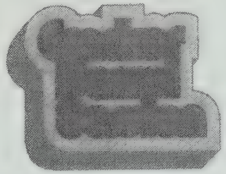
goal Writing math sentences using all four basic operations

page 309 A real challenge for anyone who is interested in trying. These puzzles need not be completed now. Pupils can return to them in their spare time.

goal Examining an early mathematics text, noting the difference in emphasis

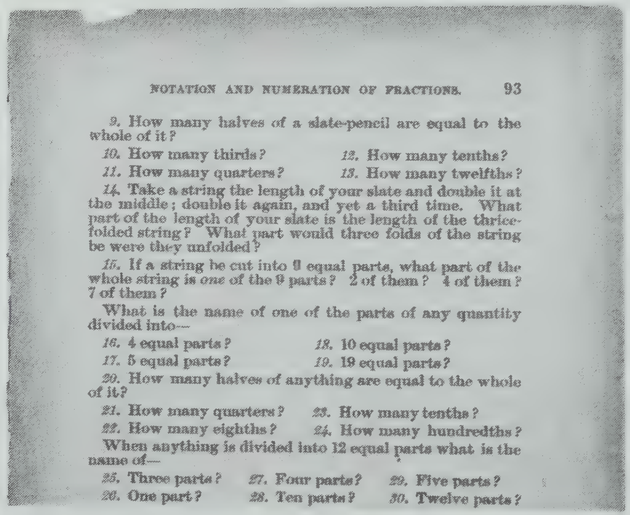
page 310 This is a fun page, which can help to put the study of mathematics in perspective in a changing world.

Let the pupils read the problems in the nineteenth-century book on their own, and make a list of the ones they think they could not handle. Then use the questions in the text as the basis for class discussion. Some interesting points about the differences between then and now may come up. Some pupils may gasp to think that there were 38 solid pages on fractions in the book of great-grandfather's day.



How would you get along with these questions?

They take up half a page in a book used in some schools before 1900. Beginning on page 92, work in fractions filled 38 pages.



NOTATION AND NUMERATION OF FRACTIONS. 93

9. How many halves of a slate-pencil are equal to the whole of it?

10. How many thirds? *12.* How many tenths?

11. How many quarters? *13.* How many twelfths?

14. Take a string the length of your slate and double it at the middle; double it again, and yet a third time. What part of the length of your slate is the length of the thrice-folded string? What part would three folds of the string be were they unfolded?

15. If a string be cut into 9 equal parts, what part of the whole string is *one* of the 9 parts? 2 of them? 4 of them? 7 of them?

What is the name of one of the parts of any quantity divided into—

16. 4 equal parts? *18.* 10 equal parts?

17. 5 equal parts? *19.* 19 equal parts?

20. How many halves of anything are equal to the whole of it?

21. How many quarters? *23.* How many tenths?

22. How many eighths? *24.* How many hundredths?

When anything is divided into 12 equal parts what is the name of—

25. Three parts? *27.* Four parts? *29.* Five parts?

26. One part? *28.* Ten parts? *30.* Twelve parts?

1. What are some of the differences between this mathematics book and yours?
   Small type, no illustrations, 38 pages on fractions
2. Which problem gives instructions that you cannot follow in a classroom today? No.
3. Do you think you can answer these questions as well as a pupil then was expected to do? If you want to try, write the number of the question and the answer.

310

People who use recipes that their grandparents liked need to know fractions. Old cookbooks always included many measurements that used fractions. With metric cookery, you may think you need to know only decimal fractions. You might be surprised! Read the recipe from a cookbook using metric measures.

BUTTER TARTS

Preheat oven to 190°C.
Prepare pastry, roll out half. Cut with a floured cookie cutter and line 15 medium-sized muffin cups. Do not prick.
Pour boiling water over
125 ml raisins
Let soak (5 minutes). Drain.
Stir together
50 ml butter or margarine
125 ml lightly packed brown sugar
Blend in
250 ml corn syrup
2 eggs
15 ml lemon juice
5 ml vanilla
Fold in raisins. Fill pastry-lined tart shells 2/3 full.
Bake in preheated 190°C oven for 20 to 25 minutes. Makes 15 tarts.

1. What fraction of the pastry is rolled out to line the muffin cups? $\frac{1}{2}$

2. What fraction of the tart shell is filled with the recipe mixture? $\frac{2}{3}$

3. Suppose that the muffin tin has only 12 cups. What fraction of the ingredients would be enough for 12 tarts? (Give the simplest name for the fraction.) $\frac{4}{5}$

4. For 12 tarts, how many millilitres would you need of:
   a raisins? 100 ml **b** butter? 40 ml **c** brown sugar? 100 ml
   d corn syrup? 200 ml **e** lemon juice? 12 ml **f** vanilla? 4 ml

5. What would you do about a fractional part of 2 eggs? It probably would not matter. Buy smaller eggs.

Now that you have done the questions, you might like to try the recipe at home.

Recipe from *How to Metricook*, courtesy of Maple Leaf Mills Limited

goal Exploring use of fractions in a recipe

page 311 Although the use of SI measurements greatly simplifies many operations and calculations, fractions will still be used in some cases. This page will serve to show the children that they will still need to know how to compute with fractions in the metric world in which they will pass the rest of their lives.

Though the actual quantities in the recipe are given in millilitres in whole numbers, some of the cooking operations call for knowledge of fractions, and using less than the whole recipe will always demand skill in fractions. This has long been a cause of trouble for cooks otherwise skilled in their art.

The children may think of other instances where computation in fractions is still necessary, even after complete conversion to the metric system.

lesson Pages 312, 313, 314, 315

goal Diagnosis of ability to add and subtract two fractions with like or unlike denominators

page 312 The practice sections that accompany this diagnostic check are spread over two pages. Since each section emphasizes a different skill, make sure that everyone follows the directions at the bottom of page 312.

It's entirely possible that some learners will require practice and help in all three sections, some in any combination of two sections, and some in only one section.

Very Important Problems

Diagnostic check on adding and subtracting fractions with like and unlike denominators

Below is part of an application form for a job at Wilson's Warehouse. They want a boy or a girl to help stock parts two hours after school. To get the job, you must be able to add and subtract fractions.

See how well you can do.

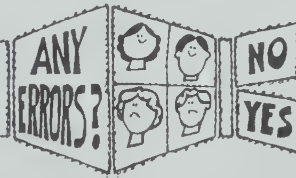
WILSON'S WAREHOUSE APPLICATION — page 2

Please complete these problems on a sheet of paper. We need to see how you work these problems.

1. $\frac{1}{3} + \frac{2}{3}$ $\frac{3}{3}$
2. $\frac{3}{4} - \frac{1}{4}$ $\frac{2}{4}$
3. $\frac{2}{3} + \frac{1}{9}$ $\frac{7}{9}$
4. $\frac{3}{8} - \frac{1}{4}$ $\frac{1}{8}$
5. $2\frac{4}{5} + 3\frac{1}{10}$ $5\frac{9}{10}$
6. $2\frac{7}{9} - 1\frac{2}{3}$ $1\frac{1}{9}$

Wilson's Warehouse wants no errors. Would you get the job? Check your answers with the key on page 336.

ADDITION AND SUBTRACTION OF FRACTIONS



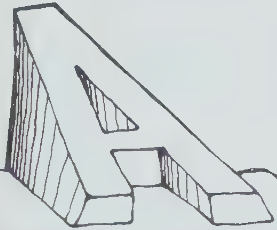
NO — GO TO PAGE 315

YES — READ DIRECTIONS BELOW

If you missed problem 1 or 2, do section A on the next page.

If you missed problem 3 or 4, do section B on the next page.

If you missed problem 5 or 6, do section C on page 314.



Adding and subtracting fractions with like denominators

Here is some help in adding and subtracting fractions when the denominators are the same.

a $\frac{1}{3} + \frac{1}{3} = ?$ $\qquad \frac{1}{3} + \frac{1}{3} = \frac{1+1}{3} = \frac{2}{3}$

b $\frac{5}{6} - \frac{2}{6} = ?$ $\qquad \frac{5}{6} - \frac{2}{6} = \frac{5-2}{6} = \frac{3}{6}$

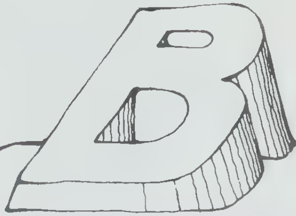
Try doing these.

1. $\frac{1}{4} + \frac{1}{4} = \underline{?}$ $\left(\frac{2}{4}\right)$ $\frac{1}{2}$ **2.** $\frac{1}{6} + \frac{1}{6} = \underline{?}$ $\left(\frac{2}{6}\right)$ $\frac{1}{3}$

3. $\frac{1}{7} + \frac{1}{7} = \underline{?}$ $\frac{2}{7}$ **4.** $\frac{7}{8} + \frac{1}{8} = \underline{?}$ $\left(\frac{8}{8}\right)$ 1

5. $\frac{3}{5} - \frac{1}{5} = \underline{?}$ $\frac{2}{5}$ **6.** $\frac{5}{6} - \frac{1}{6} = \underline{?}$ $\left(\frac{4}{6}\right)$ $\frac{2}{3}$

7. $\frac{7}{12} - \frac{5}{12} = \underline{?}$ $\left(\frac{2}{12}\right)$ $\frac{1}{6}$ **8.** $\frac{7}{10} - \frac{2}{10} = \underline{?}$ $\left(\frac{5}{10}\right)$ $\frac{1}{2}$



Skill: Adding and subtracting fractions with unlike denominators

Did you miss problem 3 or 4 on the application? If you answer no, skip to section C. If you answer yes, complete this.

This will help you remember how to write equivalent fractions.

a $\frac{1}{2} = \frac{?}{4}$ $\qquad \frac{1}{2} = \frac{1 \cdot 2}{2 \cdot 2} = \frac{2}{4}$

b $\frac{2}{5} = \frac{?}{25}$ $\qquad \frac{2}{5} = \frac{2 \cdot 5}{5 \cdot 5} = \frac{10}{25}$

Find the missing numbers. Make each of these true.

1. $\frac{3}{5} = \frac{?}{10}$ [illegible] **2.** $\frac{2}{3} = \frac{?}{6}$ 4 **3.** $\frac{3}{4} = \frac{?}{8}$ [illegible] **4.** $\frac{1}{3} = \frac{?}{9}$ 3

Now add. $\frac{3}{5} + \frac{1}{10}$

Rename and *rewrite.* $\frac{6}{10} + \frac{1}{10} = \underline{?}$ $\frac{7}{10}$

Now subtract. $\frac{7}{8} - \frac{1}{2}$

Rename and *rewrite.* $\frac{7}{8} - \frac{4}{8} = \underline{?}$ $\frac{3}{8}$

You're on your own. Find the common denominator *first*. Then compute.

5. $\frac{2}{3} + \frac{7}{12}$ $\left(\frac{15}{12}\right)$ $1\frac{1}{4}$ **6.** $\frac{5}{6} - \frac{1}{3}$ $\left(\frac{3}{6}\right)$ $\frac{1}{2}$ **7.** $\frac{2}{3} + \frac{1}{12}$ $\left(\frac{9}{12}\right)$ $\frac{3}{4}$ **8.** $\frac{7}{8} - \frac{3}{4}$ $\frac{1}{8}$

goal Practice in adding and subtracting two fractions with like or unlike denominators

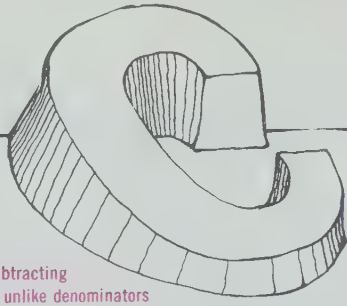
page 313 Check learners directed to section A. Make sure that they are not confusing addition and subtraction with multiplication, and that they don't try adding or subtracting denominators as well as numerators. These are common errors:

$$\frac{1}{3} + \frac{1}{3} = \frac{2}{6} \qquad \frac{5}{6} - \frac{2}{6} = \frac{3}{0}$$

Prerequisite skills for section C (page 314) are the ability to rename a given fraction as an equivalent fraction and the ability to determine a common denominator when adding or subtracting. Pupils who are weak in these skills, as well as those pupils who missed problems 3 or 4 on the application, should complete section B.

goal Practice in adding and subtracting mixed numbers with unlike denominators

page 314 With pupils directed to section C, analyze examples **a** through **d**. The renaming skill is used again. This skill can be used to make the change to a common denominator, as well as to make subtraction possible. Some answers in addition can be renamed. Whether renaming answers is important is your decision.



Skill: Adding and subtracting mixed numbers with unlike denominators

Sometimes you must rename when working with mixed numbers.

Study these examples.

a $\begin{array}{r} 3\frac{1}{6} \\ +\ 1\frac{1}{2} \\ \hline \end{array} \rightarrow \begin{array}{r} 3\frac{1}{6} \\ +\ 1\frac{3}{6} \\ \hline 4\frac{4}{6} \text{ or } 4\frac{2}{3} \end{array}$

b $\begin{array}{r} 5\frac{1}{3} \\ -\ 1\frac{2}{3} \\ \hline \end{array} \rightarrow \begin{array}{r} 4\frac{4}{3} \\ -\ 1\frac{2}{3} \\ \hline 3\frac{2}{3} \end{array}$

c $\begin{array}{r} 5 \\ -\ 1\frac{1}{2} \\ \hline \end{array} \rightarrow \begin{array}{r} 4\frac{2}{2} \\ -\ 1\frac{1}{2} \\ \hline 3\frac{1}{2} \end{array}$

This one is started. You finish it.

d $\begin{array}{r} 4\frac{1}{2} \\ -\ 2\frac{3}{4} \\ \hline \end{array} \rightarrow \begin{array}{r} 4\frac{2}{4} \\ -\ 2\frac{3}{4} \\ \hline \end{array} \rightarrow \begin{array}{r} 3\frac{6}{4} \\ -\ 2\frac{3}{4} \\ \hline ?\ 1\frac{3}{4} \end{array}$

Here are some practice problems.

1. $\begin{array}{r} 5\frac{1}{2} \\ +\ 2\frac{1}{4} \\ \hline 7\frac{3}{4} \end{array}$ **2.** $\begin{array}{r} 3\frac{5}{6} \\ +\ 1\frac{1}{12} \\ \hline 4\frac{11}{12} \end{array}$ **3.** $\begin{array}{r} 2\frac{2}{3} \\ -\ 1\frac{2}{9} \\ \hline 1\frac{4}{9} \end{array}$ **4.** $\begin{array}{r} 7\frac{4}{5} \\ -\ 2\frac{4}{5} \\ \hline 5 \end{array}$ **5.** $\begin{array}{r} 9\frac{11}{100} \\ -\ 7\frac{3}{100} \\ \hline (2\frac{8}{100})\ 2\frac{2}{25} \end{array}$ **6.** $\begin{array}{r} 1\frac{1}{8} \\ -\ \frac{3}{4} \\ \hline \frac{3}{8} \end{array}$

7. $\begin{array}{r} 5\frac{7}{10} \\ +\ 4\frac{7}{100} \\ \hline 9\frac{77}{100} \end{array}$ **8.** $\begin{array}{r} 9\frac{1}{6} \\ -\ 3\frac{1}{3} \\ \hline 5\frac{5}{6} \end{array}$ **9.** $\begin{array}{r} 6 \\ -\ 1\frac{1}{3} \\ \hline 4\frac{2}{3} \end{array}$ (Hint: $1 = \frac{3}{3}$) **10.** $\begin{array}{r} 4 \\ -\ 3\frac{2}{7} \\ \hline \frac{5}{7} \end{array}$ **11.** $\begin{array}{r} 8 \\ -\ 5\frac{3}{4} \\ \hline 2\frac{1}{4} \end{array}$

Mixed Bag

Draw twelve 2 cm squares to form the figure at the right. Copy the fraction printed in each square.

Each answer in the exercise below needs to be renamed to its simplest form. Compute and rename the answer. Look for the final answer on your drawing. Circle the fraction that is your final answer.

When you complete the exercise, each fraction on your drawing should be circled. The final answer to the first problem is already circled for you.

Recopy the questions in vertical form if you wish. Watch for the operation signs.

	$\frac{1}{2}$	$\frac{1}{3}$	
$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{2}{5}$
$\frac{3}{5}$	$\frac{4}{5}$	$\frac{1}{10}$	$\frac{3}{10}$
	$\frac{7}{10}$	$\frac{9}{10}$	

1. $\frac{9}{10} - \frac{7}{10}$ $\left(\frac{2}{10}\right)$ $\frac{1}{5}$ **2.** $\frac{19}{100} - \frac{9}{100}$ $\left(\frac{10}{100}\right)$ $\frac{1}{10}$ **3.** $\frac{27}{40} + \frac{3}{40}$ $\left(\frac{30}{40}\right)$ $\frac{3}{4}$ **4.** $\frac{2}{15} + \frac{7}{15}$ $\left(\frac{9}{15}\right)$ $\frac{3}{5}$

5. $\frac{5}{6} - \frac{1}{3}$ $\left(\frac{3}{6}\right)$ $\frac{1}{2}$ **6.** $\frac{3}{4} - \frac{1}{20}$ $\left(\frac{14}{20}\right)$ $\frac{7}{10}$ **7.** $\frac{3}{4} + \frac{1}{20}$ $\left(\frac{16}{20}\right)$ $\frac{4}{5}$ **8.** $\frac{1}{3} + \frac{17}{30}$ $\left(\frac{27}{30}\right)$ $\frac{9}{10}$

9. $\frac{1}{5} + \frac{1}{20}$ $\left(\frac{5}{20}\right)$ $\frac{1}{4}$ **10.** $\frac{3}{4} - \frac{9}{20}$ $\left(\frac{6}{20}\right)$ $\frac{3}{10}$ **11.** $\frac{5}{6} - \frac{13}{30}$ $\left(\frac{12}{30}\right)$ $\frac{2}{5}$ **12.** $\frac{1}{20} + \frac{17}{60}$ $\left(\frac{20}{60}\right)$ $\frac{1}{3}$

goal Practice in adding and subtracting fractions, and in renaming in simplest form

page 315 This page is not a must for everyone. It provides extra practice in addition and subtraction of fractions with both like and unlike denominators. The device of circling the answers in the grid allows for self-checking, and at the same time introduces a game element to liven up the practice for those who need it most and who may be growing weary of it.

315

lesson Pages 316, 317, 318

goal Diagnosis of ability and practice in multiplying with fractions and mixed numbers

page 316 Three multiplication skills are checked. Use this chart to help youngsters who are making errors in computation with zero.

Problems missed on diagnostic check	Examine examples below
1, 2	a, b, c
3, 4	d, e, f
5, 6	Begin at middle of page 317

Pupils who miss only 5 and 6 in the diagnostic check may go directly to page 317. The purpose of this chapter is to practice only those skills that have not yet been mastered.

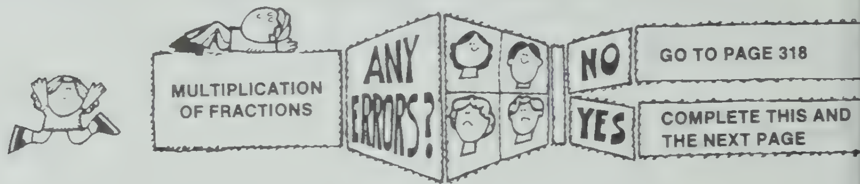
Very Important Problems

You have just finished practice in addition and subtraction with fractions.

Try some multiplication

1. $3 \times \frac{1}{5} = \underline{?}$ $\frac{3}{5}$
2. $\frac{3}{8} \times 4 = \underline{?}$ $(\frac{12}{8})$ $1\frac{1}{2}$
3. $\frac{1}{3} \times \frac{1}{4} = \underline{?}$ $\frac{1}{12}$
4. $\frac{2}{5} \times \frac{1}{2} = \underline{?}$ $(\frac{2}{10})$ $\frac{1}{5}$
5. $2 \times 1\frac{2}{3} = \underline{?}$ $\frac{10}{3}$ $3\frac{1}{3}$
6. $3\frac{1}{2} \times 4\frac{3}{4} = \underline{?}$ $(\frac{133}{8})$ $16\frac{5}{8}$

Check your answers with the answer key on page 336.



Reminder! Here's how you multiply with fractions

a $3 \times \frac{2}{7} = \underline{?}$ $\quad 3 \times \frac{2}{7} = \frac{3 \cdot 2}{7} = \frac{6}{7}$

b $\frac{2}{5} \times 2 = \underline{?}$ $\quad \frac{2}{5} \times 2 = \frac{2 \cdot 2}{5} = \frac{4}{5}$

c $4 \times \frac{5}{6} = \underline{?}$ $\quad 4 \times \frac{5}{6} = \frac{4 \cdot 5}{6} = \frac{20}{6} = 3\frac{2}{6} = \underline{?}$ $3\frac{1}{3}$

d $\frac{1}{2} \times \frac{1}{4} = \underline{?}$ $\quad \frac{1}{2} \times \frac{1}{4} = \frac{1 \cdot 1}{2 \cdot 4} = \underline{?}$ $\frac{1}{8}$

e $\frac{2}{3} \times \frac{4}{5} = \underline{?}$ $\quad \frac{2}{3} \times \frac{4}{5} = \frac{2 \cdot 4}{3 \cdot 5} = \underline{?}$ $\frac{8}{15}$

f $\frac{5}{9} \times \frac{3}{4} = \underline{?}$ $\quad \frac{5}{9} \times \frac{3}{4} = \frac{5 \cdot 3}{9 \cdot 4} = \underline{?}$ $\frac{15}{36}$

318

Skill: Multiplying fractions

Complete for practice

1. $2 \times \frac{1}{3} = \underline{?}$ $\frac{2}{3}$ 2. $4 \times \frac{1}{6} = \underline{?}$ $(\frac{4}{6})$ $\frac{2}{3}$ 3. $\frac{1}{3} \times \frac{1}{2} = \underline{?}$ $\frac{1}{6}$

4. $\frac{3}{10} \times \frac{1}{2} = \underline{?}$ $\frac{3}{20}$ 5. $3 \times \frac{2}{5} = \underline{?}$ $(\frac{6}{5})$ $1\frac{1}{5}$ 6. $\frac{2}{3} \times \frac{1}{3} = \underline{?}$ $\frac{2}{9}$

7. $6 \times \frac{3}{4} = \underline{?}$ $(\frac{18}{4})$ $4\frac{1}{2}$ 8. $\frac{3}{4} \times \frac{2}{5} = \underline{?}$ $(\frac{6}{20})$ $\frac{3}{10}$ 9. $\frac{2}{3} \times \frac{5}{6} = \underline{?}$ $(\frac{10}{18})$ $\frac{5}{9}$

Do you remember how to multiply mixed numbers?
If you know how to multiply fractions, mixed numbers will be no problem. Here is what you do.

- **A** Change each mixed number to a fraction.
- **B** Multiply the fractions.
- **C** Write the product as a mixed number.

Here is an example:

$$3\frac{2}{3} \times 2\frac{3}{5} = \underline{?}$$

Can you explain each step in finding the answer?

$$3\frac{2}{3} \times 2\frac{3}{5} = \frac{11}{3} \times \frac{13}{5} = \frac{143}{15} = 9\frac{8}{15}$$

Skill: Multiplying mixed numbers

Try doing these exercises now

1. $1\frac{1}{2} \times 1\frac{2}{3} = \underline{?}$ $(\frac{15}{6})$ $2\frac{1}{2}$ 2. $2\frac{1}{2} \times 1\frac{3}{4} = \underline{?}$ $(\frac{35}{8})$ $4\frac{3}{8}$

3. $3\frac{2}{5} \times 1\frac{2}{3} = \underline{?}$ $(\frac{85}{15})$ $5\frac{2}{3}$ 4. $2\frac{2}{3} \times 2\frac{2}{3} = \underline{?}$ $(\frac{64}{9})$ $7\frac{1}{9}$

5. $1\frac{5}{6} \times 5\frac{1}{2} = \underline{?}$ $(\frac{121}{12})$ $10\frac{1}{12}$ 6. $4\frac{3}{4} \times 1\frac{3}{5} = \underline{?}$ $(\frac{152}{20})$ $7\frac{3}{5}$

317

goal Practice in multiplying fractions and mixed numbers

page 317 Pupils who made errors in problems 1 through 4 on the previous page should complete the problems at the top of this page.

Pupils who missed problems 5 and 6 need not do those first nine problems, but they should complete the rest of the page.

goal Practice in multiplying mixed numbers in a game format

things markers

page 318 This game is a bit like checkers, but each player will probably need a pencil and some scratch paper for the computation. The game can be played more than once because your gamesmen will develop some excellent game strategies by the second or third game. Then your sharpies should be challenged to make up numbers for another game board. But beware! This task is not as easy as it sounds.

Pieces of colored paper, beans, or buttons can be used as markers. There are only two requirements—each person should be able to identify his own marker, and it should be small enough to fit in the squares.

Mixed Bag

FRANTIC FRACTIONS GAME

Up to four can play. Everyone sits on the same side of the playing board.

Each player has a token. Place your token on a vacant START.

Everyone plays in turn.

First turn. Move your token straight forward onto the first square.

Other turns. Move your token one square in any direction. (No jumps.) You must move to a vacant square.

After you move each time, challenge any other player who has a token on a square to give the product of your number and his number.

Imagine you have just challenged someone. Your token is on $1\frac{1}{2}$.

His token is on $1\frac{1}{3}$. He must give the product of $1\frac{1}{2} \times 1\frac{1}{3}$.

If his answer is right, your turn is over.

Suppose his answer is wrong. He goes back to START (any vacant START).

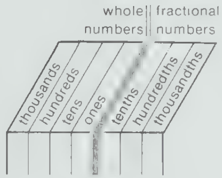
Then your turn is over. It's the next person's move.

Have fun!

WIN	WIN	WIN	WIN
$5\frac{1}{3}$	$6\frac{2}{3}$	$2\frac{4}{7}$	$3\frac{3}{4}$
$1\frac{1}{7}$	$3\frac{1}{3}$	$2\frac{2}{5}$	$1\frac{5}{6}$
$2\frac{1}{4}$	$1\frac{1}{5}$	$1\frac{1}{4}$	$3\frac{1}{2}$
$1\frac{1}{2}$	$1\frac{1}{3}$	$2\frac{1}{2}$	$1\frac{2}{3}$
START	START	START	START

318

Our number system is put together in such a way that there is no "biggest number" and no "smallest number." The numbers just go on and on and on.



$1\frac{1}{10}$ can be written 1.1

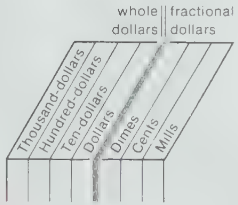
$1\frac{1}{100}$ can be written 1.01

$1\frac{1}{1000}$ can be written 1.001

1 can be written as $\frac{1}{1}$ or $\frac{10}{10}$ or $\frac{100}{100}$ or $\frac{1000}{1000}$ or 1.0.

10 means 1 *ten*; .1 means 1 *tenth.*
100 means 1 *hundred*; .01 means 1 *hundredth.*
1000 means 1 *thousand*; .001 means 1 *thousandth.*

You have worked with money for a long time. Look at the chart. Is there anything there that you don't know about? We don't hear much about a mill, but it is $\frac{1}{1000}$, or 1 thousandth, of one dollar. Use the place-value chart to help answer these questions.



1. One dime is what fractional part of one dollar? $\frac{1}{10}$
2. One cent is what fractional part of one dollar? $\frac{1}{100}$
3. How many mills would be the same as one cent? 10
4. How many cents would be the same as one dime? 10
5. How many dimes would be the same as one dollar? 10
6. How many dimes would be the same as ten dollars? 100
7. How many dimes would be the same as one hundred dollars? 1000

319

lesson Page 319

goal Comparing money notation to decimal place value

page 319 This is a review, but the idea of place value is extended and then applied to money situations. Mills are sometimes used when computing taxes—especially school taxes. Make sure that you emphasize that there are no coins that represent one mill. Decimal notation compares to money notation with the dollar sign removed.

Contrasts are presented, notation is emphasized, and some interesting, challenging questions are asked. This page is worth everyone's attention and work.

lesson Pages 320, 321, 322

goal Diagnosis of ability to add and subtract decimals (hundredths only)

page 320 The diagnosis separates problems with no renaming from those with renaming. This may help you pinpoint computation trouble that relates to basic computation skills rather than to troubles directly tied to computation with decimals.

The bottom half of the page will help youngsters zero in on the cause for their error(s). Those who are having trouble in addition and subtraction should have already come to your attention earlier in the chapter when the skills were checked with whole numbers. At this level these youngsters will require either teacher or peer tutoring.

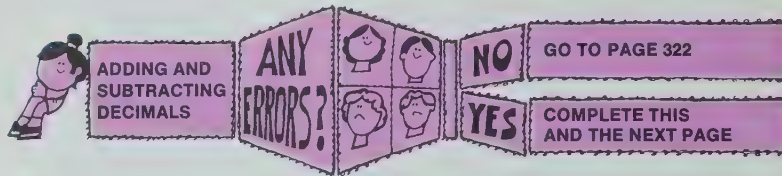
Diagnostic check on adding and subtracting decimals (hundredths only)

Very Important Problems

Compute. Watch the signs.

1. $4.24 + 1.63	**2.** 10.78 + 8.22	**3.** $5.79 − 2.53	**4.** 8.28 − 3.79
$5.87	19.00	$3.26	4.49

Check your answers on page 336.



Look back at the problems you missed. Find out *why* you made a mistake. Use these questions as a checkout list.

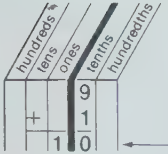
1. Did your answer have the dollar sign and the decimal in the correct place? If it didn't, why didn't it?
   Think about this.
   You gave a clerk a five-dollar bill to pay for a fifty-cent item.

   $$\begin{array}{r} \$5.00 \\ -\quad .50 \\ \hline \$4\ 50 \end{array}$$

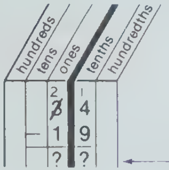
   The clerk gave you $450 in change. Would the clerk get in trouble? Would you stand for it if the clerk gave you 45 cents in change? The decimal is important. You bet; no

2. Did you make an error in adding or subtracting? If you did, you need practice with whole numbers. You won't have success with decimals until you get good with whole numbers.

Before you start, think about place value.



You are using an addition fact.
But remember the place value in tenths.



You must rename. Same idea that you know for whole numbers.
You rename 1 one as tenths.

COMPLETE THESE.

Add. Skill Adding decimals, (tenths and hundredths)

1. 0.7 + 0.9 = 1.6	**2.** 9.5 + 1.2 = 10.7	**3.** 3.7 + 6.8 = 10.5	**4.** 27.3 + 13.6 = 40.9	**5.** 47.89 + 12.10 = 59.99

Subtract. Skill Subtracting decimals (tenths and hundredths)

6. 1.8 − 0.7 = 1.1	**7.** 2.3 − 0.9 = 1.4	**8.** 51.6 − 13.7 = 37.9	**9.** 10.1 − 9.7 = 0.4	**10.** 20.00 − 4.91 = 15.09

Copy and compute. Watch the signs. Skill Adding and subtracting decimals (tenths and hundredths)

11. 1.2 + 3.56 4.76 **12.** 21.5 − 17.2 4.3 **13.** 25 − 1.25 23.75

goal Practice in adding and subtracting decimals (tenths and hundredths)

page 321 Pupils who can add and subtract whole numbers should be able to work independently. The quick review at the top of the page should be all that is necessary for any decimal-placement difficulty.

goal Application of SI measurements of temperature

page 322 If pupils complete this page with a little more familiarity with the Celsius thermometer, it will have achieved its purpose. Those with some experience in baking will especially enjoy discussing the questions. These can be done either individually or in a class discussion. If the former, it should be followed by general discussion in which pupils may ask questions or volunteer information about the reasons for different oven temperatures for different dishes. A relaxing year-end exercise in relating mathematics to real life.

Mixed Bag

Water freezes at 0°C. (Remember ° stands for degrees; C stands for Celsius.) It boils at 100°C at sea level.

Oven temperatures for baking are almost always higher than the boiling point of water. The chart on this page gives metric baking temperatures and the time needed for several products.

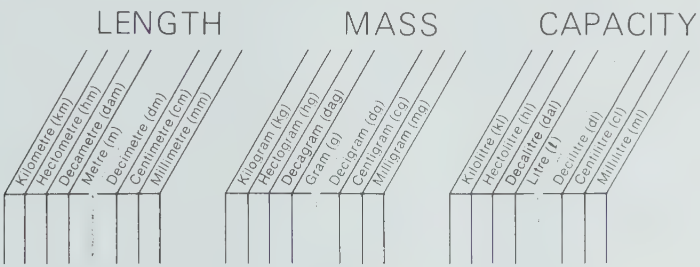
1. What is the lowest oven temperature on the chart? What can be baked at that temperature? How many hours will it take? 150° Rice pudding 2 - 3
2. What is the highest oven temperature? 230° What two products may require that temperature? Which of the two needs less time? Pastry shells and 2-crust pies and tea biscuits (any 2) Pastry shells
3. What is the range of baking temperatures on the chart? (Remember that the *range* is the difference between lowest and highest.) 80°
4. Name two egg, meat, milk, or cheese dishes that require the same oven temperature. If the times differ, which dish would you put in the oven first? Custard, meat loaf, scalloped potatoes, macaroni (any 2) The longer-cooking one

METRIC BAKING TEMPERATURES

Type of Product	Oven Temperature	Time (minutes)
BREADS, etc.		
Cream puffs, popovers	190°C	60
Muffins	200°C - 220°C	20 to 25
Quick breads	180°C - 190°C	60 to 75
Tea Biscuits	220°C - 230°C	10 to 15
Yeast Bread	200°C	30 to 40
Yeast Rolls: plain	200°C	20 to 25
sweet	190°C	20 to 30
CAKES WITH FAT		
Cup	180°C - 190°C	15 to 25
Layer	180°C - 190°C	20 to 35
Square, rectangle	180°C	45 to 60
CAKES WITHOUT FAT		
Angel and sponge (tube pan)	170°C - 180°C	50 to 60
COOKIES		
Drop	180°C - 200°C	8 to 15
Rolled	190°C	8 to 10
EGG, MEAT, MILK, and CHEESE DISHES		
Cheese soufflé, custards (baked in a pan of hot water)	180°C	30 to 60
Macaroni and cheese	180°C	25 to 30
Meat Loaf	180°C	60 to 90
Meat Pie	200°C	25 to 30
Rice Pudding (raw rice)	150°C	120 to 180
Scalloped Potatoes	180°C	60
PASTRY		
One Crust Pie (unbaked shell)	200°C - 220°C	30 to 40
Meringue on cooked filling in prebaked shell	180°C *or* 220°C	12 to 15 *or* 4 to 4 1/2
Shell only	230°C	10 to 12
Two Crust Pies with uncooked filling	200°C - 220°C	45 to 55
Two Crust Pies with cooked filling	220°C - 230°C	30 to 45

Courtesy of Maple Leaf Mills Limited

Our system of measure is really very simple.
It is organized the same way as our number system is.
The language of our measurement system is as simple as its organization.
It uses the same prefixes in measurements of length, mass, and capacity.



Do these charts look like a place-value chart? Yes
Use these charts to help you answer these questions.

LENGTH
1. How many metres in a decametre? a hectometre? a kilometre? 10; 100; 1000
2. A millimetre is what part of a centimetre? a decimetre? a metre? $\frac{1}{10}$; $\frac{1}{100}$; $\frac{1}{1000}$

MASS
1. How many grams in a decagram? a hectogram? a kilogram? 10; 100; 1000
2. How many milligrams in 1 gram? 1000
3. How many centigrams in 1 gram? 100

CAPACITY
1. How many litres in a kilolitre? 1000
2. Is a centilitre larger or smaller than a decalitre?

The units of length you need to remember are centimetre, metre, and kilometre.
The units of mass you should remember are gram and kilogram.
The units of capacity to remember are millilitre, litre, and kilolitre.

lesson Page 323

goal Relating the organization of metric measures to place value in our system of numeration

page 323 All the metric units of measure that have been used by the pupil thus far are summarized in three highly organized charts. You'll want to talk about this page. The youngsters should be able to easily relate the unit of measure charts to a place-value chart.

This is an opportunity to give a sales pitch for the metric system. Computation with metric measurements is greatly simplified because of their close relationship to place value in our number system.

lesson Pages 324, 325

goal Diagnosis of ability to change measurements to related units

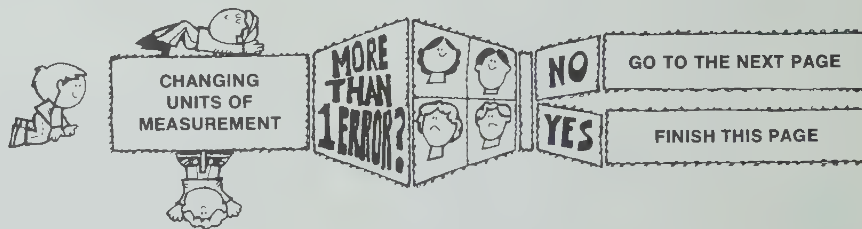
page 324 Only those who have difficulty with the first half of the page need to do the second half as well. For those who have trouble here too, give individual help.

Very Important Problems

Replace the ? by the correct measurement.

1. 7.5 cm = ? mm 75
2. 7.555 kg = ? g 7555
3. 4584 ml = ? ℓ 4.584
4. 6.08 m = ? cm 608
5. 844 cm = ? m 8.44
6. 3.075 km = ? m 3075
7. 4448 mm = ? m 4.448
8. 4448 mm = ? cm 444.8

Check your answers with the key on page 336.



If you made an error on these problems, you may not know measurement units well enough. You may also need practice on the "sliding" decimal point.

Copy and complete the tables.

1.

mm	cm
75	? 7.5
94 ?	9.4
904	? 90.4

2.

cm	m
608	? 6.08
3057	? 30.57
4598 ?	45.98

3.

ml	ℓ
2995	? 2.995
3750 ?	3.750
5117 ?	5.117

4.

g	kg
7555	? 7.555
9030 ?	9.080
9530	? 9.530

5.

m	km
9007	? 9.007
9405 ?	9.405
9999	? 9.999

Tell which of the units of measure
is probably the better one.

1.
You are told that a man is 2 ? (centimetres or metres) tall.

2.
A cat at your friend's house weighs 2 ? (grams or kilograms).

3.
Your mother's sewing needle is 5 ? (centimetres or metres) long.

4.
A large bottle of pop holds about 5 ? (decilitres or litres).

5.
The race driver said his top speed was 200 ? (metres or kilometres) per hour.

6.
Your finger is about 1.5 ? (centimetres or millimetres) wide.

7.
A pill contains 250 ? (milligrams or grams) of vitamin C.

8.
One of the events at the Olympics was a 5000 ? (metre or kilometre) run.

9.
The border between the United States and Canada is 6416 ? (metres or kilometres) long.

10.
The gas tank of a small compact car holds about 5 ? (dekalitres or litres) of gasoline.

goal Selecting an appropriate metric unit of measure

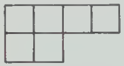
page 325 Each person should be able to work independently. Make sure that you encourage the insecure pupil to turn back to a table of measure or to chapter 9 for any hints he may need to complete the page.

lesson Pages 326, 327, 328

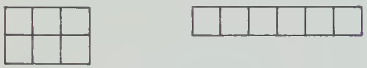
goal Finding prime and composite numbers, using arrays

memo Prime and composite numbers were identified earlier in this level. The pupils identified composite numbers by using division. Pages 326 through 328 review these ideas, using an activity approach.

page 326 Time out from computation—but stop and examine the definition of an **array** first. Remember—each row must contain the same number of squares. Rows go across. Is this an array?



Can it be changed into an array by rearranging the squares?

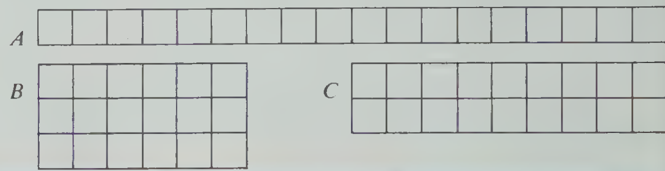


Two more are possible. Challenge your students to find them. (Turn each array vertically.)

Your pupils might like making their arrays with various manipulatives such as poker chips, dried beans, or paper disks. Shading in arrays on graph paper is another possibility.

You need a break from all that computation you've been doing. Here is a new topic for you.

The whole number 18 can be shown by three different arrays.



Now try these yourself. Make other arrays for each number shown, if you can. Arrays will vary. Examples are given.

1.

2.

3.

4. No array

5. No array

6.

7.

8.

9. Which numbers could you make other arrays for? 6, 8, 9, 10, 15, 16;
Which ones could be shown only by one array? 7, 11

Whole numbers that can be shown only by a one-row array are called **prime numbers.**

Whole numbers that can be shown with more than one array are called **composite numbers.**

10. Are the numbers 2, 3, 4, and 5 prime or composite? Draw arrays to prove it.
2, 3, and 5 are prime. 4 is composite.

Make a complete 10x10 grid like this.
Make it large enough so that you can enter 2-digit numbers in each box.
There is room in your grid for 100 numbers written in order from 1 to 100.
You will be putting numbers in order into the grid, but *not* every box will be filled. There will be some numbers missing. They will be the important ones. Follow the directions below carefully.

a Enter the number 1 in the first box of the first row.

b Enter the number 2 in the second box of the first row. The proper order of numbers has started. Make sure it continues.

c Enter every number that is a multiple of 2 in its correct location.

d Enter the number 3 in the third box in the first row.

e Enter every multiple of 3 in its correct location.

f 4 has already been entered. So enter 5 in the fifth box.

g Enter every multiple of 5 in its correct location.

h 6 has been entered. So enter 7 in the seventh box.

i Enter every multiple of 7 in its correct location.

j Have 8, 9, and 10 been entered? They should have been.

k Make a list of the numbers that should be in the empty boxes.
11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

1	2	3	4	5	6	7	8	9	10
	12		14	15	16		18		20
21	22		24	25	26	27	28		30
	32	33	34	35	36		38	39	40
	42		44	45	46		48	49	50
51	52		54	55	56	57	58		60
	62	63	64	65	66		68	69	70
	72		74	75	76	77	78		80
81	82		84	85	86	87	88		90
91	92	93	94	95	96		98	99	100

goal Finding prime numbers, using a number chart.

memo Pages 327 and 328 work together.

page 327 You may want to recall that **multiples** are the products that result when multiplying.

Careful—step **c** can be a bit tricky. Make sure each pupil understands that he must leave a box blank for 3, enter 4, skip box 5, enter 6, skip 7, and so on. This is done because 3 is not a multiple of 2 but 4 is, 5 is not a multiple but 6 is.

You're right, this is a version of the Sieve of Eratosthenes. Don't tell the youngsters now—this will be discussed on page 328.

goal Identifying the primes less than 100

page 328 Examine the numbers 2, 3, 5, and 7. Is each a prime or a composite? What about those numbers left on the list after step **h** of problem 1? (All prime) To have a complete list of primes less than 100, what numbers need to be added to the list? (2, 3, 5, 7).

1.

Look at your list of numbers.

a The number 1 isn't on your list. 2 isn't there either.

b Can you divide any number on your list by 2 and not have a remainder? No A number like that does not belong on your list!

c Is any number on your list divisible by 3? No

d Is any number on your list divisible by 5? No

e Is any number on your list divisible by 7? No

f Is any number on your list divisible by 11? Which one? What is the quotient? Yes; 11; 1 11 belongs on your list. But no multiple of 11 belongs there.

g Is any number on your list divisible by 13, other than 13? No multiples of 13 No belong on your list.

h Check the rest of your numbers in the same way. No multiples allowed.

Any number that is not divisible by any number other than itself and 1 is a prime number. All other whole numbers are composite numbers.

2.

How many prime numbers are there between 1 and 100? 25

3.

Is 100 a prime or composite number?

4.

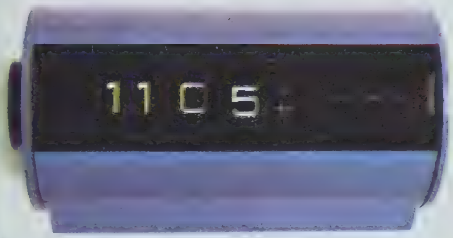
Is 101 a prime or composite number?

5.

How many prime numbers do you think there are? Infinite number

More than 2000 years ago a Greek mathematician called Eratosthenes did just about the same thing you just did. He crossed out multiples from a grid of numbers. But he was the first to do it. His grid is called the sieve of Eratosthenes by those people who can pronounce it. Since his grid was named after him, you can name *your* list after *you*. Fair enough?

Mixed Bag



Have you ever seen a digital clock or watch?

Only one digit shows in each of the openings. How many different digits have to be available for any of the openings? 10

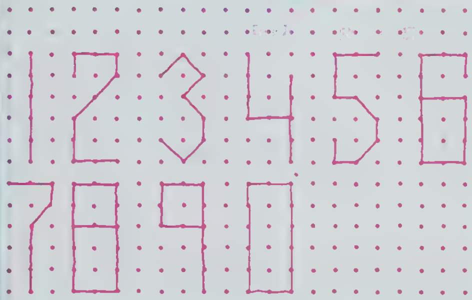
Does 0 have to be available for the first opening on the left? No

Huge outdoor clocks are often digital clocks. The numerals are formed by a series of light bulbs flashing on.

From a distance these lighted bulbs would look like the numeral 1.

Many times all the bulbs that are needed to show any digit are arranged in an array. The bulbs that are turned on to show any given numeral are controlled by a machine.

Answers will vary. Example:



Your job is to figure out one array of bulbs that could show any one of the numerals from 0 through 9. The numerals sometimes have a bit different shape than the ones we write. Be sure to show what each numeral would look like.

329

lesson Page 329

goal Application of arrays

page 329 This activity is a real challenge. There are so many possible answers, the pupils will need lots of time and lots of paper. Good answers will be determined by putting the work on display and then viewing it from a distance. Could you read the time if you were going by in a bus and just had a few seconds to look at the numerals? The curious will probably want to examine digital clocks more closely after completing this page.

lesson Pages 330, 331

goal Identifying plane geometric figures and solid geometric shapes in real-world objects

memo Pages 330 and 331 work together.

page 330 The annotations on the reduced pupil page will give only a few of the possible answers. Make sure that you take time to talk about any unusual answer. The youngster may be able to convince everyone that what appears to be a wrong answer is really legitimate. That youngster may use words for his argument, he may make a tracing, or he may find a real object that looks like an object in the picture to prove his point.

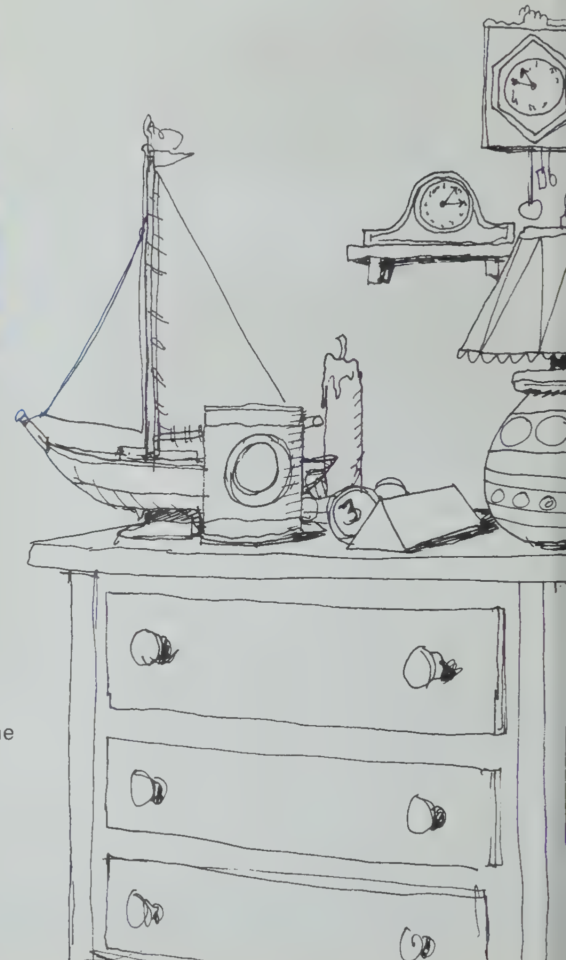
Cracker Barrel Session

Find the following shapes if you can. Tell what object reminded you of each of them. Start with some plane figures.

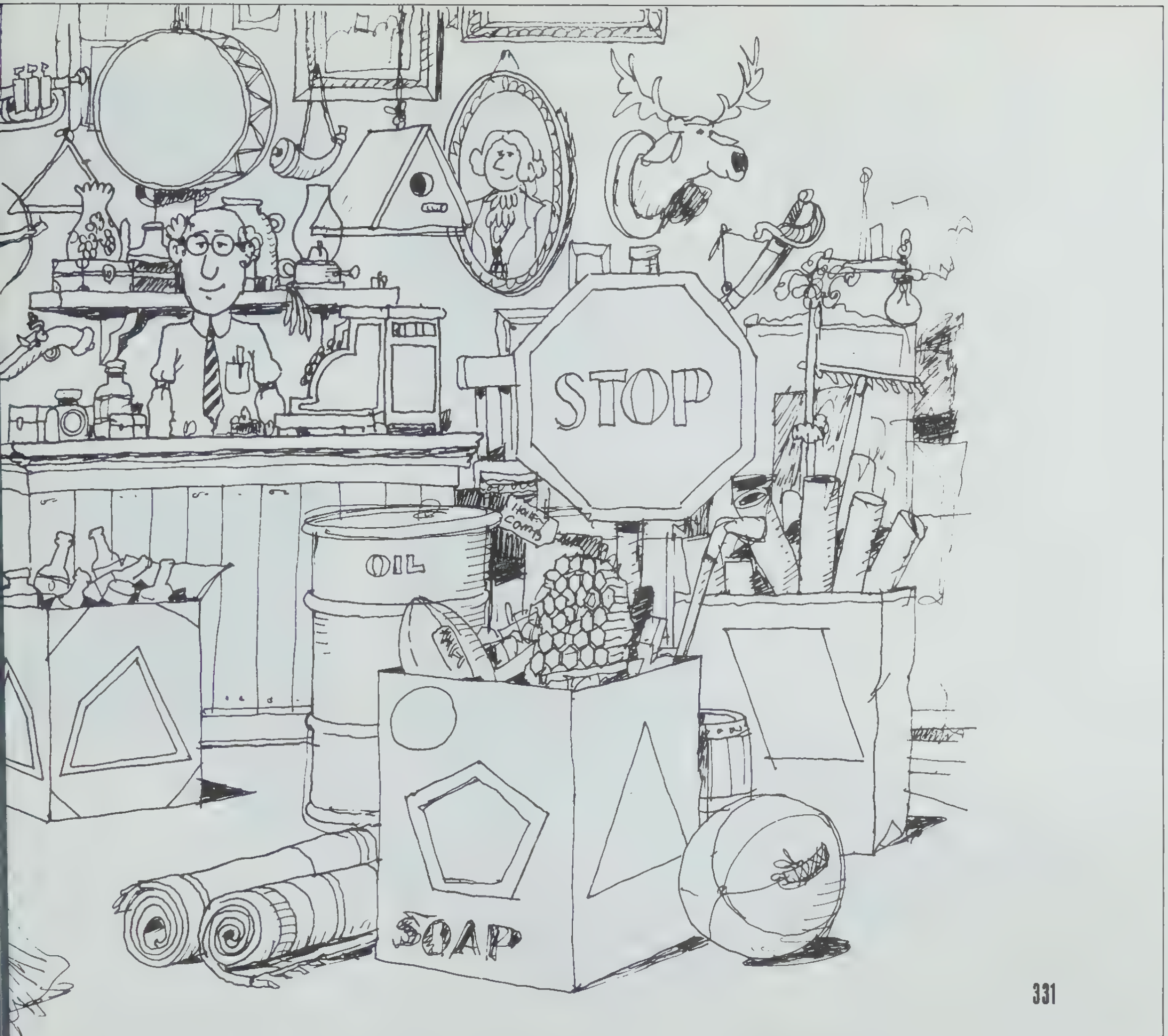
1. Squares or rectangles
2. Triangles
3. Parallelograms or quadrilaterials that are not rectangles
4. Circles
5. Pentagons or hexagons or octagons

Now look for some solid shapes. Be sure to name the object that reminded you of the shape.

6. Cubes or rectangular prisms
7. Cylinders
8. Spheres



330



goal Identifying plane geometric figures and solid geometric shapes in real-world objects *(continued)*

page 331 Sharing answers can serve as a learning situation for everyone.

lesson Pages 332, 333

goal Making a symmetrical figure and identifying the line of symmetry

page 332 A fun way to review a previously developed concept. Each person should be able to operate independently. Some of the results will probably be works of art. Be sure to display them.

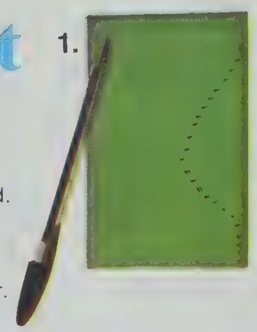
If anyone really gets going on this activity, why not encourage him to try a different approach? Poke the holes but then use some yarn (maybe more than one color) to complete the design. The guys will enjoy stitchery just as much as the gals.

Very Important Problems

Fold a piece of paper in half. Keep it folded.
Find something with a sharp point.
The end of a ballpoint pen will do.

Make a number of holes through the paper.
Make them about 1 cm apart.
The holes should begin and end near the fold.

1.



2.

Open the paper.

3.

Connect each pair of holes.

4.

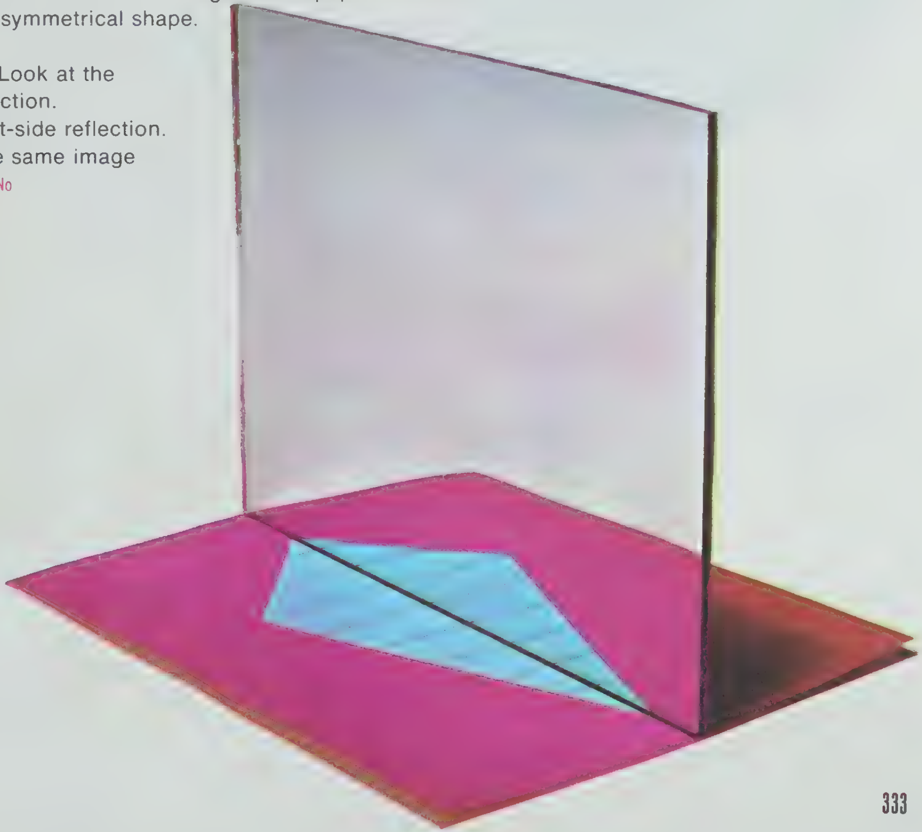
Then connect each point.

5. Does the left half of your picture match the right half? It should.
Where is a line of symmetry? The fold line
Is there another line of symmetry? No

6. Repeat this activity. This time make up a shape of your own. Do you know in advance if it will be symmetrical? Why? Yes; the fold line is a line of symmetry.

There is another way to see symmetrical shapes.
Put a mirror up to part of a shape.
The edge of the mirror becomes the line of symmetry.
You will see a reflection of the image on the paper.
You will see a symmetrical shape.

Find a mirror. Look at the right-side reflection.
Look at the left-side reflection.
Do you see the same image both times? No



333

goal Finding symmetrical shapes in a mirror reflection

things mirrors

page 333 Some youngsters will not believe unless they try and see for themselves—and what they actually see may be far different from what they expect to see.

lesson Pages 334, 335

goal Dividing a circular region into twelfths by paper folding

things coffee filters

page 334 Coffee filters are ready-made circular regions. Help with directions if you need to.

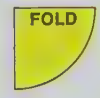
Mixed Bag

Look at the clock in your room. Most clockfaces have the shape of a circle. How many equal parts is the circular clockface divided into? You'd better say 12 or you're in trouble.

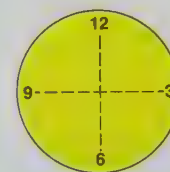
You're pretty good with folding. Trace around the bottom of a can or jar. Cut out the circular region. Divide it into 12 equal parts. It's not hard.

Fold in half.

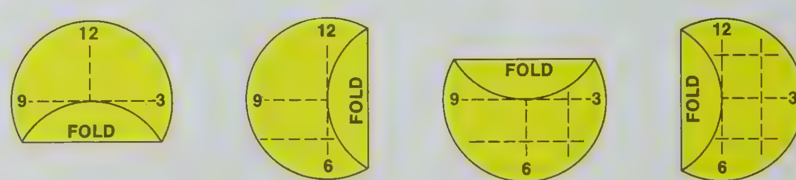
Then fold in half again.

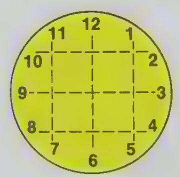


Open. Mark 3, 6, 9, and 12.



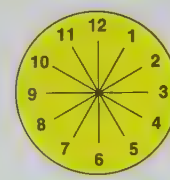
Four hours marked. How many to go? 8





Finish marking the hours. Your clock is complete. But our goal was to divide a circular region into 12 parts. So finish. Draw lines from the center to each number.

All 12 hours are marked.



Go on to the next page if you have time.

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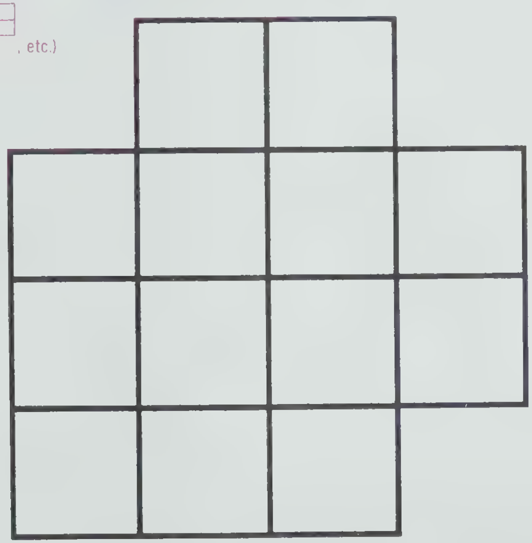
Another **Mixed Bag**

How many squares can you find? 20

(Pupil may want to prove answer by tracing and showing the square counted. For example

, etc.)



goal Identifying squares in a collection of squares

page 335 Simple and fun for youngsters of all abilities. Use when a break is needed or whenever time allows.

memo Here is the last page of the pupil book. It is appropriate that this page contains all the answers. Wouldn't it be nice if it also contained all the answers to each individual child's learning needs? Perhaps you can help with them.

A major goal for each learner was to master the objectives for this level. Not every child finished that job. It would be helpful for future work if you could find time to talk to each child about his achievements. You can give praise for those skills he has mastered, and you can put the learning tasks yet to be accomplished into a positive context.

The next level's work will let everyone have a fresh start. There will still be time and opportunity to pick up those needed skills.

We should have devoted one whole page to words of praise for you. You have persisted through 336 pages of mathematics. Some of the pages were hard, some were easy, some were fun, and you probably found a few that were boring. But you made it!

Thank you

ANSWERS

294 a 8950; 6960; 5900 b 8900; 7000; 5900
c 9000; 7000; 6000

298 a 600; 570 b 7000; 7223 c 1500; 1534

300 a 200; 209 b 3000; 3164 c 3000; 3263

303 a 800 b 648 c 14 000 d 17 064
e 60 000 f 83 232

307 a 3 b 3 c 12 d 12 e 10 or 11 f 11

312 1. $\frac{3}{3}$ 2. $\frac{2}{4}$ 3. $\frac{7}{9}$ 4. $\frac{1}{8}$ 5. $5\frac{9}{10}$ 6. $1\frac{1}{9}$

316 1. $\frac{3}{5}$ 2. $1\frac{1}{2}$ 3. $\frac{1}{12}$ 4. $\frac{1}{5}$ 5. $3\frac{1}{3}$ 6. $16\frac{5}{8}$

320 1. \$5.87 2. 19.00 3. \$3.26 4. 4.49

324 1. 75 mm 2. 7555 g 3. 4.584 ℓ 4. 608 cm
5. 8.44 m 6. 3075 m 7. 4.448 m 8. 444.8 cm

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RESOURCES

additional learning aids

notation – chapter objective 1

SRA products

Mathematics Learning System, Activity Masters, level B, SRA (1974)
Spirit masters: W-1, 2, 9, 23, 28
Skill through Patterns, level 5, SRA (1974)
Spirit master: 13

operation – chapter objectives 2, 3, 4, 5, 7, 8, 9

SRA products

Mathematics Learning System, Activity Masters, level B, SRA (1974)
Spirit masters: W-3, 4, 10, 11, 12, 13, 14, 17, 18, 20, 25, 26, 29; F-1, 3, 4, 6, 7, 8, 9, 10, 11, 12, 14, 15
Arithmetic Fact Kit, SRA (1969)
Cards: all
Computapes, SRA (1972)
Module 1, 2, 3, 4; Lessons: all
Module 5, Lessons: FR 1-20
Module 6, Lessons: DP 1-5
Computational Skills Development Kit, SRA (1965)
Whole Number cards: all
Fraction cards: Addition 1-16, Subtraction 1-9, Multiplication 1-8
Decimal cards: Addition 1-7, Subtraction 1-5
Cross-Number Puzzles (Fractions), SRA (1966)
Addition cards: all
Subtraction cards: all
Multiplication cards: 1-20
Cross-Number Puzzles (Whole Numbers), SRA (1966)
Cards: all
diagnosis: an instructional aid—Mathematics Level B, SRA (1972)
Probes: M 1-8, 15
Math Applications Kit, SRA (1971)
Everyday Things cards: 24, 39
Mathematics Involvement Program, SRA (1971)
Cards: 285, 16, 66, 96, 166, 296
Skill through Patterns, level 5, SRA (1974)
Spirit masters: 11, 12, 17, 18, 19, 28, 29, 30, 31, 32, 36, 37, 41, 47, 54, 62, 66, 67, 69, 70, 71, 72

geometry – chapter objectives 12, 13

SRA products

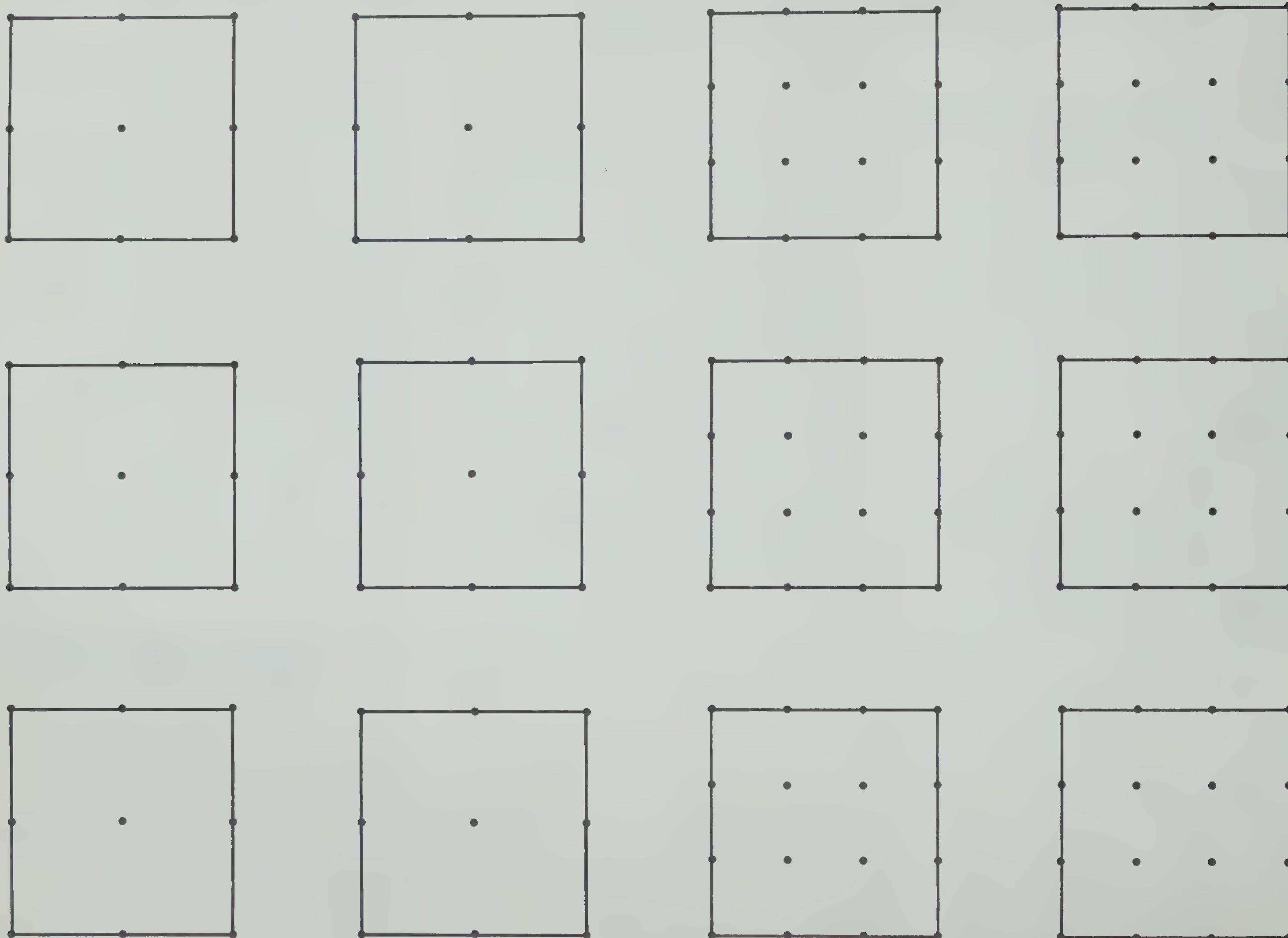
Mathematics Learning System, Activity Masters, level B, SRA (1974)
Spirit masters: W-3; P-2
diagnosis: an instructional aid—Mathematics Level B, SRA (1972)
Probe: M-31
Skill through Patterns, level 5, SRA (1974)
Spirit masters: 33, 34, 35

measurement – chapter objectives 6, 10

Mathematics Learning System, Activity Masters, level B, SRA (1974)
Spirit master: P-13
diagnosis: an instructional aid—Mathematics Level B, SRA (1971)
Probe: M-25
Skill through Patterns, level 5, SRA (1974)
Spirit masters: 10, 13, 43, 44

name ____________________

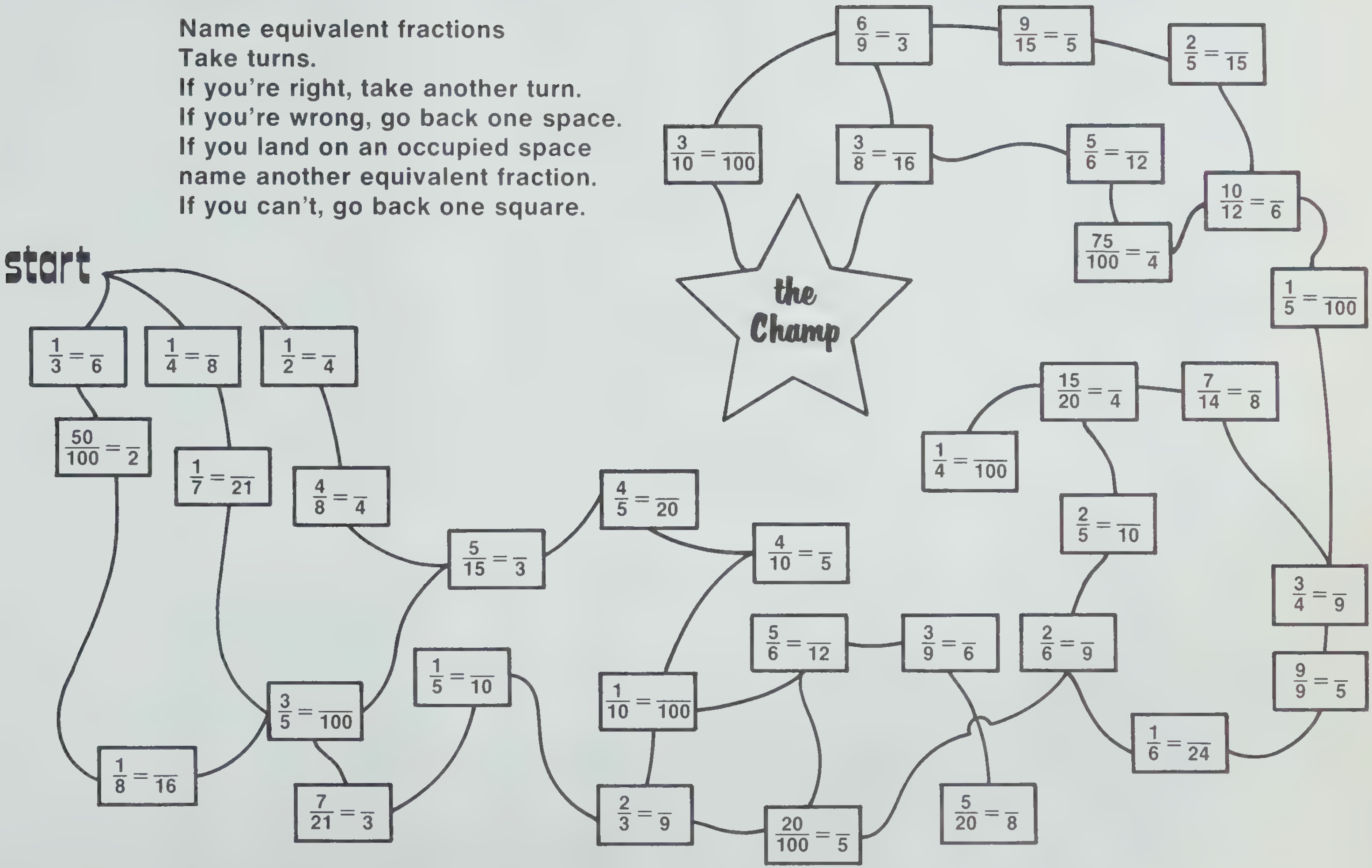
Connect dots to make fractional parts. Try to make each region look different.
Then go back. Label each part.



Pick a partner

pick a path

Name equivalent fractions
Take turns.
If you're right, take another turn.
If you're wrong, go back one space.
If you land on an occupied space
name another equivalent fraction.
If you can't, go back one square.



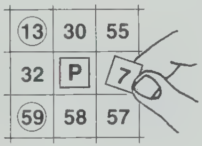
factors and multiples

Pick a partner. Get a game board. Put it in front of you. You'll need ten plain squares, and ten plain triangles. Your partner will need ten shaded squares, and ten shaded triangles.

You take your square marked P. (P means prime.) Play $\boxed{P}$ on the playing board on any prime number.

Your partner plays his $\boxed{P}$ on another prime number.

Your turn again. Play $\boxed{2}$ on one of the eight numbers next to your $\boxed{P}$.



You must play $\boxed{2}$ on a multiple of 2. (See the diagram. 56 is a multiple of 2. This means 2 is a factor of 56.)

Always play on an uncovered number. Never on top of another piece.

It's now your partner's turn. He plays his $\boxed{2}$ on a multiple of 2 next to his $\boxed{P}$.

Your turn. Play $\boxed{3}$ on a multiple of 3 next to your $\boxed{2}$.

Continue taking turns this way. Always play your squares in order: $\boxed{P}$, $\boxed{2}$, $\boxed{3}$, $\boxed{4}$, $\boxed{5}$, $\boxed{6}$, $\boxed{7}$, $\boxed{8}$, $\boxed{9}$, $\boxed{10}$.

Always play next to the last piece you played.

Suppose you run out of squares. Start using your triangles.

Suppose you can't play next to the square you played last. Start using triangles.

Start with $\triangle P$ when starting triangles. Play $\triangle P$ on a prime number. Play next to the last square you played.

Continue playing triangles. Don't go back to squares.

Play triangles just like you did squares: $\triangle P$, $\triangle 1$, $\triangle 2$, and so on. Keep on taking turns.

Suppose you can't play next to the triangle played last. Stop. Your done.

Your partner continues until he must stop too. The game is over then.

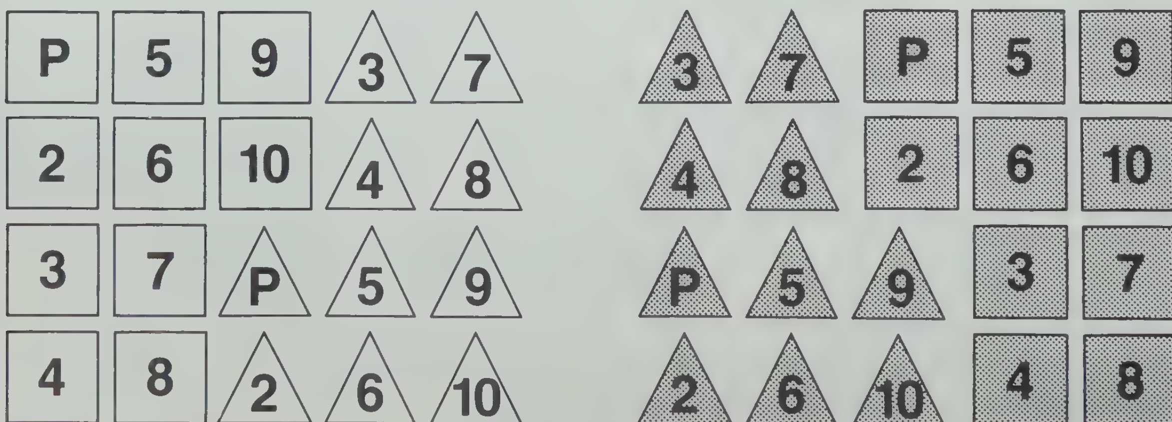
Score 2 points for each square played. Score 1 point for each triangle played.

The winner is the player with more points.

Suppose you become a good player. Don't play single games then. Play 3-game and 5-game matches.

Special rule for matches: You cannot play $\boxed{P}$ or $\triangle P$ on a prime number that has been used before in the match. You must play on another prime.

squares and triangles for factors and multiples

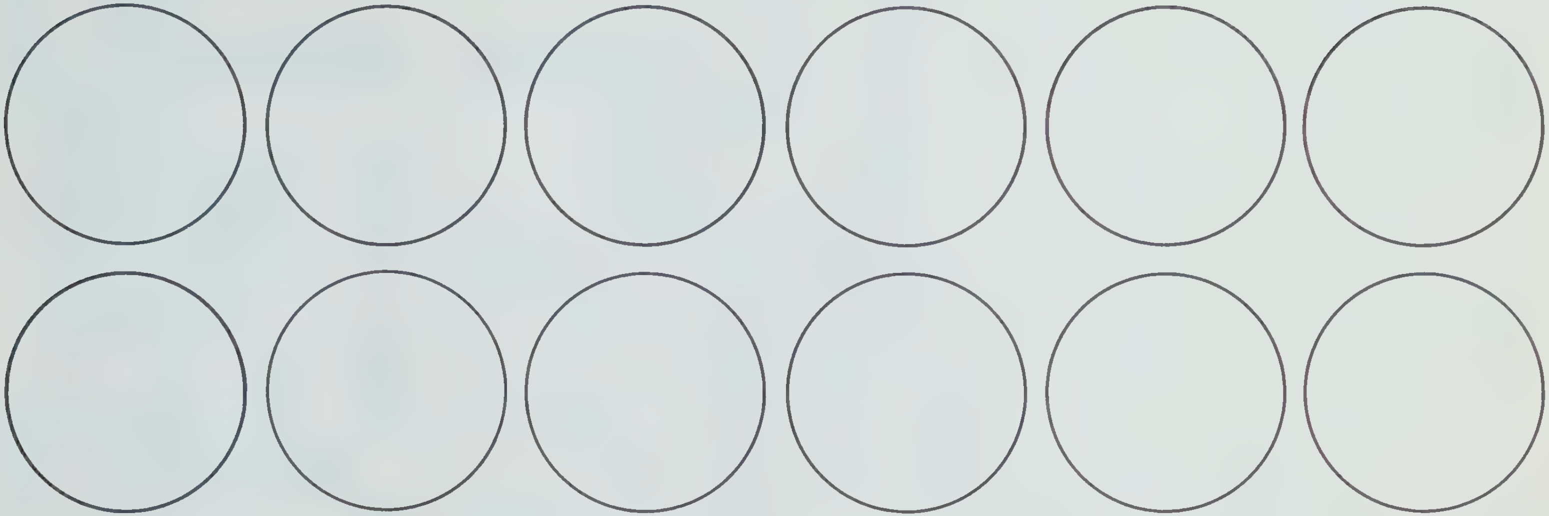


playing board for factors and multiples.

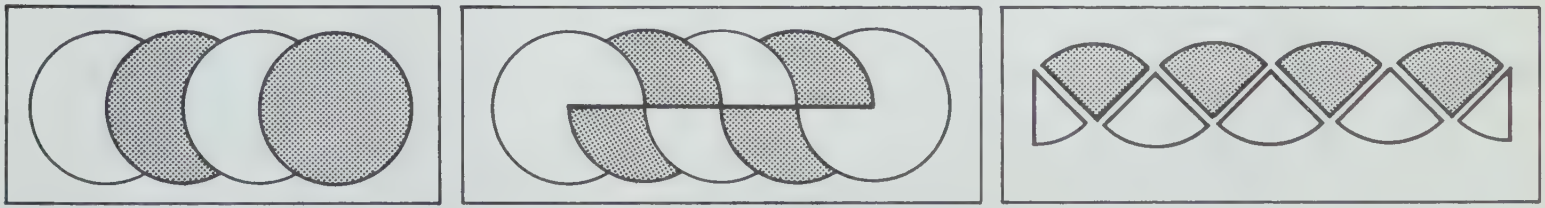
73	74	75	76	77	78	79	80	81	82
72	43	44	45	46	47	48	49	50	83
71	42	21	22	23	24	25	26	51	84
70	41	20	7	8	9	10	27	52	85
69	40	19	6		2	11	28	53	86
68	39	18	5	4	3	12	29	54	87
67	38	17	16	15	14	13	30	55	88
66	37	36	35	34	33	32	31	56	89
65	64	63	62	61	60	59	58	57	90
100	99	98	97	96	95	94	93	92	91

336f

Get 2 pieces of different color paper. Paste 1 strip of circles on one color. Paste the other strip on the second color. Cut out the circles.



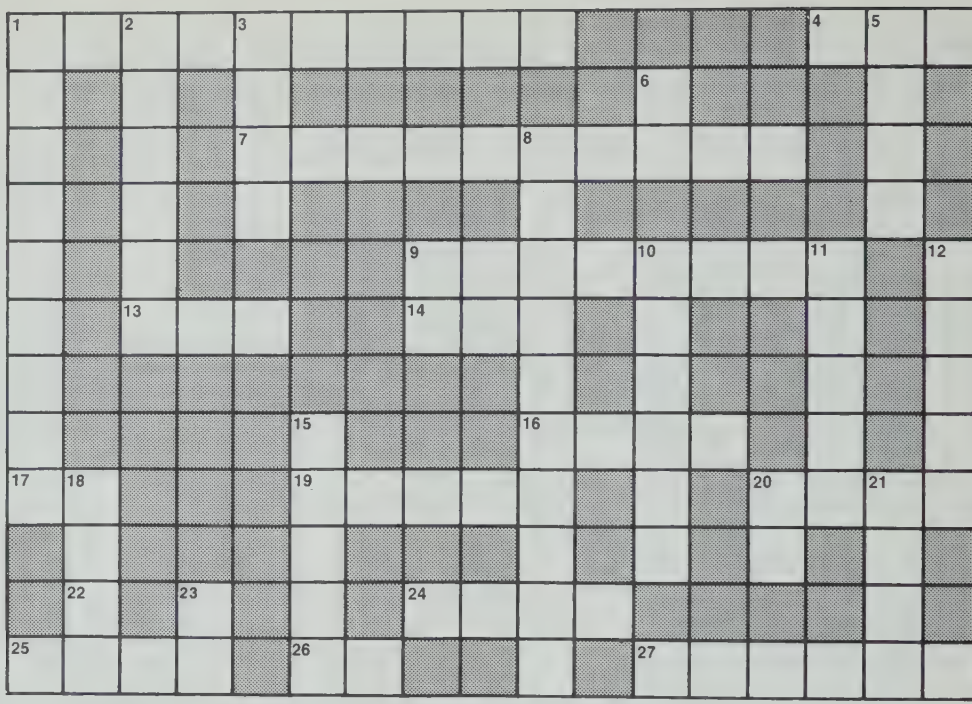
Use the color side of the circles. Make designs like those below. (The shaded part will show you the second color.) You will have to fold and cut and paste. Don't get fooled.



Can you think of other designs? Cut more circles. Try it.

name ____________________

Think units of measure.
Do the puzzle.
Can you do a better one?
Try it.
Share it.



ACROSS

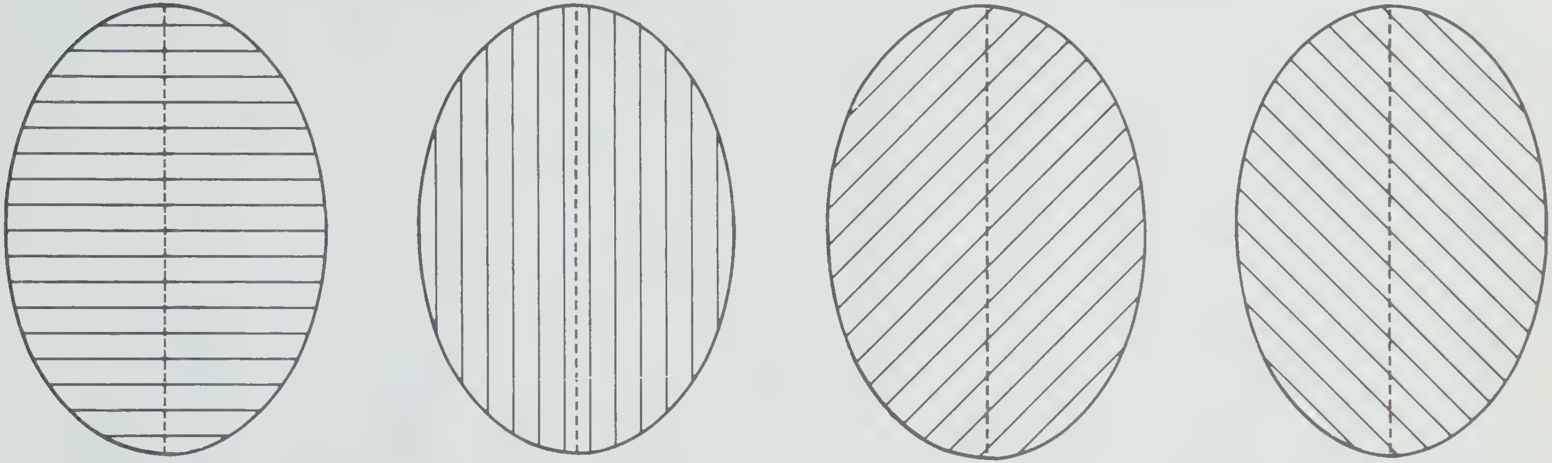
1 1000 of these in a litre
4 abbreviation for a unit of liquid measure
7 more than 2 of these in an inch
9 1000 g
13 abbreviation for a unit of time
14 symbol for a metric unit of length
16 measure for distance between cities
17 symbol for a short metric unit of length
19 unit of weight
20 unit of money
22 symbol for a metric unit of capacity
24 length for a measuring stick
25 3 make a yard
26 symbol for a metric unit of length
27 unit of time

DOWN

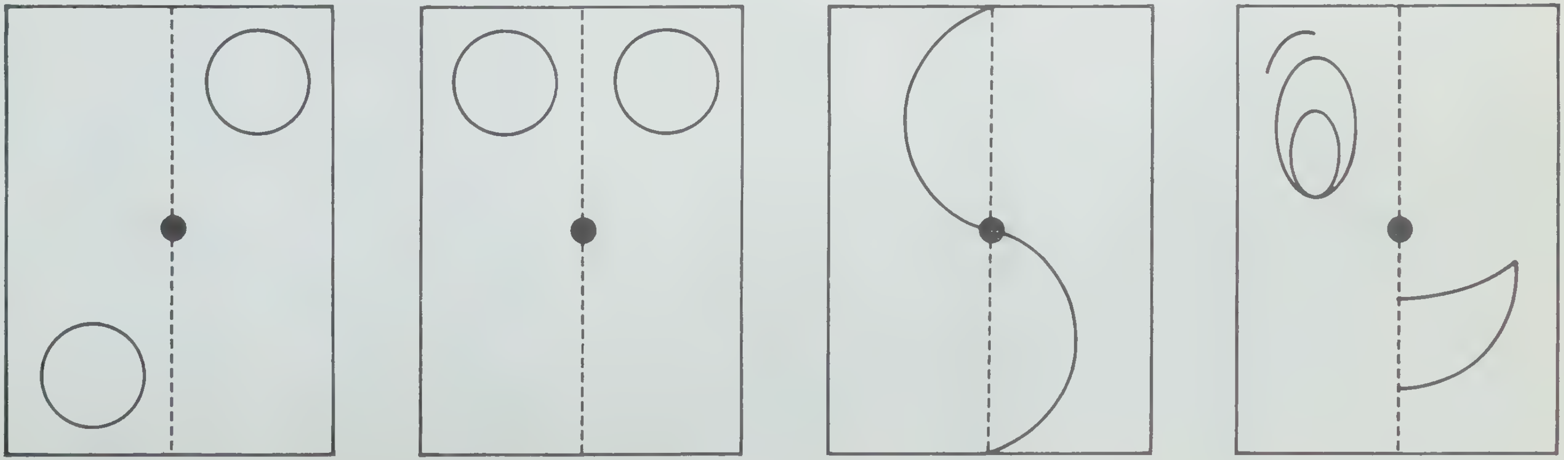
1 1000 of these in a gram
2 unit of metric capacity
3 unit of measure on a ruler
5 land measure
6 abbreviation for a unit of length
8 unit of metric length
10 you can get milk in this size container
11 length for metric measuring stick
12 milk comes in this size, too
15 this much candy would be good to have now
18 unit of length already used in this puzzle
20 symbol for a metric length
21 word that describes the metric system
23 abbreviation for a liquid unit of measure

Find a mirror.

Put it on a dotted line. Look carefully. Do you see a symmetrical figure?
Is it the same as the figure printed on the sheet?
Look at the image on the other side of the dotted line. Has anything changed?



Try rotating the mirror around on the dot. Can you find symmetical shapes now?



my name____________________ my teacher's name________________

	I can do it everytime.	I will be able to do it soon.
I can tell the value of these roman numerals: XIX XLIII CDX CDXL	______	______
I can tell the value of each digit in a number as great as 357 062 941.	______	______
I can round the numbers in problems like these and estimate the answers: a) $342 + 479$ b) $521 - 219$ c) 48×4 d) 77×24	______	______
e) 6)359 f) 7)4379 g) 42)5629 h) 59)1711	______	______
I can estimate the answers and find the exact answers to problems like these: a) 327×20 b) 506×35 c) 847×57 d) 425×42	______	______
e) 50)2739 f) 35)385 g) 24)3625 h) 19)3990	______	______
I can tell you the answers to problems like these without using pencil and paper: a) 40×6 b) 80×3 c) 700×4 d) 200×7	______	______
I can check whether 36 is the correct answer to 23)828 by using multiplication.	______	______
I can find answers to word problems such as this: Mark made and sold small trays. He needed 37 tiles for each tray. There were 185 tiles all together. How many trays could Mark make?	______	______
And I can write a math sentence to show that my answer to a word problem is correct.	______	______
I can rename pairs of fractions like these so that they have the same denominator: a) $\frac{3}{5}$ $\frac{2}{10}$ b) $\frac{3}{4}$ $\frac{5}{8}$ c) $\frac{1}{3}$ $\frac{3}{4}$ d) $\frac{1}{4}$ $\frac{5}{6}$	______	______
I can tell which of these fractions are written in simplest form and can rename those that are not in simplest form. a) $\frac{5}{7}$ b) $\frac{4}{6}$ c) $\frac{3}{9}$ d) $\frac{2}{3}$	______	______
I can rename fractions like these as whole or mixed numbers: a) $\frac{15}{3}$ b) $\frac{27}{4}$ c) $\frac{7}{6}$ d) $\frac{18}{8}$	______	______
I can also rename a whole or mixed number as a fraction.	______	______

I can find answers to problems like these:

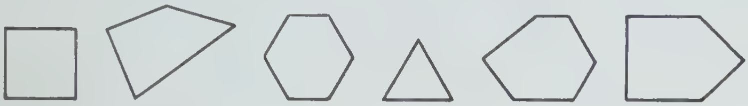
a) $\frac{2}{5}+\frac{3}{4}$ b) $\frac{2}{3}+\frac{3}{9}$ c) $1\frac{3}{5}+2\frac{5}{6}$ d) $4\frac{3}{4}+1\frac{1}{8}$ . . . . ________ ________

e) $\frac{5}{7}-\frac{2}{5}$ f) $\frac{3}{4}-\frac{1}{8}$ g) $4\frac{1}{9}-2\frac{1}{3}$ h) $7 \quad 2\frac{1}{5}$ . . . . ________ ________

i) $\begin{array}{r} 14.8 \\ +\ 7.6 \\ \hline \end{array}$ j) $\begin{array}{r} 35.25 \\ +\ 8.46 \\ \hline \end{array}$ k) $\begin{array}{r} 37.6 \\ -\ 4.9 \\ \hline \end{array}$ l) $\begin{array}{r} 4.06 \\ -2.48 \\ \hline \end{array}$ . . . . ________ ________

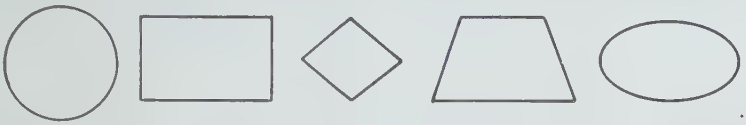
m) $\frac{1}{2}\times\frac{1}{2}$ n) $\frac{2}{5}\times\frac{1}{3}$ o) $\frac{3}{4}\times\frac{1}{6}$ p) $\frac{2}{3}\times\frac{1}{2}$ . . . . ________ ________

I can sort shapes like these by the number of sides:



. . ________ ________

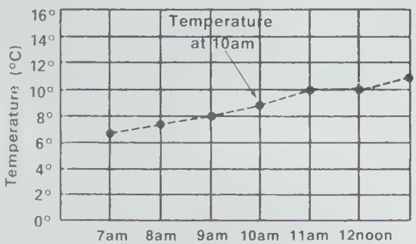
I can find the line(s) of symmetry of figures like these by folding:



. . ________ ________

I can choose a unit of measure for each
of the following:
a) Length of a trailer
b) Mass of an airmail letter
c) Width of a table
d) Contents of a bottle of shampoo
e) Distance from Edmonton, Alberta . . . . ________ ________

I can tell what information is shown in this chart:



. . . . ________ ________

I can find the average of the heights of five of my friends
and myself. . . . . ________ ________

my name ______________________________

date ______________________________

individual progress chart for ______________________
(pupil's name)

recorded by ______________________ — fifth-level teacher

evaluation of major learning goals	date of 1st try	date of 2d try	comments
Can rename whole numbers (p. 13)			
Knows how to subtract with renaming (p. 15)			
Knows how to add with renaming (p. 17)			
Knows how to estimate sums and differences (p. 20)			
Can write the standard numeral for a roman numeral and a roman numeral for a standard numeral (p. 22*)			
Can tell the value of any digit of a 7-digit numeral (p. 22*)			
Can add or subtract two 4-digit numbers with renaming (p. 22*)			
Knows how to write the standard numeral for numerals written as words (p. 20, 22*)			
Knows how to estimate and find the product for a 3-digit number multiplied by a 1- or 2-digit number (p. 34, 38, 52*)			
Knows how to estimate and find the quotient for a 3- or 4-digit number divided by a 1-digit number (p. 46, 52*)			
Knows how to estimate and find the quotient for a 3- or 4-digit number divided by a multiple of 10 (p. 52*)			
Can sort polygons by the number of sides (p. 72*)			
Can write the fraction associated with a fraction model (p. 80)			
Can compare two fractions or mixed numbers with like numerators or denominators (p. 85, 98*)			
Can add and subtract fractions with like denominators (p. 95, 98*)			
Can rename fractions (p. 89, 95, 98*)			
Can divide a multiple of 10 by a multiple of 10 (p. 104)			
Can round numbers to the nearest ten or hundred (p. 122*)			
Can estimate products and quotients (p. 122*)			
Can divide a 3- or 4-digit number by a 2-digit number (p. 113, 117, 122*)			
Can identify true, false, and open math sentences (p. 134)			
Can identify and write true math sentences (p. 144*)			
Can solve a word problem and write a math sentence to show that the answer is correct (p. 144*)			
Can subtract mixed numbers with unlike denominators (p. 155, 157)			
Can add fractions with unlike denominators (p. 162)			
Knows how to multiply fractions (p. 168, 172)			
Knows how to add and subtract mixed numbers with unlike denominators and rename the answers in simplest form (p. 175*)			
Can rename a decimal as a fraction and vice versa (p. 182)			
Can compare and order decimals (p. 189*)			
Knows how to add and subtract decimals (p. 186, 189*)			
Can select an appropriate unit of measure (p. 216*)			
Knows how to add and subtract two measurements (p. 215)			
Can rename fractions and mixed numbers in simplest form (p. 239*)			
Knows how to add or subtract fractions and mixed numbers with unlike denominators and rename the answer in simplest form (p. 229, 239*)			
Knows how to multiply fractions and mixed numbers (p. 235, 239*)			
Can locate information on a map (p. 268*)			
Can find the mean for a set of numbers (p. 268*)			
Can find a line(s) of symmetry (p. 288*)			

*Checkout

Other Learning Aids

whole-number notation

Abacus board (Creative Publications) Counting board useful for teaching place value
Chip Trading (Scott Scientific) Game to develop an understanding of place value
Place Value I and II (Creative Publications) Self-correcting cards to provide practice in reading numbers through hundred millions

whole-number operations

Dial-A-Matic Adding Machine (Sigma Scientific) A simple calculator for practice in addition and subtraction
Dividing Machine (Developmental Learning Materials) Self-checking machine to be used for practice with the division facts
Good Time Mathematics (Holt, Rinehart & Winston) A multimedia program to give activity-based learning experiences
I Win (Scott, Foresman) [sets 1, 2, and 3] Card game for practice in basic operations
Japanese Abacus (Creative Publications) An abacus for place value and basic operations
Mathfacts Games (Milton Bradley) Self-checking games for multiplication and division facts
Napier's Rods (Sigma Scientific) Rods for practice in multiplication
Numble (Sigma Scientific) Crossword-type number game for basic operations
Orbiting the Earth (Scott, Foresman) Game to provide practice in all four operations
Prime-O (Creative Publications) Card game to provide practice in prime factorization
Rally with Remainders (Math Shop) A self-correcting game providing division practice
Sequence (Math Shop) Puzzle-type game that uses addition combinations to find patterns
Ting (SEE) A jigsaw puzzle for multiplication
Triscore (Creative Publications) Games for practice in the basic operations
Veri-Tech Senior (ETA) [addition, subtraction, and multiplication books] A self-checking device that provides practice with operations
Winning Touch (Ideal) Game for reinforcement of multiplication facts

fractional-number notation

Decimal Fraction Dominoes (Mind/Matter Corp.) Game for practice in recognizing relationships of fractions and decimals
Decimal/Fraction Matching Cards (SEE) Cards to aid in learning about decimals and fractions
Experiments in Fractions (Math Shop) Activities for notation and operations
The Fat Fraction Game (ETA) Card game for drill in simplifying fractions
Fraction Bars Student Activity Book (Creative Publications) Games and activities to teach fractions and their operations
Fraction Dominoes (SEE) Game involving matching a fractional numeral with its model
Fraction Line Set (Sigma Scientific) Activity to help visualize operations by computing with fraction strips
Fraction Tally (Math Shop) Game for practice in addition and subtraction
Fractional Number Cards (Math Shop) Cards used with a geoboard for finding equivalent fractions
Tripletts (Math Shop) A rummy-type card game for identifying equivalent fractions

fractional-number operations

Action Fraction Games (Constructive Playthings) Game to develop concepts and skills
Fraction Multifax (Math Shop) Game to reinforce multiplication of fractional numbers
Mathimagination (Math Shop) [book D—Fractions] Puzzles to reinforce operations

geometry

Geoboard Activity Cards (Creative Publications) [intermediate set] Activities for the geoboard
Geoboard Kit (Cuisenaire) Plastic geoboards and activity cards that show geometric concepts
Geoboards and Motion Geometry (Scott, Foresman) Resource book dealing with congruence, coordinates, transformations, and area
Great Shapes (Cuisenaire) Game to develop principles of patterns, symmetry, and so on
Learn to Fold—Fold to Learn (Lyons & Carnahan) Workbook of paper-folding activities to demonstrate geometric figures and symmetry
Mira (Creative Publications) An aid for investigating properties of plane geometry
Mira Math for Elementary School (Creative Publications) Activities for the Mira
Mirror Magic (Lyons & Carnahan) Workbook activities for exploring the concept of symmetry
Paper and Pencil Geometry (Lyons & Carnahan) Activities to develop geometric concepts
Polygons (Math Shop) Cards for understanding of basic properties of geometric figures
Rotation, Translation and Reflection Kit (Invicta) Activity cards dealing with motion geometry
Shape Tracers (Math Shop) Set of basic geometric shapes
Tangrams (Creative Publications) Puzzle to aid in the discovery of geometric properties
Tangramath (Creative Publications) Book of tangram shapes to assist in learning concepts of shapes, congruence, similarity, and area

measurement

The Fatal Foot (Math Shop) Game for drill in addition of linear measures
Geometric Ruler (Math Shop) Folding rule to demonstrate perimeter-area relationships
The Ghastly Gallon (Math Shop) Game for practice in operations with liquid measures
Introducing the Metric System with Activities (Math Shop) Activities to develop basic understanding of the metric system
Learning about Measurement (Lyons & Carnahan) A workbook of activities using the metric and customary systems
Metric Place Value Chart (Ideal) Chart for meaning of metric system of measures
The Perilous Pound (Math Shop) Game for practice in regrouping weight measures

statistics and probability

Block Graph (ESA) Demonstration set for introducing bar graphs, averages, and so on
Histogram Board (ESA) Board for making bar diagrams—to be used with Stern Unit Cubes
Making and Using Graphs and Nomographs (Lyons & Carnahan) A workbook for skills in making and reading graphs
Probability Maze (ESA) A board to illustrate probability and statistics
Probability Set (Invicta) Apparatus to demonstrate simple probability
Towards Probability (Cuisenaire) Book of easy experiments dealing with probability

problem solving and applications

Foo (Cuisenaire) Card game for order of operation in open math sentences
Heads Up (Creative Publications) Game providing practice with equations
True or False (ESA) Game for deciding true or false statements

Bibliography

the study of numbers

whole-number concepts

children's books

Adler, Irving and Ruth. *Numbers Old and New.* New York: John Day, 1960. (4–5)
———. *Numerals: New Dresses for Old Numbers.* New York: John Day, 1964 (4–5)
Alexander, Arthur. *The Magic of Words.* Englewood Cliffs, N. J.: Prentice-Hall, 1962. (5)
Bendick, Jeanne, and Levin, Marcia. *Take a Number.* New York: McGraw-Hill, 1961. (4–6)
Carona, Philip B. *True Book of Numbers.* Chicago: Childrens Press, 1964. (4)
Lauber, Patricia. *The Story of Numbers.* New York: Random House, 1961. (5)
Lerch, Harold H. *Numbers in the Land of Hand.* Carbondale, Ill.: Southern Illinois Univ. Press, 1966. (4–5)
Luce, Marnie. *Counting Systems: The Familiar and the Unusual.* Minneapolis, Minn.: Lerner, 1969. (5)
Selfridge, Oliver G. *Fingers Come In Fives.* Boston: Houghton Mifflin, 1966. (4–5)
Simon, Leonard, and Bendick, Jeanne. *The Day The Number Disappeared.* New York: McGraw-Hill, 1963 (4–5)
Waller, Leslie. *Numbers: A Book to Begin On.* New York: Holt, Rinehart & Winston, 1960. (4–5)

films, filmstrips* and slides

**Computer Series 1: An Introduction to Computers:* "History of Computing Devices." Color w-cassettes. BFA Educ. Media. (4–6)
Macmillan Math Film Loops: "Exponents," "General Order Relations," "Place Value." Super-8mm cartridges. Macmillan. (4–6)
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**Using Modern Mathematics,* Group 5: "Numeration: Base Five." Color w/captions. Singer/SVE. (6)

whole-number operations

children's books

Grant, Eldon. *Twenty White Horses, A Book of Division.* New York: Holt, Rinehart & Winston, 1964. (4–5)
Jonas, Arthur. *More New Ways in Math.* Englewood Cliffs, N. J.: Prentice-Hall, 1964. (5)
Pink, Heinz-Guenther. *Multiplication Hula.* Honolulu: Institute of Simplified Mathematics, 1972. (5)
Whitney, David C. *The Easy Book of Division.* New York: Watts, 1970. (4–6)
———. *The Easy Book of Multiplication.* New York: Watts, 1969. (4–6)
———. *Let's Find Out About Subtraction.* New York: Watts, 1968. (4)

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fractional-number concepts

children's books

Dennis, J. Richard. *Fractions Are Parts of Things.* New York: Thomas Y. Crowell, 1971. (5)
Whitney, David C. *The Easy Book of Fractions.* New York: Watts, 1970. (4–6)

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Elementary Mathematics for Students: "Between the Whole Numbers," "Equivalent Fractions," "Comparing Rational Numbers." 16mm, sound, color. Developed by NCTM, distributed by Silver Burdett. (4–6)
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fractional-number operations

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**Using Modern Mathematics,* Group 5: "Addition and Subtraction of Fractions," "Multiplication of Fractions." Color w/captions. Singer/SVE (4–6)

geometry

children's books

Adler, Ruth and Irving. *Directions and Angles.* New York: John Day, 1969. (4–6)

Bendick, Jeanne, and Levin, Marcia. *Take Shapes, Lines and Letters.* New York: McGraw-Hill, 1962. (4–5)

Diggins, Julia E. *String, Straightedge and Shadow:* The Store of Geometry, New York: Viking, 1965. (4–6)

Hogben, Lancelot. *The Wonderful World of Mathematics,* rev. ed. New York: Doubleday, 1968. (4–6)

Juster, Norton. *The Dot and the Line.* New York: Random House, 1963. (4–6)

Kettelkamp, Larry. *Kites.* New York: William Morrow, 1959. (4–5)

Luce, Marnie. *Points, Lines, and Planes.* Minneapolis, Minn.: Lerner, 1969. (5–6)

Murray, William D., and Rigney, Francis J. *Paper Folding for Beginners.* New York: Dover, 1960. (5)

Ravielli, Anthony. *An Adventure in Geometry.* New York: Viking, 1957. (4–6)

Razzell, Arthur G., and Watts, K. G. *Circles and Curves.* New York: Doubleday, 1969. (4–6)

———. *Symmetry.* New York: Doubleday, 1968. (5–6)

Russell, Solveig Paulson. *Lines and Shapes.* New York: Walck, 1965. (4–5)

Sitomer, Mindel and Harry. *Lines, Segments, Polygons.* New York: Thomas Y. Crowell, 1971. (4)

———. *What Is Symmetry?* New York: Thomas Y. Crowell, 1970. (4–5)

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Harbrace Mathematics Instructional Slides: "Geometry, Measurement, and Graphing." Cartridge of 140 slides, color w/captions. Harcourt Brace Jovanovich. (1–6)

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**Using Modern Mathematics,* Group 5: "Geometry: Perimeters, Areas, Space Figures." Color w/captions. Singer/SVE. (6)

measurement

children's books

Adler, Irving. *The Giant Golden Book of Mathematics.* New York: Golden Press, 1960. (4–6)

Asimov, Isaac. *Realm of Measure.* Boston: Houghton Mifflin, 1960. (4–5)

Bendick, Jeanne. *How Much and How Many: The Story of Weights and Measures.* New York: McGraw-Hill, 1947. (4–6)

———. *Measuring.* New York: Watts, 1971. (4–6)

Branley, Franklyn M. *Think Metric!* New York: Thomas Y. Crowell, 1972. (4–6)

Carona, Philip B. *Things That Measure.* Englewood Cliffs, N.J.: Prentice-Hall, 1962. (4–5)

Epstein, Sam and Beryl. *The First Book of Measurement.* New York: Watts, 1960. (5–6)

Friskey, Margaret, ed. *About Measurement.* Chicago: Melmont, 1965. (5)

Kadesch, Robert R. *Math Menagerie.* New York: Harper & Row, 1970. (6)

Lieberg, Owen S. *Wonders of Measurement.* New York: Dodd, Mead, 1972. (4–6)

Luce, Marnie. *Measurement: How Much? How Many? How Far?* Minneapolis, Minn.: Lerner, 1969. (5–6)

Myller, Rolf. *How Big Is a Foot?* New York: Atheneum, 1962. (4)

Page, Chester H., and Vigoureux, Paul, eds. *The International System of Units (SI).* (National Bureau of Standards Special Publication 330.) Washington: U.S. Government Printing Office, 1972. (4–6)

Russell, Solveig Paulson. *Size, Distance, Weight: A First Look at Measuring.* New York: Walck, 1968. (4–5)

Simon, Leonard. *Stretching Numbers.* New York: Holt, Rinehart & Winston, 1964. (4)

Weyl, Peter K. *Men, Ants, and Elephants: Size in the Animal World.* New York: Viking, 1959. (4–6)

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**Using Modern Mathematics,* Group 5: "Using Measures." Color w/captions. Singer/SVE (4–6)

statistics and probability

children's books

Linn, Charles F. *Probability.* New York: Thomas Y. Crowell, 1972. (4–6)

Lowenstein, Dyno. *First Book of Graphs.* New York: Watts, 1969. (5–6)

Razzell, Arthur G., and Watts, K. G. *Probability—The Science of Chance.* New York: Doubleday, 1967. (5–6)

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Predicting Through Sampling. 16 mm, sound, color. BFA Educ. Media. (4–6)

The Probabilities of Zero and One. 16mm, sound, color. BFA Educ. Media. (4–6)

Probability: An Introduction. 16mm, sound, color. BFA Educ. Media. (4–6)

**Using Modern Mathematics,* Group 5: "Graphs: Pictographs, Bar, Line, Number Pairs, Maps." Color w/captions. Singer/SVE. (4–6)

problem solving

children's books

Barr, George. *Entertaining with Number Tricks.* New York: McGraw-Hill, 1971. (4–6)

Brooke, Maxey. *One Hundred and Fifty Puzzles in Crypt-Arithmetic.* New York: Dover, 1972. (5–6)

Charosh, Mannis. *Mathematical Games for One or Two.* New York: Thomas Y. Crowell, 1972. (4)

Dudeney, Henry E. *Five Hundred Thirty-Six Puzzles and Curious Problems.* New York: Scribner, 1967. (5–6)

Feravola, Rocco. *The Wonders of Mathematics.* New York: Dodd, Mead, 1963. (4–6)

Gardner, Martin. *Perplexing Puzzles and Tantalizing Teasers.* New York: Simon & Schuster, 1969. (6)

Jacobs, Allan D. and Leland B., eds. *Arithmetic in Verse and Rhyme.* Champaign, Ill.: Garrard, 1971. (4–5)

Jonas, Arthur. *New Ways in Math.* Englewood Cliffs, N. J.: Prentice-Hall, 1962. (4)

Kettelkamp, Larry. *Puzzle Patterns.* New York: Morrow, 1963. (6)

Kohn, Bernice. *Secret Codes and Ciphers.* Englewood Cliffs, N. J.: Prentice-Hall, 1968. (6)

Linn, Charles F. *Estimation.* New York: Thomas Y. Crowell, 1972. (4–6)

GLOSSARY

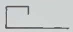
arithmetic mean An average

Add the numbers. $\begin{array}{r} 10 \\ 3 \\ +\ 5 \\ \hline 18 \end{array}$ Divide by number of numbers. $3\overline{)18}$ with quotient 6. 6 is the mean of 10, 3, and 5.

composite number A number with a pair of factors other than 1 and the number

$6 = 3 \times 2$
6 is composite.

congruent Two figures are congruent if they are the same size and shape

 is congruent to 

curve A figure drawn without lifting the pencil

curve closed curve simple closed curve

data Numbers collected from events by counting or observing

decimal Another name for some fractions

$\frac{23}{100} = .23$ $1.3 = 1\frac{3}{10}$

decimal point decimals

division

$$\begin{array}{r} 22 \text{ R2} \\ 13\overline{)288} \\ \underline{260} \\ 28 \\ \underline{26} \\ 2 \end{array}$$

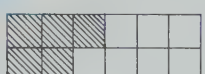
quotient → 22 (2 + 20), R2 ← remainder

divisor → 13, 288 ← dividend

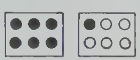
260 (20 × 13)

26 (2 × 13)

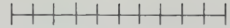
fraction A number that tells how much



$\frac{5}{12}$



$1\frac{1}{6}$



0 .7 1

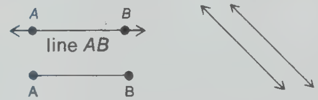
$\frac{5}{7}$ ← numerator, ← denominator

$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$ equivalent fractions

$\frac{12}{18} = \frac{12 \div 6}{18 \div 6} = \frac{2}{3}$ simplest name

$\frac{1}{5} = \frac{2}{10}$
$\frac{7}{10} = \frac{7}{10}$ common denominator

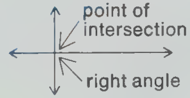
line



line *AB*

line segment *AB* (part of a line)

parallel lines (never meet)



point of intersection

right angle

perpendicular lines (form square corners)

math sentence

true: $3 + 6 < 10$
open: $6 + \square = 10$
$6 + 5 = 10$

replacement for $\square$

false: $6 - 3 = 4$
open: $6 + \square = 10$
true: $6 + 4 = 10$

solution

mixed number Another name for some fractions

$2.4 = 2\frac{4}{10}$ mixed numbers $3\frac{1}{3} = \frac{10}{3}$

multiplication

$\frac{2}{3} \times \frac{1}{5} = \frac{2}{15}$

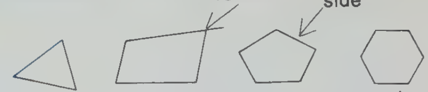
factors product

$3 \times 7 = 21$
$4 \times 7 = 28$
21 and 28 are multiples of 7.

operation Using a pair of numbers to get another number

addition + (plus)	multiplication × (times)
subtraction − (minus)	division ÷ (divided by)

polygon A simple closed curve made with line segments



vertex side

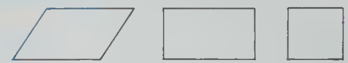
triangle quadrilateral pentagon regular hexagon

All sides and angles are congruent in a regular polygon.

prime number A whole number greater than 1 whose only pair of factors is 1 and itself. 2, 3, 5, 7, 11 are some prime numbers.

probability A way of finding how likely something is to happen

quadrilateral A four-sided polygon



parallelogram rectangle square

ray A part of a line that starts at a point and continues without end in one direction

endpoint  ray *PQ*

P *Q*

relation symbols $=, \neq, >, <$

$7 > 3$
7 is greater than 3

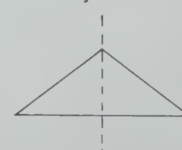
$2 + 4 = 6$
2 + 4 equals 6

$6 < 8$
6 is less than 8

$4 \neq 6$
4 is not equal to 6

statistics Computing with data, such as finding the arithmetic mean for a set of numbers

symmetry When a plane figure can be folded on a line to match exactly



line of symmetry

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